The Wavemeter

1 Background

The goal of this experiment is to utilize the sensitivity of the Michelson interferometer, used here in a wavemeter, to precisely determine the frequency and wavelength of an"unknown" laser. Due to the wavelike nature of light, the basic Michelson interferometer, as depicted in Fig. 1, produces dark and bright fringes at the photodetector (PD) as the length of one arm is continuously varied by an amount Δ. These fringes are the result of constructive and destructive interference of the fields from each arm of the interferometer. For a more detailed description of the Michelson interferometer, see [1].

2 Theory

The intensity at the output of a Michelson interferometer, see Fig. 1, that is recorded by the photodiode is given by:

$$I = \frac{I_o}{2}\left[1 + \cos(2\pi\frac{2(d_2 - d_1)}{\lambda})\right]. \tag{1}$$

The photodetector will see one fringe go by for a displacement of one half wavelength of M1. If we translate M1 by a known distance d, and count the number of fringes N on the photodetector, we can then determine λ from the relationship

$$d = N\lambda/2 \tag{2}$$

by measuring N and d. If for instance $d = 30$ cm and $N = 10^6$ fringes, then $\lambda = 0.6$ micron. There are, however, two limiting factors on the precision with which we can determine λ. 1) How well we know N, and 2) how accurately we can determine d. For instance, even if d is known with absolute accuracy, simple fringe counting would only yield an accuracy of 10^{-6} for the determination of the wavelength, limited by the lack of information from rounding off to the nearest fringe integer. A higher precision is possible by recording the interference fringes and calculating the position between fringes after the last full fringe has been counted, such that N is known to a certain number of decimal places. N is in fact the round-off of $\arccos(2I/I_o)$:

$$N = \mathrm{INT}[\arccos 2I/I_o - 1/2]. \tag{3}$$

Determining N with a higher precision will help, but in reality the distance d cannot be measured with the required accuracy. One could try to translate M1 uniformly and to record accurately the fringe count rate. Knowing the speed of translation, one could determine the wavelength from the fringe counting rate (or frequency). The problem with that approach is in achieving the required uniformity of motion. One could use synchronous motors (the frequency stability of the 60 Hz network is fairly good). However, the error in the reducing gears and leadscrew of the translation stage make it impossible to achieve uniform translation speed. In fact, a chart recording of the fringes of an interferometer using a synchronous motor driven translation stage will indicate periodic changes in "wavelength", which can be used to identify the gear sizes and defects in the reduction box. Therefore, a simple way to determine the quality of such a drive mechanism is to record closely spaced fringes (chart the motor on a low speed relative to the periodicity of the interferences), and copy them on a transparency. The

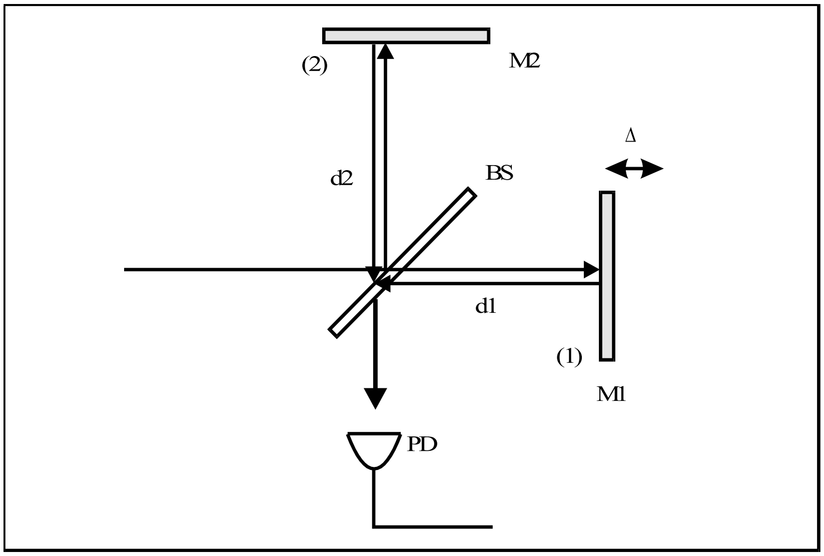

Figure 1: Michelson Interferometer

"Moire" pattern observed when the transparency is superimposed on the original recording is an indication of the speed variations in the motor drive.

