Gap Solitons: An Introduction

Amarin Ratanavis

Department of Physics and Astronomy, University of New Mexico

May 5, 2005

Gap solitons refer to intense nonlinear optical pulse propagation in periodic variation fiber. This paper is a brief survey and updates some of recent research in gap solitons. The theory of gap solitons by D. Mills [1] has been reviewed. The first experiment demonstration of gap (Bragg) solitons by Eggleton et al [2] has been discussed.

I. Introduction

Solitons are the localized concentrations of field energy, basing on the interplay and balance of dispersive and nonlinear effects [3]. In two significant scientific advances of the 1960s: the development of the mathematical theory of solitons in 1965 and the development of the laser are the root of optical solitons [3]. In a nonlinear material, light propagates and changes the index of refraction of the medium. This leads to self-focusing. The self-focusing rivals with diffractive effects. At sufficient intensities, a soliton is the development of a structure in which diffraction and self-focusing exactly balance [3]. A typical length at which this balance is achieved varies from a few hundred meters to several kilometers [4]. An exciting soliton research area is soliton-like states, called gap solitons, which propagate in periodic media. The gap solitons can propagate at any speed between zero to the velocity of light theoretically [3][4].

This paper is a brief survey of significant researches on optical gap solitons from the first sight in 1987 (Chen and Mills) until recent work.

II. Theory of Gap Solitons in Nonlinear Periodic Structures [1]

In this section, we explore a classic work of Mills [1], which summarize the original work of Chen, and Mills [5]. We try to illustrate concept of gap solitons in nonlinear periodic structures; however, we cannot present this work in full detail. Some equations are unavoidable but we hope to give them as fairly referable on the subject as we can.

A super lattice consists of a periodic array of dielectric films. The structure can be imagined to … ABABAB… where the symbol A, B refers to a thin film of thickness $d_A$, $d_B$ with dielectric constant $\varepsilon_A$, $\varepsilon_B$ respectively. The spatial period (d) is defined to be equal $d_A + d_B$. However, before we consider the optical nonlinearities of periodic dielectric structures, we need to review and understand the nature of wave propagation in the linear limit. Now, we turn our attention to simple plane wave propagation parallel to the z direction. The electric field $E(z, t)$ with frequency $\omega$ obeys the linear Maxwell’s equation and leads to

$$\frac{d^2}{dz^2} E(z) + \frac{\omega^2}{c^2} \varepsilon(z)E(z) = 0$$

(1)

Where $\varepsilon(z) = \varepsilon_A$ when we are in a film of type A and $\varepsilon(z) = \varepsilon_B$ when we are in a film of type B

Here $\varepsilon_A$ and $\varepsilon_B$ are assumed to be real. Another word $\varepsilon(z)$ is any periodic function with period d. Any solution of $E(z)$ in this periodic structure must yield the intensity of wave, $|E(z)|^2$ to be identical in each unit cell. Therefore, $E(z)$ and $E(z + d)$ can differ only a phase factor,

$$E(z + d) = \exp(i\phi)E(z)$$

(2)
Assume a superlattice with a large number of unit cells \( N \) and require \( E(z) \) to be identical for the choice \( n = 1 \) and \( n = N \). The interval \( 0 < z < d \) and \( Nd < z < (N+1)d \) are identical, then

\[ \exp(iN\phi) = 1 \]

Where \( \phi = \frac{2\pi m}{N} \); \( m = 0,1,2,\ldots(N-1) \)

Define \( K_m = \frac{2\pi m}{L} \) and note that \( K \) is some called as Bloch wave number ; \( L = Nd \), the solution can be written as

\[ E_{km}(z) = \exp(iK_mX_n)\phi_{km}(z) \quad , nd < z < (n+1)d \]

Where \( \phi_{km}(z) \) is perfectly periodic function with period \( d \), and \( X_n = nd \).

In the limit \( L \rightarrow \infty \), \( K \) may be viewed as a continuous variable. Choose \( K \) to lie in the parameter regime \( \frac{-\pi}{d} < K < \frac{\pi}{d} \). Then the solution, as \( L \rightarrow \infty \) is shown

\[ E_k(z) = \exp(iKX_n)\phi_k(z) \quad , nd < z < (n+1)d \]

Where \( \phi_k(z) = \phi(z + md) \), i.e. \( \phi(z) \) is periodic in \( z \). This object is a Bloch function [1].

The most general form for \( \phi_k(z) \) is

\[ \phi_k(z) = \begin{cases} \frac{1}{A}e^{ikAz} + \frac{1}{B}e^{-ikBz} & 0 < z < d_A \\ \frac{1}{A}e^{ikAz} + \frac{1}{B}e^{-ikBz} & d_A < z < d_A + d_B \end{cases} \]

Where \( k_A = \frac{\omega\sqrt{\varepsilon_A}}{c} \) and \( k_B = \frac{\omega\sqrt{\varepsilon_B}}{c} \).

