Formula Sheet PHYC/ECE 463 Advanced Optics I UNM/Fall 2007

harmonic plane wave: $E = \mathrm{Re}\{E_0 \exp(i\phi)\}$ with $\phi \equiv \omega t - k.r + \varphi$, $k = \frac{n\omega}{c} = \frac{2\pi n}{\lambda_0}$

Poynting vector $S = \frac{1}{\mu_0} E \times B$ Irradiance: $I = \langle S \rangle = \frac{\varepsilon_0 nc}{2} |E_0|^2$

$n_i \sin(\theta_i) = n_t \sin(\theta_t)$ *Snell's Law*

$$\rho_\pi = \frac{n_t \cos(\theta_i) - n_i \cos(\theta_t)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \qquad \tau_\pi = \frac{2n_i \cos(\theta_i)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)}$$

$$\rho_\sigma = -\frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad \tau_\sigma = \frac{2n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)}$$

$1 - \rho^2 = \tau\tau'$ $\quad R = |\rho|^2$ $\quad T = |\tau|^2 \frac{n_t \cos(\theta_t)}{n_i \cos(\theta_i)}$ $\quad$ *R+T=1*

If n is complex then $n \to \tilde{n} = n - i\kappa \equiv \sqrt{\varepsilon / \varepsilon_0} \equiv \sqrt{1+\chi}$ in above expressions

Absorption coefficient (K or α) and skin depth (δ): $\alpha \equiv \frac{2}{\delta} = \frac{4\pi\kappa}{\lambda_0}$

Classical Electron Oscillator Model:

$$\chi = \frac{\omega_p^2}{\omega_0^2 - \omega^2 + i\omega / \tau} \quad \text{where} \quad \omega_p = \sqrt{\frac{Nq^2}{m_0 \varepsilon_0}} \text{ (plasma frequency)}$$

Drude model for metals: $\omega_0 \to 0$

Group Velocity $v_g = \frac{d\omega}{dk}$

Light pressure (on perfecting absorbing surface) $P = \frac{I}{c}$

Prism with apex angle α at minimum deviation angle θ_D:

$$n = \frac{\sin\left(\frac{\alpha + \theta_D}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

Numerical Aperture (NA) of an optical fiber: *f# (f-number)=f/D*

$$n_0 \sin(\theta_{\max}) = \sqrt{n_f^2 - n_c^2}$$

Lens-makers' formula: Gaussian imaging formula (thin lens) (refractive sphere)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \frac{1}{f} = \frac{1}{S} + \frac{1}{S'} \qquad \frac{n'-n}{R} = \frac{n}{S} + \frac{n'}{S'}$$

Paraxial Ray Tracing Matrices:

Propagation $\begin{pmatrix} 1 & 0 \\ d/n & 1 \end{pmatrix}$ (length d and index n)	Refractive surface $\begin{pmatrix} 1 & -P \\ 0 & 1 \end{pmatrix}$ *where P=(n'-n)/R*
Mirror with radius of curvature R $\begin{pmatrix} 1 & -2/R \\ 0 & 1 \end{pmatrix}$	Thin lens of focal length *f* $\begin{pmatrix} 1 & -1/f \\ 0 & 1 \end{pmatrix}$
Thick lens (Thickness D_l , index n_l) $\begin{pmatrix} 1-\frac{P'D_l}{n_l} & -P-P'+\frac{PP'D_l}{n_l} \\ \frac{D_l}{n_l} & 1-\frac{PD_l}{n_l} \end{pmatrix}$ $P=(n_l-n)/R$, $P'=(n'-n_l)/R'$	Separated Doublet $\begin{pmatrix} 1-\frac{d}{f_2 n_b} & -\frac{1}{f_1}-\frac{1}{f_2}+\frac{d}{f_2 f_1 n_b} \\ \frac{d}{n_b} & 1-\frac{d}{f_1 n_b} \end{pmatrix}$

between conjugate planes $\begin{pmatrix} m_\alpha \frac{n'}{n} & \tilde{M}_{12} \\ 0 & m_x \end{pmatrix}$	between principal planes $\begin{pmatrix} 1 & M_{12} \\ 0 & 1 \end{pmatrix}$
for telescopic Systems $\begin{pmatrix} m_\alpha \frac{n'}{n} & 0 \\ M_{21} & m_x \end{pmatrix}$	position of principal planes $D = \left(\frac{n}{M_{12}}\right)(1-M_{11})$ $D' = \left(\frac{n'}{M_{12}}\right)(1-M_{22})$

Contact Doublet Achromatization:

$$f_1V_1 + f_2V_2 = 0 \quad \text{where} \quad V = \frac{n_d - 1}{n_f - n_c} \quad \text{is the Abbe number}$$

