All-fiber Optical Parametric Oscillator

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Abstract—All-fiber optical parametric oscillator (FOPO) has received considerable attention throughout these years. In this paper, we review the basic theory—four wave mixing (FWM) in the optical fibers, which leads to the parametric amplification. Phase matching condition is then discussed, with emphasis on the phase matching near the zero dispersion wavelength. In the end, recent developments on FOPO are introduced, with two reports on the demonstration of cw FOPOs.

Index Terms—fiber optical parametric oscillator, parametric amplification, phase matching
I. Introduction

Parametric amplification is a well-known phenomenon in materials providing $\chi^{(2)}$ nonlinearity and optical parametric oscillators that use $\chi^{(2)}$ nonlinear crystals have become widely available tools for use in optical research and development. However, parametric amplification and oscillation can also be obtained in optical fibers exploring the $\chi^{(3)}$ nonlinearity. In FOPOs, crystals are replaced with optical fibers and the configurations can be made totally fiber integrated, virtually eliminating the need for alignment and allowing for increased compactness [1]. Thus, it has become a promising alternative to solid-state lasers. Since the nonlinearity of commonly available fibers is fairly low, obtaining sufficient gain for making a parametric oscillator with such fibers was difficult for a long time. Recently, with the availability of highly-nonlinear fibers and new high power light sources, interest has been increasing in such FOPOs.

II. Theory[2]

Parametric amplification with $\chi^{(3)}$ nonlinearity relies on highly efficient four wave mixing (FWM) phenomenon. We will describe it from three view angles.

In an intuitive approach, the non-degenerated process starts with two waves at frequencies $\omega_1$ and $\omega_2$ that co-propagate together through the fiber. As they propagate, they will continuously beat with each other. The intensity modulated beat note at frequency $\omega_2 - \omega_1$ will modulate the intensity dependent refractive index of the fiber. When a third wave at frequency $\omega_3$ is added, it will become phase modulated with frequency $\omega_2 - \omega_1$, due to the modulated $n$. From the phase modulation, the wave at $\omega_3$ will develop sidebands at the frequencies $\omega_3 \pm (\omega_2 - \omega_1)$. The amplitude of the sidebands will be proportional to the amplitude of the signal at $\omega_3$. In the same way, $\omega_3$ will beat with $\omega_1$ and phase modulate $\omega_2$, which will
generate sidebands at $\omega_2 \pm (\omega_3 - \omega_1)$, where $\omega_2 + (\omega_3 - \omega_1)$ will coincide with the previously mentioned $\omega_3 + (\omega_2 - \omega_1)$. It should be noted that from a FWM process including three incident waves, nine new frequencies will be generated. Fig. 1 shows all nine frequencies.

It also shows that some FWM-products will overlap with the signal frequency. These products will result in a gain for the signal, i.e., provide parametric amplification. In general, the remaining weaker frequencies are usually neglected except the frequency component at $\omega_i = \omega_3 + (\omega_2 - \omega_1) = \omega_2 + (\omega_3 - \omega_1)$. The two overlapping components are referred to as generated idler.

In the degenerated case with only one pump, $\omega_2$ and $\omega_3$ will coincide. For the rest of the paper, we will focus on the degenerated case including one pump at $\omega_p$, one signal at $\omega_s$ and one idler at $\omega_i$. And also we assume the pump power is high enough to be un-depleted. Fig. 2 shows the parametric amplification in the degenerated case. In this figure, the wave at frequency $2\omega_p - \omega_s$ is the idler and it can be noticed that the idler and the signal are positioned symmetrical to the pump. Another wave at frequency $2\omega_s - \omega_p$ is weak enough to be neglected.
Parametric amplification can be viewed from a quantum mechanical picture. Here, the degenerated parametric amplification is manifested as the conversion of two pump photons at frequency $\omega_p$ to a signal and an idler photon at frequencies $\omega_s$ and $\omega_i$. The conversion needs to satisfy the energy conservation and the quantum-mechanical photon momentum conservation relation, which is here interpreted as phase-matching condition.

From an electromagnetic point of view, we may consider the interaction of three stationary co-polarized waves at angular frequencies $\omega_p$, $\omega_s$ and $\omega_i$ characterized by the slowly varying electric fields with complex amplitudes $A_p$, $A_s$ and $A_i$, respectively. This theory about FWM in optical fibers has been explored fully by Stolen and Bjorkholm[3] and is summarized along with experimental results by Agrawal[4]. One obtains the following coupled amplitude equations that describe FWM in the fiber[5]:

$$\frac{\partial A_p}{\partial z} = -\frac{\alpha}{2} A_p + i\gamma |A_p|^2 A_p$$ \hspace{1cm} (1-a)

$$\frac{\partial A_{s(i)}}{\partial z} = -\frac{\alpha}{2} A_{s(i)} + i\gamma [2 |A_s|^2 A_{s(i)} + A_p^2 A_{s(i)}^* \exp(-i\Delta\beta z)]$$ \hspace{1cm} (1-b)

In Eqs. (1-a) and (1-b), $\alpha$ is the attenuation coefficient of the fiber according to $P(z) = P(0) \exp(-\alpha z)$, where $P$ is power and $z$ is the propagation distance. $\Delta\beta$ is the linear phase mismatch, which is given by $\Delta\beta = \beta(\omega_s) + \beta(\omega_i) - 2\beta(\omega_p)$. The
nonlinear coefficient, \( \gamma \), is related to the nonlinear refractive index, \( n_2 \), by
\[ \gamma = n_2 \omega / A_{c_{\text{eff}}} \]
where \( A_{c_{\text{eff}}} \) is the effective mode area of the fiber. We have assumed here the pump is much stronger than the input signal. Furthermore, the frequencies are assumed to be similar such that \( \gamma \) are equal for the three light waves.