Apply boundary conditions by requiring \( \phi_k(z) \) and \( d\phi_k/dz \) be continuous at \( z = d_A \) and \( \phi_k(z = 0) = \exp(iKd)\phi_k(z = d) \) similarly for \( d\phi_k/dz \). One can find [1],

\[ \cos(Kd) = \cos(k_Ad_A)\cos(k_Bd_B) - \frac{1}{2} \left( \frac{\varepsilon_A}{\varepsilon_B} \right)^{1/2} + \left( \frac{\varepsilon_B}{\varepsilon_A} \right)^{1/2} \sin(k_Ad_A)\sin(k_Bd_B) \]

To proceed, we define

\[ \cos(Kd) = \Phi(\omega) \]

In the limit of very low frequencies, \( k_Ad_A << 1, k_Bd_B << 1 \), we have the solution with the wave vector \( K \) is real. As \( \omega \) increases, we have frequency bands where \( \Phi(\omega) > +1 \) or \( \Phi(\omega) < -1 \).

When \( |\Phi(\omega)| > 1 \), there is no solution for the dispersion solution for real value of \( K \), which is forbidden gaps in the dispersion relation. This is shown in Fig.1 for the forbidden gaps. The calculations assume \( \varepsilon_A = 2.25, \varepsilon_B = 4 \) and \( d_A = d_B = d/2 \). In the regions at \( |\Phi(\omega)| > 1 \), there are the gaps in the dispersion relation. The propagation electromagnetic field whose frequency lies within the forbidden gap must decay to zero exponentially inside the structure. Another word, since no energy can be propagated through the structure, the reflectivity of the super lattice must be unity.

Fig. 1. The function \( \Phi(\omega) \) as a function of the variable \( \omega d / c \) (The figure is reproduced from [1])
We are now to turn to the effect of optical nonlinearities on electromagnetic propagating in the periodic structures. We examine the equations for solutions of the form.

\[ E(z, t) = E(z) e^{-i\omega t} + c.c. \]

The nonlinear Maxwell’s equation yields,

\[
\frac{d^2 E(z)}{dz^2} + \frac{\omega^2}{c^2} \varepsilon(z) E(z) + \frac{12\pi \omega^2}{c^2} \chi^{(3)}(z) |E(z)|^2 E(z) = 0
\]

(8)

The dielectric constant \( \varepsilon(z) \) has a spatial modulation of sinusoidal form.

\[ \varepsilon(z) = \varepsilon + \Delta \varepsilon \cos \left( \frac{2\pi}{d} z \right) \]

(9)

Within the gap, \( \omega_- < \omega < \omega_+ \), the solutions of the gap solitons could be defined as,

\[ E_+(z) = f(z) e^{i\phi(z)} \]

\[ E_-(z) = f(z) e^{-i\phi(z)} \]

(10)

Full details of derivation of gap soliton solutions had been shown by Mills [1] but for the completeness we will show here only the results.

The type I, gap solitons exist only when \( \chi^{(3)} < 0 \). For this case, we have [1]

\[ \phi_1(z) = n\pi - \tan^{-1} \left( \frac{(\omega^2 - \omega_0^2)^{1/2}}{(\omega_+^2 - \omega_0^2)^{1/2}} \tanh \left( \frac{z}{d(\omega)} \right) \right) \]

\[ f_1^2(z) = \frac{\varepsilon}{8\pi \chi^{(3)}} \left( \frac{\omega^2 - \omega_0^2}{\omega_0^2} \right) \frac{\sec h^2 (z / d(\omega))}{1 + \left( \frac{\omega_+^2 - \omega_0^2}{\omega_+^2 - \omega_0^2} \right) \tanh^2 \left( \frac{z}{d(\omega)} \right)} \]

(11)

The type II gap solitons exist when \( \chi^{(3)} > 0 \). We have, for this solution [1]

\[ \phi_2(z) = \left( n + \frac{1}{2} \right)\pi + \tan^{-1} \left( \frac{(\omega_+^2 - \omega_0^2)^{1/2}}{(\omega_0^2 - \omega_-^2)^{1/2}} \tanh \left( \frac{z}{d(\omega)} \right) \right) \]

\[ f_2^2(z) = \frac{\varepsilon}{8\pi \chi^{(3)}} \left( \frac{\omega^2 - \omega_0^2}{\omega_0^2} \right) \frac{\sec h^2 (z / d(\omega))}{1 + \left( \frac{\omega_+^2 - \omega_0^2}{\omega_+^2 - \omega_0^2} \right) \tanh^2 \left( \frac{z}{d(\omega)} \right)} \]

