# Micro Lens Arrays

## 1 Introduction

Micro lens arrays (MLAs) are a standard tool in wavefront measurements. When a perfect beam (plane wavefront) is sent onto a surface of interest, and the reflection (or transmission) is projected onto a CCD array with a micro lens array, imperfections of the surface can be retrieved from the phase front of the beam via the deviation of each focal spot from the ideal condition. This lab is designed to familiarize you with micro lens arrays, and its use in diagnostics of phase front distortions.

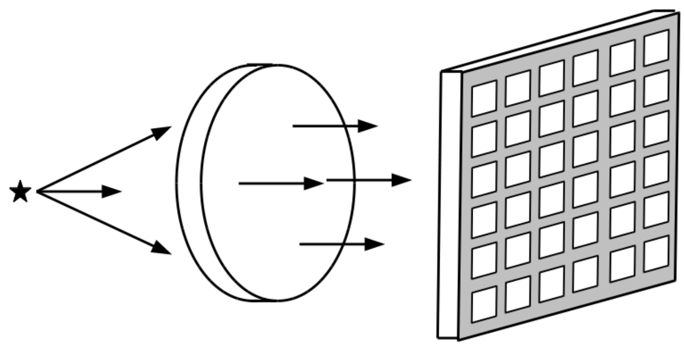

Figure 1: Three dimensional sketch of a plane wave illuminating micro lens array.

## 2 Theory

For the most part, MLAs are imaging systems doing very much the same job as other imaging systems do with a few peculiarities of their own. When a collimated beam illuminates a periodic structure, such as a diffraction grating, the object will image itself periodically as well. This self-imaging phenomenon was first observed by a British scientist named H. F. Talbot in 1836 [1], and since then has been the subject of extensive study [2, 3, 4]. In the following two sections, two approaches to understanding how they work are presented.

## 3 A grating and the Talbot effect

Consider the transmission grating illuminated at normal incidence by a plane wave. The zero order diffraction travels straight. The diffracted rays, of order $|n| > 0$, are emitted in the direction $\theta$ given by the relation:

$$n\lambda = a \sin\theta, \tag{1}$$

where $\lambda$ is the wavelength, $a$ the period of the grating. If $a \gg \lambda$, the *paraxial* approximation applies, i.e. $\sin\theta \approx \theta \approx \theta$. It can easily be seen from Fig. 2 that all the rays issues from each facet of the grating are crossing with the same angles on a plane located at a distance $a/\theta$. The Talbot distance is defined as $\ell = 2a^2/\lambda$. It is clear that the original grating is imaged (real image) on all planes situated at multiple of the half Talbot distances $\ell/2$, $2\ell/2$, $3\ell/2$, ... You will notice also the patterns at quarter Talbot distances: an image of the grating shifted by 1/2 of the grating period.

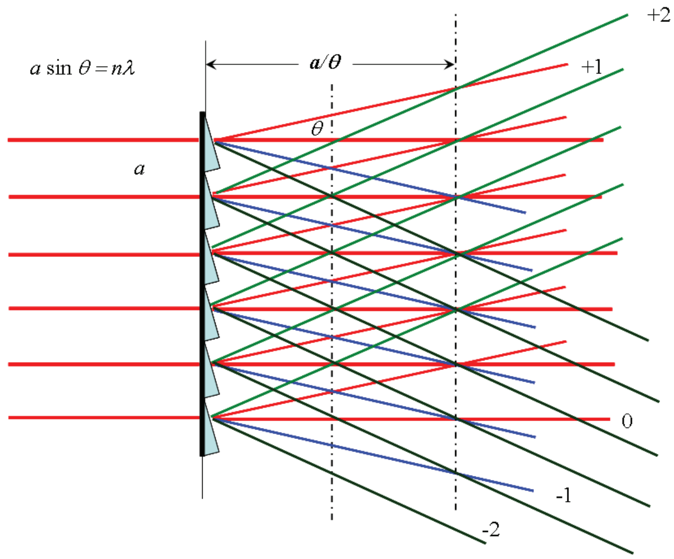


Figure 2: Ray tracing for a transmission grating.

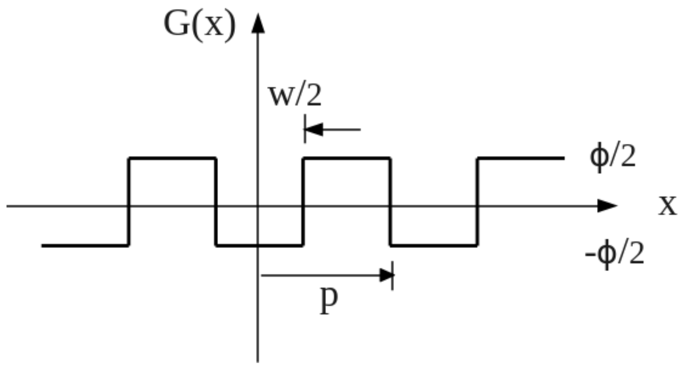


Figure 3: Functional representation, along the $x$ direction, of a phase grating of groove width $w$ and period $p$ with modulation depth $\Phi$.

