**Formula Sheet**

*PHYC/ECE 464 (Laser Physics I)- University of New Mexico*

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## Hermite-Gaussian Beams

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right)H_p\left(\frac{\sqrt{2}y}{w(z)}\right)\frac{w_0}{w(z)}\exp\left(-i\frac{kr^2}{2q(z)}\right)\times\exp\left(-i\left[kz-(1+m+p)\tan^{-1}(z/z0)\right]\right)$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)}, \quad w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right), \quad R(z) = z\left(1+\frac{z_0^2}{z^2}\right), \quad z_0 = \frac{\pi n w_0^2}{\lambda_0} \quad k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0}$$

Irradiance: $I = <S> = \frac{nc\varepsilon_0}{2}E_0^{\ 2}$

Fresnel's reflectivities:

$$r_{\parallel} = \frac{n_t\cos(\theta_i) - n_i\cos(\theta_t)}{n_t\cos(\theta_i) + n_i\cos(\theta_t)} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)}$$

$$r_{\perp} = -\frac{n_i\cos(\theta_i) - n_t\cos(\theta_t)}{n_i\cos(\theta_i) + n_t\cos(\theta_t)} = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$$

Snell's Law $n_i\sin(\theta_i) = n_t\sin(\theta_t)$

Intensity (Power) reflectivity: $R=|r|^2$

Brewster angle (from 1 to 2): $\theta_B = \tan^{-1}(n_2/n_1)$

Critical angle (from 1 to 2): $\theta_c = \sin^{-1}(n_1/n_2)$

$n \rightarrow \tilde{n} = n + i\kappa$ *in n complex*

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| Lens Transformation of a Gaussian beam: $\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$ | $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ Lens-makers' formula: |

## Fabry-Perot Transmission and Reflection (with gain or loss)

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| $T(\theta,G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$ | *Finesse* $F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}}$ |
| $R(\theta,G_0) = \frac{\left(\sqrt{R_1} - G_0\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$ | *Free Spectral Range:* $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$ |
| $2\Delta\theta_{1/2} = \frac{1-G_0\sqrt{R_1R_2}}{G_0^{\ 1/2}\sqrt[4]{R_1R_2}}$ | *Photon Lifetime:* $\tau_p = \frac{\tau_{RT}}{1-R_1R_{21}} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$ |
| $\theta = kd = \frac{\omega nd}{c}$ | *General Resonance Condition: roundtrip phase change= q2π* |

**ABCD Matrices** $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ AD-BC=1 $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

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| Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$ | Dielectric interface (from $n_1$ to $n_2$) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$ | ABCD rule for Gaussian Beams $q_2 = \frac{Aq_1+B}{Cq_1+D}$ *where* | $q(z) = z + iz_0$ *or* $\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w(z)^2}$ |
| medium of length d and index $n_2$=n immersed in vacuum ($n_1$=1). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$ | Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$ | Gaussian pulse propagation (broadening) in dispersive media $\tau_p^2(z) = \tau_{p0}^2\left(1+\frac{z^2}{\ell_0^2}\right)$ where $\ell_0 = \frac{\tau_{p0}^2}{2\lvert\beta_2\rvert}$ and group velocity dispersion (GVD) $\beta_2 = \frac{\lambda^3}{2\pi c^2}\frac{d^2n}{d\lambda^2} = \frac{\lambda^2}{2\pi c}D$ | |
| Mirror with radius of curvature R $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$ | Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$ | Photon Density (Photon Number per Volume) $\frac{N_p}{V} = \frac{I}{h\nu c/n_g}$ | |

Gain in a two-level system: $\gamma(\nu) = \sigma(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right]$ Gain cross section: $\sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)$

Lineshape Normalization: $\int g(\nu)d\nu = 1$ Beer's Law: $\frac{1}{I}\frac{dI}{dz} = -\alpha(I) + \gamma(I)$

Gain or absorption saturation in a homogenously-broadened system:

$\gamma(I) = \frac{\gamma_0}{1 + I/I_s}$ or $\alpha(I) = \frac{\alpha_0}{1 + I/I_s}$ $I_s(\nu) = \frac{h\nu}{\sigma(\nu)\tau_2}$

Einstein's relation: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$ $g_2 B_{21} = g_1 B_{12}$ $\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-(E_2-E_1)/kT}$

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| *Lorentzian line shape:* $g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_h/2)^2}$ | *Doppler broadened line shape* $g(\nu) = \left(\frac{4\ln 2}{\pi}\right)^{1/2}\frac{1}{\Delta\nu_D}\exp\left[(-4\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right]$ with $\Delta\nu_D = \left(\frac{8kT\ln 2}{Mc^2}\right)^{1/2}\nu_0$ |

$\frac{dN_p}{dt} = \frac{G^2S-1}{\tau_{RT}}N_p + N_2 c\sigma$ (Photon number dynamics due to *stimulated* and *spontaneous* emission)

$S$ (survival factor)=$R_1R_2$ for a simple two mirror linear cavity, $G^2$=roundtrip gain ($\exp(2\gamma L_g)$)
Threshold condition: $SG^2=1$ (linear cavity), $SG=1$ (ring cavity)

Schawlow-Townes limit for laser linewidth: $\Delta\nu_{osc} \approx 2\pi\frac{h\nu}{P_{out}}(\Delta\nu_{1/2})^2$

At steady-state: $\gamma = \gamma_{th} = \frac{\gamma_0}{1 + I/I_s}$ (for homogenously broadened)

Inside the gain medium: $I \approx I^+ + I^- \approx 2I^+$ for a high-Q linear (standing-wave) or bidirectional ring cavity, $I \approx I^+$ for a unidirectional ring cavity.
$I_{out} = T_a..T_2 I^+$ ($T_2$ is the output coupling transmission and $T_a$… are the transmission of other optical surfaces in the path)

Q-Switching and Gain-Switching: $\Delta t_p \approx \tau_p$ (cavity photon lifetime)

Modelocking: Repetition Rate $=1/T_{rt}=2Ln_g/c$ (linear cavity), Pulsewidth: $\Delta t_p \approx > 1/\Delta\nu$

Threshold current density in a diode laser: $J_{th} = eN_{eh}^{th}d/\tau_r$

Physical Constants

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| $c=2.998 \times 10^8$ m·s$^{-1}$ | $h=6.63 \times 10^{-34}$ J·s | $e=1.6 \times 10^{-19}$ C |
| $k_B=1.380 \times 10^{-23}$ J·K$^{-1}$ | $\varepsilon_0=8.85\times10^{-12}$ F/m | $m_e=9.1 \times 10^{-31}$ kg |

**1G = $10^{-4}$ T , 1 eV = $1.602 \times 10^{-19}$ J, 1 dyne = $10^{-5}$ N, 1 erg = $10^{-7}$ J**