PHYC/ECE 463  Advanced Optics I
Fall 2007
Midterm Exam, Closed Book, 1.5 hours

Class Score Chart (9 students)

Average (with highest and lowest scores removed): 69
Problem 1. (35 points)
Consider the spherical fishbowl (radius \( |R| \)) filled with water having an index \( n \). An object (e.g. a fish), indicated by the arrow, is located near the bowl wall (at a distance \( 2|R| \) from vertex \( V \)). Refraction due to thin glass walls can be ignored.

(a) Find the location of the image and discuss its real or imaginary nature with its dependence on \( n \).

(b) Derive an expression for the angle \( \alpha \) subtended by the (virtual) image as seen by an observer situated a distance \( D \) from \( V \) (as shown in the figure). The result should be in terms of \( n \), \( |R| \), \( D \) and \( h \) (the height of object). Assume small angle.

(c) Assume \( |R| = 10 \text{ cm} \), and the bowl is filled with water \( (n=1.3) \). Find the distance \( D \) for which the observer sees the fish 25% larger than when it swims to the front side of the bowl (right behind vertex \( V \)).
Problem 2. (25 points) (same as HW 8)
Design an achromatic doublet with the contours indicated in the Fig. below. Use glass 510:635 as the crown component and the glass 620:364 as the flint component. Treat the lenses as thin lenses with zero separation and find a combination of radius of curvature that will produce a lens having a focal length of 100 cm.

![Diagram of an achromatic doublet]

Note: Glass type is xxx:yyy where xxx=(n_r-1)\times1000 and yyy=10\times V

Problem 3. (15 points)
Your are given the following four thin lenses with focal lengths $f$ and diameter $D$:

<table>
<thead>
<tr>
<th>Lens #</th>
<th>$f$ (cm)</th>
<th>$D$ (cm)</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>5</td>
<td>plano-convex</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>3</td>
<td>plano-convex</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>6</td>
<td>equi-convex</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>5</td>
<td>equi-convex</td>
</tr>
</tbody>
</table>

Which one would you use to light a match faster by focusing the sunlight? Explain.

Problem 4. (25 points)
Briefly (using less than 35 words for each) and using diagrams, equations (when necessary) and examples answer only 2 of the following 3 questions.

(a) Describe the spherical aberration. Give example for a thin lens.

(b) Describe the frustrated total internal reflection.

(c) Why are metals highly reflective at certain wavelengths?
harmonic plane wave:  \( E = \text{Re}\{E_0 \exp(\text{i} \omega t - \text{i} k r + \varphi)\} \)

\( k = \frac{n \omega}{c} = \frac{2 \pi n}{\lambda_0} \)

Poynting vector \( S = \frac{1}{\mu_0} E \times B \)

Irradiance: \( I = \langle S \rangle = \frac{\varepsilon_0 n c}{2} \left| E_0 \right|^2 \)

\( n \sin(\theta_i) = n_s \sin(\theta_s) \quad \text{Snell's Law} \)

\[
\begin{align*}
\rho_\sigma &= \frac{n_s \cos(\theta_i) - n \cos(\theta_s)}{n_s \cos(\theta_i) + n \cos(\theta_s)} = \frac{\tan(\theta_s - \theta_i)}{\tan(\theta_s + \theta_i)} \\
\rho_\sigma &= -\frac{n_s \cos(\theta_i) - n \cos(\theta_s)}{n_s \cos(\theta_i) + n \cos(\theta_s)} = \sin(\theta_i - \theta_s)
\end{align*}
\]

\( 1 - \rho^2 = \frac{R}{T} \quad R \approx |\rho|^2 \)

\( R + T = 1 \)

If \( n \) is complex then \( n \rightarrow \bar{n} = n - \text{i} \kappa = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{1 + \chi} \) in above expressions

