Problem 1.
(a) Show that \( d_{	ext{eff}} \), as defined by
\[
P = 4d_{	ext{eff}} E_1 E_2,
\]
is related to the \( d \) tensor via:
\[
d_{	ext{eff}} = \hat{e}_3 \cdot d \hat{e}_1 \hat{e}_2
\]
where \( \hat{e}_j \) (\( j=1,2,3 \)) is the unit vector associated with \( E_1, E_2, \) and \( P_3 \).
(b) For a given geometry, \( d_{	ext{eff}} \) is usually calculated in terms of \( d_{ij} \)'s and the angles \( \phi \) and \( \theta \) as described in the figure below. Here \( x, y \) \textit{and} \( z \) are the crystal axis and \( X, Y, \) and \( Z \) (laboratory frame) are optical propagation axes (e.g. \( k_2 \parallel k_1 \parallel Z \)).

(i) Derive expressions for \( d_{	ext{eff}} \) for a class \( 3m \) crystal (e.g. \( \text{LiNbO}_3 \)) where \( \hat{e}_1 = \hat{e}_2 = Y, \) and \( \hat{e}_3 = X \). (As we will see later on, this is called type-I condition).

(ii) Repeat the above calculation for type-II condition where \( \hat{e}_1 = Y, \hat{e}_2 = X \) and \( \hat{e}_3 = X \).

(iii) Find a geometry (i.e. \( \theta \) and \( \phi \)) that accesses the largest \( d_{ij} \) element in \( \text{LiNbO}_3 \). (see table 1.5.3 in Boyd).

Problem 2. Consider an \textit{isotropic} nonlinear material. We are concerned here with the self-phase modulation case where the frequency arguments of \( \chi^{(3)}_{ijkl} \) is of the type \( (\omega;\omega,-\omega,\omega) \) and all the other possible permutations.

(a) Show that \( \chi^{(3)}_{xxxx} = \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxx} + \chi^{(3)}_{xxyy} \)
(b) Let the incident field be represented by \( \tilde{E} = E_0 e^{-i\omega t} (\hat{x} + e^{i\phi} \hat{y}) \) where \( \phi \) is the phase difference between \( x \) and \( y \) components. Note that \( \phi = m\pi \) and \( \phi = (m+1/2)\pi \) describe the linear and circular polarization receptively. Let us now define an effective nonlinear susceptibility through:
\[
\tilde{F}^{(3)}(\omega) = \chi^{(3)}_{\text{eff}} |\tilde{E}|^2 \tilde{E}
\]

(i) Find \( \chi^{(3)}_{\text{eff}} \) as a function of the \( \chi^{(3)} \) tensor elements and the phase angle \( \phi \). Can we define such a \( \chi_{\text{eff}} \) for all values of \( \phi \) (i.e. elliptical polarization)? (Assume Kleinman Symmetry)

(ii) Find the circular/linear dichroism defined as \( \eta = \chi^{(3)}_{\text{eff}}(\phi=\pi/2)/\chi^{(3)}_{\text{eff}}(\phi=0) \)