Wavemeter v2.2

The goal of this experiment is to precisely determine the frequency and wavelength of an unknown laser by comparing it to a reference laser. This is accomplished by introducing the unknown and reference laser beams into a Michelson interferometer and studying their relative fringe patterns. Such a configuration is referred to as a wavemeter. One arm of the wavemeter interferometer has a fixed length while the other arm length varies. The length variation is implemented with a corner-cube reflector that slides on an air track with very little friction.

Background The optical intensity at the output of an interferometer is proportional to:

$$|E_1|^2 + |E_2|^2 + 2E_1 \cdot E_2 \cos \left( \frac{2\pi}{\lambda} d \right)$$

where $E_1$ and $E_2$ are the amplitude of the electric field vectors in arms 1 and 2, respectively, $\lambda$ is the wavelength, and $d$ is the relative path length change of the arms. The photodiodes used in this experiment are square-law detectors and produce a voltage proportional to this intensity. Interference is revealed as bright and dark fringes that modulate the intensity signal. This modulation arises from the third term in the above expression. When $E_1 = E_2$, fringe visibility is maximum; it ranges from a peak proportional to $4|E|^2$ and a minimum of 0, i.e. destructive interference is complete and there is no light output. It is important to note that the interference term is a dot product of the two electric field vectors. If the polarization of light in the two arms is orthogonal, no interference will take place.

The coherence length of light is an important consideration for attaining interference. A multi-mode He-Ne laser has a coherence length of the order of 10 cm, while a single-mode He-Ne laser can be three orders of magnitude larger. An incoherent white light source may have a coherence length of only 100 $\mu$m. If the path length difference $d$ exceeds the coherence length, interference disappears.

It is possible to determine the wavelength $\lambda$ of a monochromatic beam of light (laser) without using the two-color wavemeter setup. Consider a single-color Michelson interferometer. By counting the number of fringes ($N$) that are observed while translating one arm a distance $d$, the wavelength can be found from the interference term: $\lambda = 2d/N$. Assuming a wavelength of $\lambda = 600$ nm, a translation of $d = 30$ cm will generate $10^6$ fringes. Accuracy is limited by: i) the precision with which $d$ can be measured and ii) the ability to count fringes to the nearest integer. The integer fringe rounding error limits accuracy to $10^{-6}$ ($\lambda = 600$ nm $\pm$ 0.6 pm), assuming zero uncertainty in the value of $d$. Absolute accuracy for the displacement is not possible, of course, and
will contribute to the overall error. Designating the fringe count uncertainty as $\Delta N$ and the displacement uncertainty as $\Delta d$, the wavelength measurement precision of a single-color Michelson measurement is:

$$\Delta \lambda = 2 \sqrt{\left[ \frac{d \Delta N}{N^2} \right]^2 + \left[ \frac{\Delta d}{N} \right]^2}$$

The two uncertainties ($\Delta N$, $\Delta d$) are independent of each other (i.e. uncorrelated), so the cumulative error is obtained by adding them in quadrature. Refer to the error analysis lecture for more information:


To maintain a wavelength accuracy comparable to the fringe count rounding error ($\Delta N = 1$), the displacement $d$ must be measured with a precision better than 50 nm (i.e. 30 cm ± 50 nm), which is extremely difficult to do.

The wavemeter achieves high accuracy because the frequency (wavelength) of the reference laser and the speed of light ($c$) are known very precisely. The interference fringes produced by the reference laser allow the relative path length $d$ to be accurately calculated. This path length is then mapped onto the interference fringes of the unknown laser to measure its frequency.

Fig. 1 Wavemeter setup
Setup The completed interferometer setup is shown in Fig. 1. A red He-Ne laser serves as the reference and a green He-Ne laser is the unknown. The red laser is linearly polarized and the green laser is unpolarized. They must be aligned so they are collinear (propagating parallel) and have orthogonal linear polarizations. This is accomplished with a mirror and polarizing beam splitter cube. Orient the red laser for $s$-polarization. Before sending the beams into the interferometer, propagate them as far as practical in the lab and align them so that they are collinear over a path length of at least several meters. Verify beam overlap at a point close to the lasers and at a second point located a large distance away.

