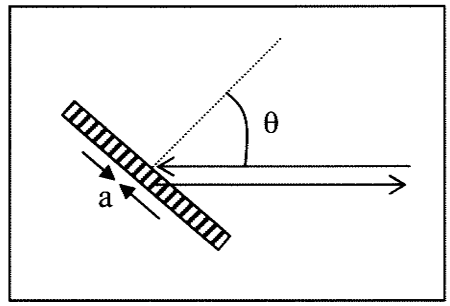
PHYC/ECE 463 Advanced Optics I
Fall 2007
Homework #11, Due Wednesday Nov. 28

1. Interference (Grating)

(a) Obtain the angle of incidence θ at which a reflection grating (grating period = *a*) retro-reflects (reflects back) the m-th order of a monochromatic incident beam at wavelength λ_0. (See figure below). *3 points*

(b) For a CO_2 laser at λ_0=10.6 μm, which of the following gratings is suitable for the above experiment: (1) 800 lines/mm, (2) 300 lines/mm, and (3) 50 lines/mm? Explain. What is (are) the order (m≠0) and the incidence angle(s) at which retro-reflection occurs? *4 points*



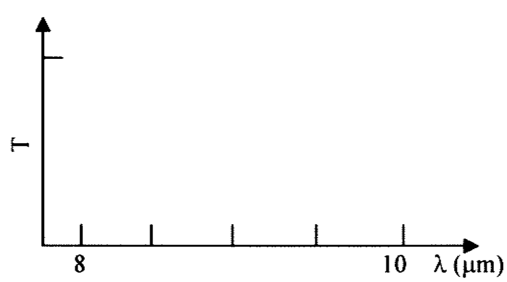
(c) In part (b), what is the highest resolution (Δλ) attainable if the CO_2 laser has a diameter of 1 cm? *3 points)*

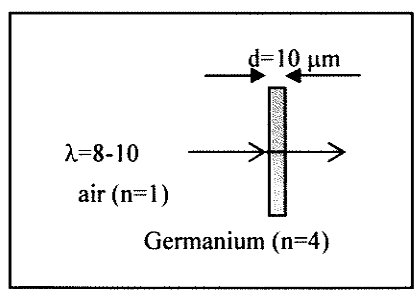
2. Interference (Fabry-Perot)

Light from an IR laser source (λ=8-10 μm) is incident on a Germanium parallel plane thin slab (d=10 μm, n=4) as shown.

(a) Plot the transmission versus wavelength (in the range of 8 to 10 μm) at normal incidence. Be quantitative and give values for T_{max}, T_{min}, Finesse, and $\Delta\lambda_{1/2}$ (assume high finesse for the latter) *(6 points)*

(b) b) Fix the wavelength at one of the transmission peaks. Can we rotate the sample by an amount Δθ such that the transmission becomes a minimum? Is this practical? *(4 points)* (Assume surface reflectivities remain constant)

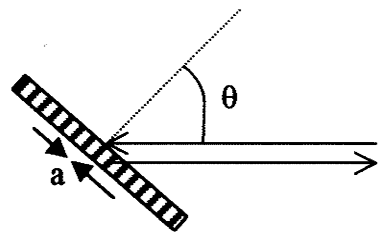




θ

1. Interference (Grating)

(a) Obtain the angle of incidence at which a reflection grating (grating period = *a*) retro-reflects (reflects back) the m-th order of a monochromatic incident beam at wavelength λ_0. (See figure below). *(10 points)*



$a(\sin\theta' - \sin\theta) = m\lambda$

retro reflector $\Rightarrow$ $\theta = -\theta'$

$-2a\sin\theta = m\lambda$

$\sin\theta = -\frac{m}{a}\frac{\lambda}{2}$

for $\theta > 0$ $m < 0$

(b) For a CO_2 laser at λ_0=10.6 μm, which of the following gratings is suitable for the above experiment: (1) 800 lines/mm, (2) 300 lines/mm, and (3) 50 lines/mm? Explain.
What is (are) the order (m) and the incidence angle(s) at which retro-reflection occurs? (Exclude m=0)
(15 points)

(1) $\sin\theta = -m\frac{10.6\times10^{-3}\times800}{2} = |-m\times8.24| > 1$ for $|m| \neq 0$ Not suitable

(2) $\sin\theta = -m\frac{10.6\times10^{-3}\times300}{2} = |-m\times1.59| > 1$ "   "

→ (3) $\sin\theta = -m\times\frac{10.6\times10^{-3}\times50}{2} = |-m\times0.265| \Rightarrow$ can be < 1 suitable

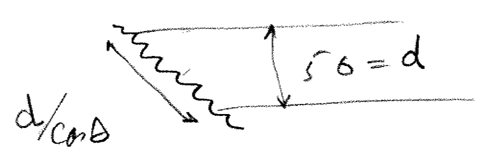
$m = -1, -2, -3$

$\theta_{-1} = \sin^{-1}(0.265) = 15.4^\circ$, $\theta_{-2} = \sin^{-1}(0.53) = 32.^\circ$, $\theta_{-3} = \sin^{-1}(.795) = 52.6^\circ$