The groove of the grating can be replaced by Fresnel lenses. No assumption in the sketch of Fig. 2 is made on the shape of the “groove”. Now we have two characteristic planes to be repeated periodically: the plane of the focal points of the lenses, and the Talbot planes. It is thus possible with a single dimensional array of lenses to make a three dimensional crystal of dots.

# 4 Wave Description

In the wave optics picture, mathematical details of the imaging mechanism emerge, as a consequence of the Fourier treatment of the system. Figure 3 shows a representation of the phase grating function $G(x)$ of groove width $w$ and grating period $p$ along the $x$ dimension. For simplicity, here we consider again the uni-dimensional case. The functional form of a phase grating can be easily described as a sum of phase shifts and given by the Fourier series [5]:

$$G(x) = \sum_m G_m e^{i2\pi mx/p}, \tag{2}$$

where $G_m$ represent the Fourier coefficients of the function, and $m$ an integer. In the Fresnel region, for a grating illuminated by a plane wave, the complex distribution of the propagating field $\mathcal{E}(x, z)$ becomes, in a paraxial approximation:

$$\mathcal{E}(x, z) \approx e^{ikz} \sum_m \{G_m e^{-i2\pi m^2 \frac{z}{z_T}}\} e^{i2\pi mx/p}, \tag{3}$$

where again $z_T = 2p^2/\lambda$ is the Talbot length. At each distance $z$, the propagated wave is represented by a different Fourier series with coefficients $G_m \exp -i2\pi m^2 z/z_T$. At $z = z_T$ (one Talbot length), the exponent is a multiple of $2\pi$ and the coefficient is reduced to $G_m$, i.e the image is replicated. At $z = z_T/2$, the coefficient becomes $G_m \exp -i\pi m^2$, which corresponds to an image of a grating with a lateral shift of $p$ (show that this is true, and what to expect at $z = z_T/4$).

Figure 4 shows the one-dimensional configuration of a 2-D phase grating array illuminated by a plane wave. At sub-multiples of the Talbot length, different periodical structures are observed. The composite of all the images formed at the sub-multiple planes forms a mosaic which is sometimes referred to as the ”Talbot carpet”.

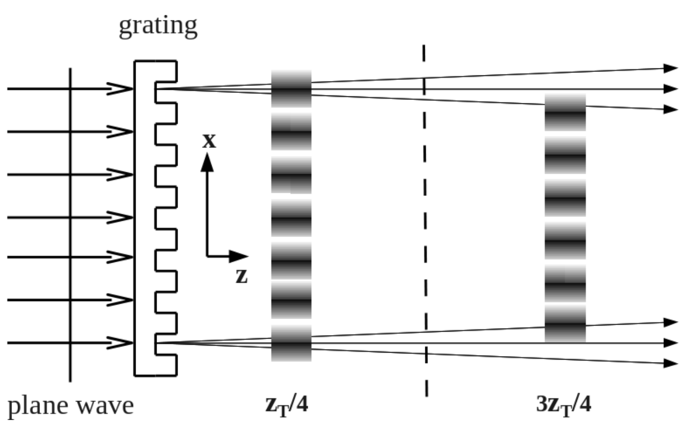


Figure 4: Two dimensional representation of a grating array illuminator in the Fresnel region, at sub-multiples distances to the Talbot length.

# 5 Experiment

## 5.1 Talbot Length

Measure the Talbot length for the lens array. Make sure that the beam is first cleaned by the spatial filter. Also to observe the fractional Talbot effect, beam hight alignment is critical. From this measurement, extract the grating period, knowing the He-Ne laser wavelength. Compare this with the actual image and the actual value. Find the focal distance of the lenses

**Food for thoughts**

- How can you be sure to have measured the focal distance, and not the distance to a subsequent plane of foci?
- What is the ratio of the Talbot distance and the focal lens.

