NONLINEAR OPTICS (PHY 555)  
Spring 2005 - Instructor: M. Sheik-Bahae  
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Homework #1, Due Thursday, Feb. 3

Problem 1. EFISH: Electric-Field Induced Second Harmonic (3 points)
Consider a centrosymmetric and isotropic material (e.g. glass) for which $\chi^{(3)}(\omega_4; \omega_3, \omega_2, \omega_1)$ is known. In an experimental arrangement (as shown in the Figure) this material is sandwiched between two parallel electrodes while an intense laser beam is propagating parallel to the electrodes.

(a) By applying a large d.c. voltage ($V$), some second harmonic generation ($2\omega$) is observed. Explain how this is possible.

(b) Assuming glass with $\chi^{(3)} \approx 10^{-14}$ esu, estimate the required voltage to produce a $\chi^{(2)}_{\text{eff}}$ equal to that of KDP ($\chi^{(2)} \approx 2 \times 10^{-9}$ esu). The electrode spacing $d=5$ mm.

(c) In the small signal regime (i.e. when the incident light intensity is very low), show that the phase of the transmitted beam is modulated by the applied voltage. Explain.

Problem 2. $\chi^{(3)}$ in non-centrosymmetric materials (5 points)
(a) Using classical electron oscillator (CEO) model derive an expression for $\chi^{(3)}(\omega_4; \omega_1, \omega_2, \omega_3)$ for a non-centrosymmetric medium. (Start with equation 1.4.7c in Boyd).

(b) Identify $\chi^{(3)}(\omega; \omega, -\omega, \omega)$ and the corresponding nonlinear refractive index $n_2(\omega)$.

(c) Obtain equivalent Miller’s rules relating $\chi^{(3)}$ to $\chi^{(1)}$, and $\chi^{(3)}$ to $\chi^{(2)}$.

(d) Compare the above results with that derived in the text (Boyd- 1.4.52) for a centrosymmetric medium. (Using $d=3$ Å and $\omega_0=1\times10^{16}$ rad/sec, estimate $\chi^{(3)}$ (esu) and compare this value with that of eqn. 1.4.56 in the text).

(e) Assuming lattice spacing, $d$, is nearly constant for most solids, find the scaling of $\chi^{(3)}$ ($\omega \rightarrow 0$) with the resonance frequency $\omega_0$.

Problem 3. Kramers-Kronig Relations (2 points)
The causality condition on the linear response $R^{(1)}(t)$ of a system can be expressed by writing

$$R^{(1)}(t) = u(t)R^{(1)}(t),$$

where $u(t)$ is the unit step function.

(a) Using Fourier analysis of the above equation derive the Kramers-Kronig (K-K) dispersion relations for $\chi^{(1)}(\omega)$.

(b) Verify the K-K relations for an RC circuit for the transfer function of the following RC circuit.