So how then can we determine d with the required precision? The answer seems almost circular: use the laser. It is usually the frequency of a laser which is known with the most precision. The absolute frequency of many well stabilized lasers has been determined by beating its frequency down, in a long chain of complicated nonlinear mixing devices, to a range in which it can be compared with the current definition of time, the ground state hyperfine splitting of cesium, which is 9 192 631 770 Hz. As of 1983, the speed of light has been *defined* to be 299 792 458 m/s. Thus by knowing the precise frequency of the laser, and using the defined value for the speed of light to determine the wavelength (in vacuum), one can calculate the distance d from Eq. (2). This is in fact how the meter is currently defined, through the definition of c and the second. This technique will then require two lasers, the frequency of one of them is well known. *The fringe count of the known laser frequency is used to calculate the distance d, which is in turn used - knowing the fringe count for the other laser - to calculate the unknown wavelength.* Several arrangements are possible. One of them is the Mach-Zehnder interferometer. The other is the double Michelson sketched in Fig. 2.

One should be aware that this method provides a wavelength ratio in the laboratory environment. Depending on how successful your experiment is, you may have to correct for the dispersion of air. The appropriate constants for that correction can be found in the "Handbook of Chemistry and Physics" (Chemical Rubber Co.)

Two other limitations are the bandwidth and stability of the reference laser, as well as that of the laser to be measured. These (bandwidth and stability) are two different factors, which might - but need not - be related.

3 Experimental Setup

Attempt to make a Michelson interferometer using an air rail as one arm (see Fig. 2). This interferometer will be used as a wavemeter by combining two lasers, a known reference (red

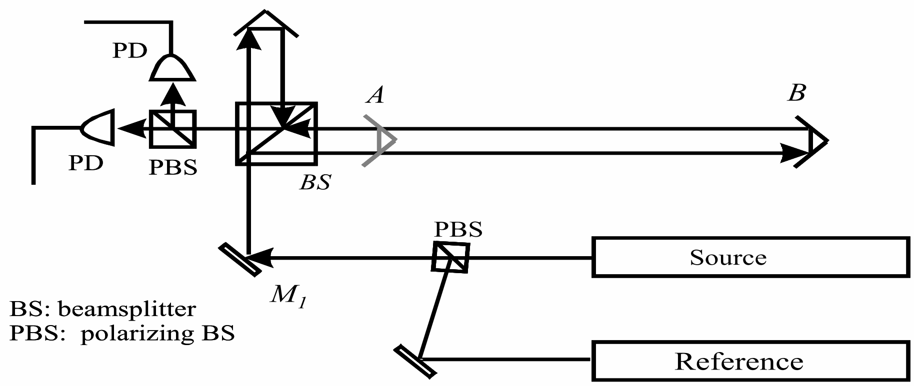

Figure 2: Wavemeter

He-Ne) and an unknown source (green He-Ne, this is the least you should determine!). The polarization of the lasers should be orthogonal to each other. Using a polarizing beamsplitter (PBS), the beams from the two lasers can be superimposed on each other and aligned through the wavemeter. This allows the two beams to propagate the same optical path length through the same interferometer. The beams are then separated at the exit of the interferometer and sent to two different photodetectors. If necessary, use color filters to further suppress the unwanted beam at the detectors. We then can obtain data from "two" interferometers.

You will notice three practical problems with this arrangement:

1. The translation axis of the variable arm has to be lined up with the beam with great accuracy;

2. Due to diffraction, it is difficult to observe uniform extinction (and therefore good fringe contrast) over the cross section of the beam

3. how to create and maintain perfect orthogonal polarization?

3.1 Alignment of the axis of the translation stage (Rail)

Do not attempt to change the alignment of the rail! The rail should be set *horizontal* for two reasons. First, the horizontal corner cube will travel at a uniform velocity between the two ends of the rail only if it is horizontal. The second reason is to preserve polarization, as explained in the subsection 3.3. It is the combined red/green laser beam that should be aligned parallel to the rail. There is one basic principle to be used in this as in any other optical alignment where a beam has to be aligned in position and direction: use the mirror the farthest away to adjust the position; use the mirror the closest to adjust the direction. In the present case, to facilitate the alignment, cover first the corner cube with a paper or index card, and mark the desired point of contact of the beam. The procedure is:

1. With the corner cuber in the close position A, use the "remote" mirror M_1 to adjust the beam on the mark on the index card.

