Formula Sheet PHYC/ECE 463 Advanced Optics I UNM/Fall 2007

harmonic plane wave: $E = \mathrm{Re}\{E_0 \exp(i\omega t - ik.r + \varphi)\}$ $\quad k = \frac{n\omega}{c} = \frac{2\pi n}{\lambda_0}$

Poynting vector $S = \frac{1}{\mu_0} E \times B$ Irradiance: $I = \langle S \rangle = \frac{\varepsilon_0 nc}{2} |E_0|^2$

$n_i \sin(\theta_i) = n_t \sin(\theta_t)$ *Snell's Law*

$$\rho_\pi = \frac{n_t \cos(\theta_i) - n_i \cos(\theta_t)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\rho_\sigma = -\frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

Fresnel's Reflectivity

$1 - \rho^2 = \tau\tau'$ $\quad R = |\rho|^2$, $\quad R + T = 1$

If n is complex then $n \to \tilde{n} = n - i\kappa \equiv \sqrt{\varepsilon / \varepsilon_0} \equiv \sqrt{1 + \chi}$ in above expressions

Absorption coefficient (K or α) and skin depth (δ): $\alpha \equiv \frac{2}{\delta} = \frac{4\pi\kappa}{\lambda_0}$

Classical Electron Oscillator Model:

$$\chi = \frac{\omega_p^2}{\omega_0^2 - \omega^2 + i\omega / \tau} \quad \text{where} \quad \omega_p = \sqrt{\frac{Nq^2}{m_0 \varepsilon_0}} \text{ (plasma frequency)}$$

Drude model for metals: $\omega_0 \to 0$

Group Velocity $v_g = \frac{d\omega}{dk}$

Light pressure (on perfecting absorbing surface) $P = \frac{I}{c}$

Prism with apex angle α at minimum deviation angle θ_D:

$$n = \frac{\sin\left(\frac{\alpha + \theta_D}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

Numerical Aperture (NA) of an optical fiber: *f# (f-number)=f/D*

$$n_0 \sin(\theta_{\max}) = \sqrt{n_f^2 - n_c^2}$$

Lens-makers' formula: $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Gaussian imaging formula (thin lens) $\frac{1}{f}=\frac{1}{S}+\frac{1}{S'}$

(refractive sphere) $\frac{n'-n}{R}=\frac{n}{S}+\frac{n'}{S'}$

Paraxial Ray Tracing Matrices:

Propagation $\begin{pmatrix}1 & 0\\ d/n & 1\end{pmatrix}$ (length d and index n)	Refractive surface $\begin{pmatrix}1 & -P\\ 0 & 1\end{pmatrix}$ *where P=(n'-n)/R*
Mirror with radius of curvature R $\begin{pmatrix}1 & -2/R\\ 0 & 1\end{pmatrix}$	Thin lens of focal length *f* $\begin{pmatrix}1 & -1/f\\ 0 & 1\end{pmatrix}$
Thick lens (Thickness D_l, index n_l) $\begin{pmatrix}1-\frac{P'D_l}{n_l} & -P-P'+\frac{PP'D_l}{n_l}\\ \frac{D_l}{n_l} & 1-\frac{PD_l}{n_l}\end{pmatrix}$ $P=(n_l-n)/R,\quad P'=(n'-n_l)/R'$	Separated Doublet $\begin{pmatrix}1-\frac{d}{f_2 n_b} & -\frac{1}{f_1}-\frac{1}{f_2}+\frac{d}{f_2 f_1 n_b}\\ \frac{d}{n_b} & 1-\frac{d}{f_1 n_b}\end{pmatrix}$

between conjugate planes $\begin{pmatrix}m_\alpha \frac{n'}{n} & \tilde{M}_{12}\\ 0 & m_x\end{pmatrix}$	between principal planes $\begin{pmatrix}1 & M_{12}\\ 0 & 1\end{pmatrix}$
for telescopic Systems $\begin{pmatrix}m_\alpha \frac{n'}{n} & 0\\ M_{21} & m_x\end{pmatrix}$	position of principal planes $D=\left(\frac{n}{M_{12}}\right)(1-M_{11})$ $D'=\left(\frac{n'}{M_{12}}\right)(1-M_{22})$

Contact Doublet Achromatization:

$f_1V_1+f_2V_2=0$ where $V=\frac{n_d-1}{n_f-n_c}$ is the Abbe number

Physical constants:

speed of light	$c\sim 2.998\times10^{8}$	m s^{-1}
electronic charge	$q\sim 1.602\times10^{-19}$	C
permittivity of vacuum	$\varepsilon_0\sim8.854\times10^{-12}$	F/m
electronic mass	$m_0\sim9.1094\times10^{-31}$	Kg
Planck constant	$h\sim6.626\times10^{-34}$	J.s

Near distance of a normal eye: d_0=250 mm