Solution

Laser Physics I (PHY 464)

Midterm Exam I, Closed Book, Single CheatSheet, Time: 90 min.

FALL 2001

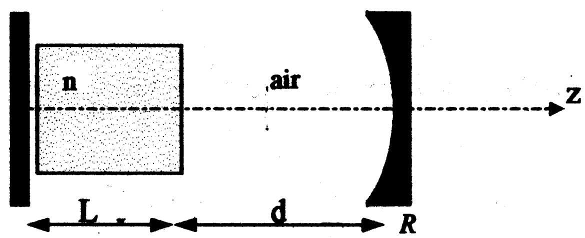
NAME ....

last *first*

grade

Please staple and return these pages with your exam.

1. Consider the laser cavity shown below constructed from a plane mirror and a concave mirror of radius R. The gain medium (having length L and index of refraction *n*) is placed adjacent to the flat mirror.



(a) Identify a unit cell and obtain the ABCD matrix for a cavity roundtrip. *(20 pts.)*

(b) Obtain the stability condition in terms of the cavity parameters *R*, d, L and n. *(5 pts.)*

(c) Identify the location of the minimum spot size and derive an expression for its magnitude (w_0). *(10 pts.)*

(d) What is the beam curvature at the concave mirror? *(3 pts.)*

(e) What is the $\Delta\nu_{FSR}$? *(7 pts.)*

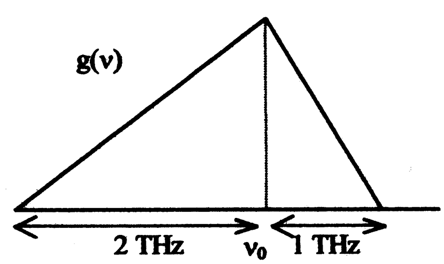
(f) What is the cavity *finesse* and *photon lifetime* if mirror reflectivities are R1, R2, and the surface reflection (for each surface) of the gain medium is R_s. (Note: we are assuming a passive cavity; i.e. no gain) *(15 pts.)*

(g) Given that Z_0 is determined from previous parts, set up the equation for finding the resonant frequency ($\nu_{q,m,p}$) of a given TEM_{mp} mode. *(10 pts.)*

2. A two level system is described by a lineshape $g(\nu)$ -shown below- and the following parameters:
Spontaneous lifetime (τ_{sp})=10 μsec
λ_0=1 μm
n=2
$N_{total}(=N_1+N_2)=10^{18}$ cm^{-3} (at thermal equilibrium all atoms are in level 1)
Degeneracy factors: $g_1=1$, $g_2=3$

(a) Find the gain (or loss) coefficient at $\nu=\nu_0$ if 20% of atoms are excited into level 2.
(15 points)

(b) Find the gain (or loss) coefficient at $\nu=\nu_0$-1THz if 80% of atoms are excited into level 2
(15 points)



Some useful formulas:

Solution for the q (at the starting point) of a cavity:

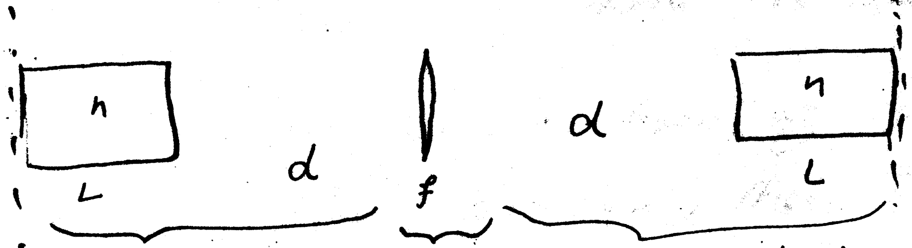
$$\frac{1}{q} = -\frac{A-D}{2B} - i\frac{\sqrt{1-\left(\frac{A+D}{2}\right)^2}}{B}$$

Longitudinal Phase of a Hermite-Gaussian mode:

$$\phi(z) = kz - (1+m+p)\tan^{-1}(z/z_0)$$

#1

(a)



$$\begin{pmatrix} 1 & \frac{L}{n}+d \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & \frac{L}{n}+d \\ 0 & 1 \end{pmatrix}$$

$$T_{RT} = \begin{pmatrix} 1 & \frac{L}{n}+d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}\begin{pmatrix} 1 & \frac{L}{n}+d \\ 0 & 1 \end{pmatrix}$$

let $D = \frac{L}{n}+d$

$$\begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & D \\ -\frac{2}{R} & 1-\frac{2D}{R} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{2D}{R} & D+D(1-\frac{2D}{R}) \\ -\frac{2}{R} & 1-\frac{2D}{R} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

(b) Since $\frac{1}{q} = \frac{1}{R} - \frac{i\lambda_0}{\pi n W^2} = \frac{A-D}{2B} - i\frac{\sqrt{1-\left(\frac{A+D}{2}\right)^2}}{B}$

$$-1 < \left(\frac{A+D}{2}\right) < 1 \Rightarrow -1 < \left(1-\frac{2D}{R}\right) < 1 \Rightarrow 0 < \left(1-\frac{D}{R}\right) < 1$$

$$\Rightarrow \boxed{0 < 1 - \frac{d+L/n}{R} < 1}$$

- Contn.