Physical constants:

speed of light	$c\sim 2.998\times10^{8}$	m s^{-1}
electronic charge	$q\sim 1.602\times10^{-19}$	C
permittivity of vacuum	$\varepsilon_0\sim8.854\times10^{-12}$	F/m
electronic mass	$m_0\sim9.1094\times10^{-31}$	Kg
Planck constant	$h\sim6.626\times10^{-34}$	J.s

Near distance of a normal eye: d_0=250 mm

Two-beam Interference

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta)$ where $\delta = \phi_1 - \phi_2$

Multiple-Beam Interference:

Wavefront splitting (N slits, waves having equal amplitudes): $I(\delta) = I(0)\left(\frac{\sin(N\delta/2)}{N\sin(\delta/2)}\right)^2$

$\delta = 2\pi\frac{a}{\lambda}(\sin\theta' - \sin\theta)$ *a*= slit separation, $\delta = 2m\pi$ (constructive interference)

Resolving power $\mathcal{R}$: $\frac{\lambda}{\Delta\lambda} = mN$

Amplitude Splitting: $\delta = \frac{4\pi}{\lambda_0} n_2 d\cos\theta_2 - \Delta\gamma$ $\Delta\gamma$=roundtrip reflection phase (if any)

Fabry-Perot: (for $R_1=R_2=R$) $T = \frac{T_{max}}{1 + F\sin^2(\delta/2)}$ where $T_{max} = \left(1 - \frac{A}{1-R}\right)^2$ and A=loss (mirror)

$F = \frac{4R}{(1-R)^2}$ *(contrast),* $\mathcal{R} = \frac{\lambda}{\Delta\lambda} = m\mathcal{F}$ *where* $\mathcal{F} = \frac{\pi\sqrt{F}}{2} \approx \frac{\pi}{1-R}$ is the Finesse

as a (laser) cavity: $\mathcal{F} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}}$ $\Delta\nu_{FSR} = c/2nd$ `

Matrix Formalism (thin film coatings):

$E_j = \begin{pmatrix} E_{lj} \\ E_{rj} \end{pmatrix}$ fields on the left side of layer j, $E'_j = \begin{pmatrix} E'_{lj} \\ E'_{rj} \end{pmatrix}$ fields on the right side of layer j

interface transition: $E'_i = H_{ij} E_j$ where $H_{ij} = \frac{1}{\tau_{ij}}\begin{pmatrix} 1 & \rho_{ij} \\ \rho_{ij} & 1 \end{pmatrix}$

propagation: $E_j = L_j E'_j$ where $L_j = \begin{pmatrix} e^{-i\beta_j} & 0 \\ 0 & e^{+i\beta_j} \end{pmatrix}$ with $\beta_j = \frac{2\pi}{\lambda_0} n_j d_j \cos\theta_j$ (note n_j may be complex if loss or gain is present).

For N-2 layers (sandwiched between medium 1 and N):

$S = H_{12} L_2 H_{23} L_3 \ldots L_{N-1} H_{N-1,N} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$, $\tau = \frac{1}{S_{22}}$, $\rho = \frac{S_{12}}{S_{22}}$

Waveguides:

Assuming: $E(r,t) = \mathcal{E}(x,y)e^{i\omega t - i\beta z}$ then $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\mathcal{E}(x,y) + (n^2 k_0^2 - \beta^2)\mathcal{E}(x,y) = 0$ **

Slab waveguide solutions: core: $\mathcal{E}(x) = \begin{cases} A\sin(hx) \\ A\cos(hx) \end{cases}$ cladding: $\mathcal{E}(x) = B\begin{cases} e^{-p(x-d/2)} \\ e^{+p(x+d/2)} \end{cases}$ where

$h = \sqrt{n_2^2 k_0^2 - \beta^2}$ $p = \sqrt{\beta^2 - n_1^2 k_0^2}$

* *h* and *p* are obtained from conditions that tangential components (E_y and $H_z \propto \partial E_y/\partial x$ for TE , H_y and $E_z \propto (1/n^2)\partial H_y/\partial x$ for TM) are continuous at the interfaces.

* For fibers the above Helmholtz equation** is solved in cylindrical coordinates. $E(r,\phi,t) = \mathcal{E}(r)e^{-il\phi}e^{i\omega t - i\beta z}$

$\mathcal{E}(r) \propto \begin{cases} J_l(k_T r) & \text{for } r < a(core) \\ K_l(\gamma r) & r > a(cladding) \end{cases}$ $\sqrt{(k_T a)^2 + (\gamma a)^2} = 2\pi\frac{a}{\lambda_0}(n_2^2 - n_1^2) = V$ (V-number)

Number of modes $M \approx \frac{4}{\pi^2}V^2$ (step index fibers)