Absorption coefficient (\( K \) or \( \alpha \)) and skin depth (\( \delta \)): \( \alpha = \frac{2}{\delta} = \frac{4 \pi \kappa}{\lambda_0} \)

Classical Electron Oscillator Model:

\( \chi = \frac{\omega_p^2}{\omega_p^2 - \omega^2 + i \omega / \tau} \)

where \( \omega_p = \sqrt{\frac{N q^2}{m_0 e_0}} \) (plasma frequency)

Drude model for metals: \( \omega_p \rightarrow 0 \)

Group Velocity \( v_g = \frac{d\omega}{dk} \)

Light pressure (on perfecting absorbing surface) \( P = \frac{\dot{I}}{c} \)

Prism with apex angle \( \alpha \) at minimum deviation angle \( \theta_D \):

\[
\sin \left( \frac{\alpha + \theta_D}{2} \right) = \frac{n_0}{n}
\]

Numerical Aperture (NA) of an optical fiber: \( f# \) (f-number) = \( f/D \)

\[
n_0 \sin(\theta_{\text{max}}) = \sqrt{n_f^2 - n_c^2}
\]
Lens-makers' formula: \[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Gaussian imaging formula (thin lens) \[ \frac{1}{f} = \frac{1}{S} + \frac{1}{S'} \]

(refractive sphere) \[ \frac{n'-n}{R} = \frac{n}{S} + \frac{n'}{S'} \]

\[ m_x = \frac{S'}{S} \] (transverse magnification) \[ m_a = \frac{n/n'}{m_x} \] (angular magnification)

Paraxial Ray Tracing Matrices:

<table>
<thead>
<tr>
<th>Propagation</th>
<th>Refractive surface</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
1 & 0 \\
\frac{d}{n} & 1 \\
\end{pmatrix}
\] (length d and index n) | \[
\begin{pmatrix}
1 & -P \\
0 & 1 \\
\end{pmatrix}
\] where \( P = (n'-n)/R \) |

<table>
<thead>
<tr>
<th>Mirror with radius of curvature R</th>
<th>Thin lens of focal length f</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
1 & -2/R \\
0 & 1 \\
\end{pmatrix}
\] | \[
\begin{pmatrix}
1 & -1/f \\
0 & 1 \\
\end{pmatrix}
\] |

<table>
<thead>
<tr>
<th>Thick lens (Thickness ( D_1 ), index ( n_1 ))</th>
<th>Separated Doublet</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
1 - \frac{P'D_1}{n_1} & -P - P' + \frac{PnP'D_1}{n_1} \\
\frac{D_1}{n_1} & 1 - \frac{PD_1}{n_1} \\
\end{pmatrix}
\] \( P = (n-n)/R \), \( P' = (n'-n)/R' \) | \[
\begin{pmatrix}
1 - \frac{d}{f_1n_b} & -\frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_2f_1n_b} \\
\frac{d}{n_b} & 1 - \frac{d}{f_1n_b} \\
\end{pmatrix}
\] |

between conjugate planes | between principal planes |
|--------------------------|--------------------------|
| \[
\begin{pmatrix}
m_x & n' \\
m_0 & m_x \\
\end{pmatrix}
\] \( \tilde{M}_{12} \) | \[
\begin{pmatrix}
1 & M_{12} \\
0 & 1 \\
\end{pmatrix}
\] |

for telescopic Systems | position of principal planes |
|------------------------|-----------------------------|
| \[
\begin{pmatrix}
m_g & n' \\
M_{21} & m_x \\
\end{pmatrix}
\] | \[
D = \frac{n}{M_{12}} (1-M_{11}) \\
D' = \frac{n'}{M_{12}} (1-M_{22}) \\
\] |