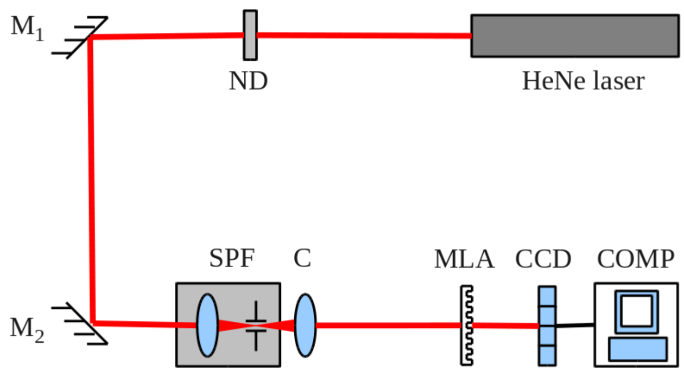


Figure 5: Schematics of the experimental setup to observe/measure the Talbot length and its sub-multiples.

- Do they have a common multiple? How would be the pattern if the focal distance was equal to 1/2 of the Talbot distance? Or 3/4 of the Talbot distance?
- What kind of crystal structure is formed by the focal spots?
- What distortion will occur in white light (be as quantitative as possible)

## 5.2 Phase Front Distortion

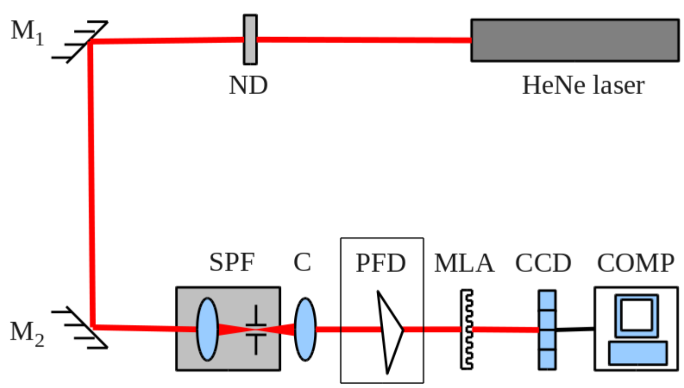


Figure 6: Schematics of the experimental setup to observe the effects of inserting an imperfection on the imaging system through the distortion of the phase front.

In order to gain some appreciation for the use of MLAs on the detection of imperfections of an element of the imaging system, a prism is inserted on the setup of Fig. 5 to create a phase front distortion.

**Measurement**

- Find the angle of the prism through the change in pattern of the diffracted light.
- Do you need to use the first plane of foci?
- Use other methods to measure the angle of the prism, and compare

### 5.3 How far can the "Talbot planes" be observed

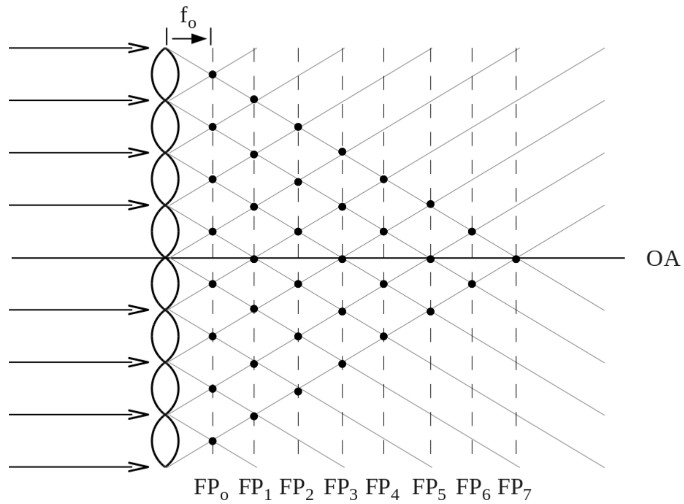


Figure 7: Ray tracing of light incident on a micro lens array of focal planes (FPs) on a 2-D representation. Each lens has a focal length $f_0$.

Place an iris diaphragm in front of the lenslet array. Determine the diameter from the aperture, from the distance at which only one focal point is observed. Verify by measuring the aperture by some other means.

# 6 Summary

1. Micro lens array can be used as an instrument for detection of wave front distortion.

2. A phenomena associated with a periodical phase altering element like a micro lens array is the Talbot effect. In the near field the structure will be "imaged" after a multiple of a certain distance, the so called Talbot length.

3. At sub-multiples of the Talbot distance, images of the of the grating are formed, where the periodicity and offset changes.

## References


[1] H. F. Talbot. Facts relating to optical science. *Philos. Mag. 9*, 4:401–407, 1836.

[2] Lord Rayleigh. On copying diffraction-gratings and on some phenomenon connected therewith. *Philos. Mag.*, 11:196 – 205, 1881.

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[4] F. Zernicke. Das phasenkontrastverfahren bei der mikroskopischen beobachtung. *Physik Z.*, 36:848, 1935.

[5] S. Sinzinger and J. Jahns. *Microoptics*. Wiley-VCH, Weinheim, 1999.