(c) In part (b), what is the highest resolution ($\Delta\lambda$) attainable if the CO_2 laser has a diameter of 1 cm? *(10 points)*

$\frac{\lambda}{\Delta\lambda} = |m| N$ $N = \frac{d_{beam}}{a\cos\theta} = \frac{10\text{ mm}}{\frac{1}{50}\cos\theta} = \frac{500}{\cos\theta_{-3}} = 823$

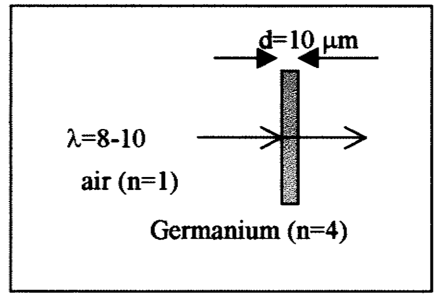
$\Delta\lambda = \frac{\lambda}{|m| N} = \frac{10.6\ \mu m}{3\times823} \approx 4.29\text{ nm}$



2. Interference (Fabry-Perot)

Light from an IR laser source (λ=8-10 μm) is incident on a Germanium parallel plane thin slab (d=10 μm, n=4) as shown.

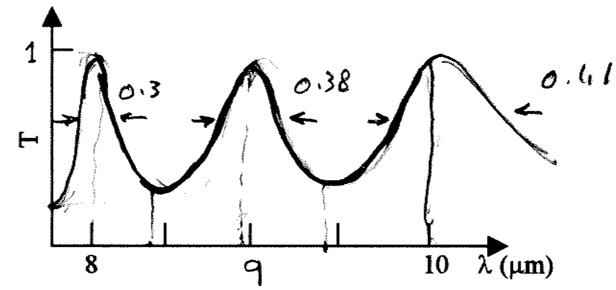
(a) Plot the transmission versus wavelength (in the range of 8 to 10 μm) at normal incidence. Be quantitative and give values for T_{max}, T_{min}, Finesse, and $\Delta\lambda_{1/2}$ (assume high finesse) *(25 points)*



$R_1 = R_2 = R = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{3}{5}\right)^2 = 0.36$

$T_{max} = 1 \qquad T_{min} = \frac{1}{1+F} = 0.22$

$F = \frac{4R}{(1-R)^2} = \frac{4 \times 0.36}{(1-0.36)^2} = 3.51$



$T = \frac{1}{1 + F\sin^2(\delta/2)}$

$\delta_{max} = \frac{4\pi}{\lambda_0} n d = 2m\pi$

$\lambda_{max} = \frac{2 \times 4 \times 10}{m}$ μm

$\lambda_{max} = \frac{80}{m}$ μm

$m = 10 \quad \lambda = 8$ μm

$m = 9 \quad \lambda = 8.88$

$m = 8 \quad \lambda = 10$

$\lambda_{min} = \frac{80}{(2m+1)/2} \qquad m = 9 \quad \lambda_{min} = 8.42, \quad m = 8 \quad \lambda_{min} = 9.41$

$\frac{\Delta\lambda_{1/2}}{\Delta\lambda_{FSR}} = \frac{\Delta\nu_{1/2}}{\Delta\nu_{FSR}} = \left(\frac{\pi\sqrt{F}}{2}\right)^{-1} = \left(\pi \times \frac{\sqrt{3.51}}{2}\right)^{-1} = 0.34$

$\Delta\lambda_{1/2} = 0.34 \times \Delta\lambda_{FSR}$

$(\Delta\lambda_{1/2})_1 = 0.34 \times 0.88$ μm

$(\Delta\lambda_{1/2})_2 = 0.34 \times 1.12$

$\Delta\lambda_{1/2} \approx 0.34$ μm

(b) Fix the wavelength at *first* one of the transmission peaks and calculate the minimum rotation angle ($\Delta\theta$) for which the transmission becomes a minimum. Explain your assumptions. *(10 points)*

$\delta = \frac{4\pi n d}{\lambda} = 2m\pi \qquad$ max $\quad \theta' = 0$

$\delta = \frac{4\pi}{\lambda} n d \cos\theta' = (2m-1)\pi \qquad$ min $\quad \theta' = \Delta\theta'$

$\cos\Delta\theta' = \frac{2m-1}{2m} \qquad$ take $m = 9 \qquad \cos\Delta\theta' = \frac{18-1}{18} \Rightarrow$ **$19.18^\circ = \Delta\theta$**

$n \sin\Delta\theta' = \sin\Delta\theta \Rightarrow \sin\Delta\theta = 1.31 > 0$

not practical.