Contact Doublet Achromatization:

\[ f_1Y_1 + f_2Y_2 = 0 \]

where \( V = \frac{n_g - 1}{n_f - n_c} \) is the Abbe number

Physical constants:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed of light</td>
<td>( \sim 2.998 \times 10^8 ) m s(^{-1} )</td>
</tr>
<tr>
<td>electronic charge</td>
<td>( \sim 1.602 \times 10^{19} ) C</td>
</tr>
<tr>
<td>permittivity of vacuum</td>
<td>( \sim 8.854 \times 10^{-12} ) F/m</td>
</tr>
<tr>
<td>electronic mass</td>
<td>( m_0 \sim 9.1094 \times 10^{-31} ) Kg</td>
</tr>
<tr>
<td>Planck constant</td>
<td>( h \sim 6.626 \times 10^{-34} ) J.s</td>
</tr>
</tbody>
</table>

Near distance of a normal eye: \( d_0 = 250 \) mm
S = 2 |R|

\[ \frac{n}{s} + \frac{1}{s'} = -\frac{1-n}{|R|} \Rightarrow \frac{n}{2|R|} + \frac{1-n}{|R|} = \frac{1}{s'} \]

\[ s' = \frac{2|R|}{2 + n} \]

\[ s' > 0 \iff n > 2 \quad \text{(real image)} \]

\[ s' < 0 \iff n < 2 \quad \text{(virtual image)} \]

\[ x' = \frac{-S'}{S} = \frac{1}{2-n} \]

\[ \alpha = \frac{x'}{-S' + D} = \frac{x'}{D + \frac{2|R|}{2-n}} = \frac{-\frac{s'}{s}}{D + \frac{2|R|}{2-n}} = \frac{1}{D + \frac{2|R|}{2-n}} \]

\[ \alpha = \frac{h}{(2-n)D + 2|R|} \]
\[ M \alpha = \frac{\alpha}{\alpha_0} = \frac{D}{(2-n)D + 2IR} \]

\[ D(1 - M(2-n)) = M \cdot 2R \]

\[ D = 2IR \times \frac{M}{1 - M(2-n)} \]

\[ M = 1.25 \quad n = 1.3 \quad IR = 10 \quad \text{cm} \]

\[ D = 2IR \times 10 = 200 \quad \text{cm} \]
Glass: $\frac{510:635}{F_{\text{Crown}}} \Rightarrow n_{d1} = 1.510 \quad V_1 = 63.5$

Flint: $\frac{620:364}{F_{\text{Flint}}} \Rightarrow n_{d2} = 1.620 \quad V_2 = 36.4$

\[
f_1, V_1, f_2 V_2 = 0.
\]

\[
\frac{1}{f_1} = \frac{-V_1}{V_2-V_1} \frac{1}{f}
\]

\[
\frac{1}{f_2} = \frac{V_2}{V_2-V_1} \frac{1}{f}
\]

\[
\frac{1}{f_1} = \frac{-63.5}{36.4-63.5} \frac{1}{f}
\]

\[
\frac{1}{f_2} = \frac{36.4}{36.4-63.5} \frac{1}{f}
\]

\[
\frac{1}{f_1} = 2.343 \times 100
\]

\[
\frac{1}{f_2} = -1.343 \times 100
\]

\[
\frac{1}{f_1} = 2.343 = (1.510-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{R_1} - \frac{1}{R_2} = 0.0459
\]

\[
\frac{1}{f_2} = -1.343 = (1.620-1)\left(\frac{1}{R_2} - \frac{1}{\infty}\right) = \frac{1}{R_2} = -0.0216
\]

\[
R_2 = -46.16 \text{ cm}
\]

\[
R_1 = 41.25 \text{ cm}
\]
#3 Find f# of each lens.

\[
\begin{array}{cccccc}
\Phi & 1 & 2 & 3 & 4 \\
D & 4 & 5 & 4.7 & 4
\end{array}
\]

So lenses 1 & 4 have the lowest f#.

However, for focusing rays from sun (S → D), a plano-convex lens works best (minimizes the spherical aberration). So lens 1 is the one to be used as

#2-1. Read the text & the class notes.