(12)

Where \( d(\omega) \) is the length, which diverges as \( \omega \) approaches either \( \omega_+ \) or \( \omega_- \), [1]

\[ d(\omega) = \frac{2d}{\pi} \left( \frac{\omega^2 - \omega_0^2}{(\omega_+^2 - \omega_0^2)^{1/2}} \right) \frac{\omega_0^2}{(\omega_+^2 - \omega^2)^{1/2}} \]

These formulae are rather complex. However, the physical picture associated with the gap soliton solutions is the following: In a periodic media, the dispersion relation for electromagnetic field, which relates the frequency \( \omega \) to the Bloch wave number \( K \), is divided into bands and separated by gaps in which propagating fields are not allowed. If the Kerr nonlinearity is included with a high-power beam having a frequency located inside a gap. It becomes possible to locally tune this photonic band gap and allows a propagation of localized concentration light pulses; i.e., gap soliton, inside the band gap.
III. Experiments of Gap solitons [2]

This section is devoted to the first experiment that demonstrated the observation of gap solitons in 1996. This observation of nonlinear propagation effects in fiber Bragg grating has been reported by Eggleton et al [2]. The results were nonlinear optical pulse compression and soliton propagation. Since the solitons occur at frequencies near the photonic band gap of the grating, they have been called as Bragg grating solitons. These solitons exist due to the balancing between self-phase modulation (SPM) and the dispersion associated with the periodic variation in the index of refraction. The grating transmission has been shown in Fig. 2.

The grating length is 55 mm. Note that the dashed curve is the power spectrum of the incident pulse. The magnitude of the average refractive index change was $\Delta n = 3 \times 10^{-4}$. The schematic diagram of experiment is shown in Fig. 3. In the experiment, pulses generated by a mode-locked and Q-switched YLF laser are compressed by a factor of about 5. The result has been shown in Fig. 4. The compression is intensity dependence and due to a combination of the negative dispersion associated with the grating and the phase shift associated with the Kerr nonlinearity.

Fig. 2. The transmission spectrum of the fiber grating (The figure is reproduced from [2])

Fig. 3. Schematic diagram of experiment (The figure is reproduced from [2])

Fig. 4. Transmitted pulses having FWHM $\approx 80$ ps when the grating is tuned far from the resonance and the compressed pulse having FWHM $\approx 15$ ps, when grating is tuned at 1052.6 nm (close to the gap). (The figure is reproduced from [2])
These results can be explained qualitatively. The fiber Bragg grating couples forward and backward propagating electric fields. As we have learned, frequencies outside a photonic band gap can propagate. However frequencies inside the gap are reflected. In the case of Bragg grating solitons, a transform limited pulse tuned to a frequency just above the upper band edge of the photonic band gap [2]. In this close to the gap edge, the grating is strongly dispersive but the reflectivity is small. At low intensities, the spectrum of the transmitted pulse will be broadened upon propagation through the grating but in the nonlinear regime, SPM, produces new frequency component that are redshifted near leading edge and blueshifted near the trailing edge [6]. However, in the regions of anomalous dispersion, the rear of the pulse (with high frequency) travels faster than the front of the pulse (with low frequency). Therefore the incident pulse width will be compressed. As the interaction continues, the pulse shape adjusts itself until the dispersion and SPM in balance [2]. This will form Bragg grating solitons.

IV. Motivation of Gap Solitons

Optical gap solitons first were well known as a theoretical concept in 1987 but it was not until nine years later that the first experimental succeeded. Propagating gap solitons have been demonstrated in fiber gratings and in ridge waveguide grating but stationary gap solitons have not been observed yet. Gap solitons research today may be assumed to be Bragg grating solitons because of the experimental difficulty to excite gap solitons in the high reflectivity of the periodic medium in the gap [7]. Therefore the gap solitons may be referred to a class of Bragg grating solitons. The suggestion to observe the stationary gap soliton has been reported by D. Mandelik et al. following [7]: In an ideal experiment for observing stationary gap solitons, one would send a high-power pulse inside the nonlinear medium at the mid gap frequency. The combination of two counterpropagating components which are its shape matching of gap solitons and then turn on the periodic structure leads in a localized, in mobile, solitons. Unfortunately, the observing stationary temporal gap solitons never succeed so far [7].

However, gap soliton is a promising research for future applications for example, nonlinear optical switches, soliton lasers, pulse compressor, optical buffers, etc [4]. We hope that this exciting area will be continued to be a great development of optical sciences, physics and mathematics problems.

V. Conclusions

The purpose of this paper is to give a review and update some of the recent research on gap soliton. This article is such a very small particle compared to gap soliton problems but my points are try to understand physical concept of gap solitons and convince the reader of how exciting this area to be a great impact science in the future.

References