2. Move the corner cube to the end of the rail (B), and adjust the beam with the beamsplitter BS.

3. repeat (1) and (2) until no more adjustment is required. This processus converges, even if, as in the present case, the "remote" mirror is fairly close to the BS.

3.2 Fighting diffraction

The beam path in the long arm can be as much as 1 m longer than in the short arm. If the beam generated by the laser is too narrow (which is the case), it will widen by diffraction. Therefore, the beam having gone through the short arm will be much narrower than the one that went through the long arm, resulting in a poor contrast of the interferences.

In order to produce interference fringes with the best possible contrast, the two beams must have the same intensity and therefore the same cross-section. This condition, for a delay variation of 2 m, implies some minimum dimension for the beam waist. Determine the minimum size of the beams, and expand the beam with an appropriate choice of lenses.

3.3 Polarization problems

The polarizing beam splitter cubes are simply dielectric coatings, with a different reflectivity for s and p polarization. In general, this type of polarizer is not working equally well in transmission or reflection. Consider horizontal polarization (p) incident on a corner cube. The various dielectric layers are all close to Brewster incidence. By tuning the angle of incidence, it is possible to find an orientation for which the reflection is zero. That means that this p polarization is perfectly transmitted. The weaker green laser is not polarized, and should use the best polarizing properties of the cube (i.e. in transmission). The red (reference) laser is polarized. It should be rotated around its axis, in such a way as to be incident on the polarizing beam splitter in s incidence, as shown on the figure. Not that some of the red s polarized light will be transmitted through the beam splitter cube, a loss that is much more affordable than if it were the green laser light.

It is essential, to maintain the polarization throughout the setup, that the beams remain in a plane parallel to the table. Show how, by moving out of the plane, the polarization can be made elliptical, or rotated by 90o.

The standard recording instrument used with a wavemeter is a frequency counter. That will be the first technique to use. By measuring the ratio of the fringe frequency from each laser as the mirror on the air rail is moved at a constant velocity, one should be able to calculate the frequency of the unknown laser.

The alternative technique that will be used is to record all the data, store them, and make appropriate manipulations to determine the number of fringes passed from each laser. This is the more direct approach. The details on how to take data from the two photodiodes, D_s and D_r, and store it on the computer, are given in Appendix **??**. With this data, you can count the number of fringes, including the partial fringe, to calculate the frequency of the unknown laser.

Before you use the frequency counter and computer observe the fringes with an oscilloscope. This will allow you to make final adjustments with your detectors. You will notice that the fringe contrast changes periodically when the length of one interferometer arm is changed. Why is that? For data taking, identify the regions on the rail where you have good interference contrast from both detectors.

When taking data by computer, turn off the compressed air on the rail, and move the delay of the previously fixed arm by turning the knob of the translation stage.

4 Summary

In this lab you should have learned how the Michelson interferometer can be used as a wavemeter to make precision measurements of a laser's frequency and wavelength. You should also try to identify some of the factors which limited the precision of your measurements.

Additional questions

1. How is the precision of this technique affected by the distance which mirror 1 is translated? What does this tell you about the physical dimensions of a high-precision wavemeter?

2. Corner cube reflectors: The metallic corner cube reflectors have an accuracy of $2''$ of arc. Will that be sufficient to keep the laser beams aligned?

3. Why use metallic corner cube reflectors, rather than glass corner cube reflectors (based on total internal reflection)?

4. Fringe visibility and diffraction: Given as a condition that the beam diameter should not vary more than 10% over the entire range,in order to provide good fringe visibility, what should the initial waist of the beam be for each laser? (Hint: This requires some basic knowledge of gaussian beam propagation.) You should estimate the size of the initial beam waist from each laser by measuring its far-field divergence angle.

5. Observe the fringe contrast of both the red and the green HeNe laser individually over the entire translation range with an oscilloscope. Sketch the contrast as a function of delay Δd and explain the main features of the your graph

6. Explain this measurement in terms of Doppler shifts at each moving mirror.

Refractive index of dry air (at sea level): Make use of this table in your data analysis and discussion.

λ (nm)	$(n-1) \times 10^7$
540	2773
550	2771
630	2760
640	2759

References

[1] Eugene Hecht and Alfred Zajac. *Optics.* Addison-Wesley, ISBN 0-201-11609-X, Menlo Park, California, 1987.