(c) location of min spot size : at flat mirror

Since $R = \infty$

Since we choose our starting point of the unit cell to be at the plane mirror, then the q parameter corresponds to that position

Note: This is verified by $\frac{1}{R} = \frac{A-D}{2B} = 0$

Thus

$$\frac{\lambda_0}{\pi W_0^2} = \frac{\sqrt{1-\left(1-\frac{2D}{R}\right)^2}}{2D\left(1-\frac{D}{R}\right)} = \frac{\sqrt{\frac{4D}{R}-\frac{4D^2}{R^2}}}{2D\left(1-\frac{D}{R}\right)} = \frac{1}{\sqrt{DR}\sqrt{1-\frac{D}{R}}}$$

$$W_0^2 = \frac{\lambda_0}{\pi}\sqrt{DR}\times\sqrt{1-\frac{D}{R}} \qquad D = d + \frac{L}{n}$$

(d) $R(\text{at } z = L+d) \equiv R$ — This is the scale of the optical Resonator

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(e) $$\Delta\nu_{FSR} = \frac{1}{T_{\text{round trip}}} = \frac{1}{\frac{L}{c/n}+\frac{d}{c}} = \frac{c}{d+nL}$$

(f) Cavity finesse $F = \frac{\pi\,(R_1R_2)^{1/4}}{1-\sqrt{R_1R_2}}$

But now the survival factor $S = R_1R_2 \times T_s^4$

$T_s = (1-R_s)$ is the transmission of each surface and the beam encounters 4 surfaces in a round trip.

Thus $$F = \frac{\pi\left(R_1R_2(1-R_s)^4\right)^{1/4}}{1-\sqrt{R_1R_2(1-R_s)^4}}, \quad \tau_{\text{Photon}} = \frac{T_{\text{roundtrip}}}{1-S} = \frac{(d+nL)/c}{1-R_1R_2T_s^4}$$

\# 1 Contin.

(g) for a round trip $2\Phi(\text{round trip}) = 2q\pi$

$$\Phi(z) = k(nL+d) - (1+m+p)\left[\tan^{-1}\frac{d+nL}{z_0}\right] = q\pi$$

$$k = \frac{2\pi}{\lambda_{q,m,p}} = \frac{2\pi\nu_{q,m,p}}{c}$$

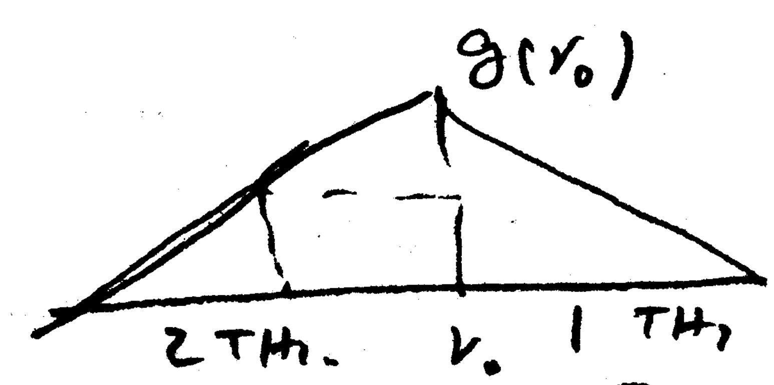
#2

$$A_{21} = \frac{1}{\tau_{sp}} = \frac{1}{10\times10^{-6}} = 10^{5}\ \text{sec}^{-1}$$

$$\lambda_0 = 1\ \mu m = 1\times10^{-4}\ cm \qquad \nu_0 = \frac{c}{\lambda_0} = 3\times10^{14}$$

$$n = 2 \qquad g_1 = 1 \qquad g_2 = 3$$

$$\sigma(\nu) = A_{21}\frac{\lambda_0^2}{8\pi n^2}g(\nu) \qquad \gamma = \sigma(\nu)\times\left(N_2 - \frac{g_2}{g_1}N_1\right)$$



Since $\int g(\nu)d\nu = 1$

$$\Rightarrow \frac{3\times g(\nu_0)}{2} = 1 \Rightarrow \boxed{g(\nu_0) = \frac{2}{3}\times10^{-12}}\ \text{sec.}$$

(a) $$\sigma(\nu_0) = 10^{5}\frac{(1\times10^{-4})^2}{8\pi\times4}\times\frac{2}{3}\cdot10^{-12} = 6.63\times10^{-18}\ cm^2$$

$$\gamma(\nu_0) = 6.63\times10^{-18}\,(0.2 - 3\times0.8)\times10^{18}$$

$$\gamma(\nu_0) = -14.6\ cm^{-1}\ \text{(absorption)}$$

$$g(\nu_0 - 2\,THz) = \frac{1}{2}g(\nu_0)$$

(b) $$\gamma(\nu_0 - 1\,THz) = \frac{6.63\times10^{-18}}{2}\times(0.8 - 3\times0.2)\times10^{18} = +0.663\ cm^{-1}$$

(gain)