Optical alignment is a process of iteration. Check the beam overlap at one point, optimize it, then move to the second point and optimize it. Go back to check the first point and repeat as needed. You will need two degrees of freedom (horizontal and vertical) to set each point, which means four mechanical adjustments in total are required. If done correctly, the alignment will converge.

When collinear beam overlap is established, introduce the beams into the interferometer as shown in Fig. 2. The beams must accurately maintain their position on the moving corner cube (CC2) as it travels back and forth on the air track. This alignment is accomplished by using a temporary screen taped to the outer housing of CC2. It is a good idea to keep the beams close to the center of CC2 because the polarization of the reflected beams will rotate with increasing spatial offset from center. Use an iterative process with a mirror and beamsplitter as described above. A corner cube is much better than a mirror in this application because it will keep the alignment while moving down the track, even in the presence of acoustic noise and vibrations. The fixed length arm of the interferometer also uses a corner cube (CC1) to introduce a reflected polarization.
consistent with the moving arm. Parallel polarizations for the individual colors in both arms is essential to produce interference fringes.

**Note:** Do not make any adjustments to the air rail.

After exiting the Michelson interferometer, the beams are separated with a second polarizing beam splitter and individually detected. Scattered light can be reduced by placing filters in front of the detectors. Use an oscilloscope to display signals from both detectors. Use DC coupling to optimize the alignment and to observe the modulation depth. Assuming proper polarization orientation, maximum fringe visibility occurs when the signal contributions from both arms are the same. You should be able to get > 200 mV signal from each arm (and each color) and a modulation depth of 30% or better.

**Note:** Beam divergence and diffraction will reduce fringe visibility. This will be apparent for laser light traveling a much longer distance in the moving arm. It can be helpful to expand and collimate the beams with a telescope to compensate for this.

**Measurement and Analysis** Once optical alignment is complete, the air track can be pressurized. Excessive pressure will cause the stage to vibrate and thus reduce signal-to-noise. The traditional method of implementing a wavemeter is with a frequency counter. A HP 5345 counter is available for this purpose. By properly setting the trigger levels on both channels, adjusting the gate width, and simultaneously observing both signals on the oscilloscope, the ratio of the two interference fringe waveforms can be measured. A longer gate time increases accuracy, but requires that the velocity of CC2 remain constant. Some regions of the rail will give much better signals than others. Can you explain why? What is the highest precision that can be attained?

The counter attempts to find the frequencies of both channels simultaneously and calculate their ratio. This can be problematic if the stage is accelerating and/or the signals are noisy. A more direct approach is to capture the waveforms with a sampling oscilloscope (Tektronix TDS 3054). A LabView program is available for this purpose. Connect the scope to the acquisition computer using an ethernet cable. When configured properly, both scope channels will display in the LabView program. Move CC2 and pause the program at the appropriate time, i.e. when two clean interference signals are observed. Data from both channels is acquired and saved to a three-column text file: i) time (s), ii) Ch 1 signal (V), and iii) Ch 2 signal (V). Analysis of the data reveals the instantaneous frequency of both channels. This can be determined by either the zero (baseline) crossings or the separation of sinusoidal peaks and valleys. With this information, the frequencies can be calculated at each half-period. This allows one to correct for time-dependent frequency changes (chirp), calculate the velocity and acceleration of the stage, and determine the accuracy of the experiment. A computer
program you write can perform this calculation at each step.

The goal is to capture as many fringes as possible within the experimental constraints. The program must accommodate the existence of noise on both signals that will add uncertainty to the measurement of the half periods. It must also account for the finite sampling rate of the scope. All these uncertainty issues should go into your error analysis. You can also correct for non-unity refractive index of air in the lab.