By rewriting Eqs. (1) in terms of powers and phases of the waves, insight can be gained. Let \( \phi_j(z) \), where
\[ A_j(z) = \sqrt{P_j} \exp(i\phi_j) \]
\[ \frac{dP_j}{dz} = 2\gamma(P_p^2 P_s P_i)^{1/2} \sin \theta \]  \hfill (2-a)
\[ \frac{dP_i}{dz} = 2\gamma(P_p^2 P_s P_i)^{1/2} \sin \theta \]  \hfill (2-b)
\[ \frac{d\theta}{dz} = \Delta \beta + \gamma(2P_p - P_s - P_i) + \gamma(2(P_p P_i / P_s)^{1/2} - 4(P_p P_i)^{1/2}) \cos \theta \]  \hfill (2-c)

Here, \( \theta(z) \) describes the relative phase difference between the four involved light waves.
\[ \theta(z) = \Delta \beta z + 2\phi_s(z) - \phi_i(z) - \phi_p(z) \]  \hfill (3)

As can be observed from (2-a)-(2-c), by controlling the phase relation, \( \theta \), we have the opportunity to control the direction of the power flow from the pump to the signal and the idler (\( \theta = \pi / 2 \), parametric amplification) or from the signal and the idler to the pump (\( \theta = -\pi / 2 \), parametric attenuation). For the general application when the pump is very intense and the idler is initially zero, we have \( \theta = \pi / 2 \) at the fiber input port [6]. Thus, if we somehow satisfy the condition
\[ \Delta \beta + \gamma(2P_p - P_s - P_i) = 0 \]  \hfill (4)
\( \theta \) will remain \( \pi / 2 \) along the fiber and both the signal and the idler will grow exponentially.

### III. Phase Matching

As discussed above, only when (4) is satisfied, can parametric amplification happen effectively. Thus, this is just the phase matching condition for parametric amplification. When it is operating in an un-depleted mode \( P_p \gg P_i \),
\[ \Delta \beta + \gamma (2P_p - P_s - P_i) \approx \Delta \beta + 2\gamma P_p = \kappa \]  \hspace{1cm} (5)

The phase mismatch parameter \( \kappa \) is first introduced by Stolen and Bjorkholm in [3]. We may see that it contains both the linear phase mismatch and the nonlinear phase mismatch.

In order to achieve phase matching, a lot of methods are used, such as using fiber modes, Birefringence matching, or phase matching near the zero dispersion wavelength[3]. The last technique is the most widely used.

Expanding \( \beta(\omega) \) in Taylor series around the zero-dispersion frequency \( \omega_0 \), (\( \beta_2(\omega_0) = 0 \)), the wavelength dependent part, \( \Delta \beta \) of the phase mismatch parameter \( \kappa \) can be rewritten as

\[ \Delta \beta = \beta_3(\omega_p - \omega_0)(\omega_p - \omega_i)^2 \] \hspace{1cm} (6)

Here, \( \beta_2 \) and \( \beta_3 \) is the second and third derivative of the propagating constant \( \beta(\omega) \) at \( \omega_0 \). (6) may be transformed to the more generally used wavelength domain

\[ \Delta \beta = \beta(\omega_p) + \beta(\omega_i) - 2\beta(\omega_p) = -\frac{2\pi c}{\lambda_0^2} \frac{dD}{d\lambda} (\lambda_p - \lambda_0)(\lambda_p - \lambda_i)^2 \] \hspace{1cm} (7)

Here, \( D = \frac{-2\pi c}{\lambda_0^2} \beta_2 \), and \( dD/d\lambda \) is the slope of the dispersion at the zero-dispersion wavelength.

Now, the phase matching condition can be written as

\[ 2\gamma P_p = -\Delta \beta \propto (\lambda_p - \lambda_0)(\lambda_p - \lambda_i)^2 \] \hspace{1cm} (8)

By positioning the pump wavelength in the anomalous dispersion regime \( (\lambda_p > \lambda_0) \), it is possible to compensate for the nonlinear phase mismatch \( 2\gamma P_p \) by the linear phase mismatch \( \Delta \beta \). Or, we can say, the parametric process establishes a balance between group velocity dispersion and the nonlinear Kerr-effect.
IV. Recent Development

Since the first FOPO was reported more than a decade ago[7], it has been attracting much interest. However, to date FOPOs have presented two major limitations: As the nonlinearity of conventional fibers is considerably lower than that of crystals, most FOPOs reported used pulsed pumps such that high peak pump powers could be achieved and the parametric threshold reached; In addition, the phase matching between the pump and the signal waves is achieved with the pump near the fiber’s zero-dispersion wavelength, which limits the operation of FOPOs to wavelength longer than 1300nm, where the zero dispersion of silica is located[1].

Recently, FOPO configurations were reported that somewhat overcome one or both of the abovementioned limitations.

In one experiment[8], a cw FOPO was demonstrated at ~1550nm through the use of fiber with a small core diameter that resulted in increased pump intensities and therefore in a decreased parametric threshold.

Last year, A team from Imperial College UK and the Danish firm Crystal Fibre claims to have made the first continuous wave (cw) all-fiber optical parametric oscillator (OPO) using a holey fiber[1]. The oscillator they made operates at 1550nm and can yield an oscillation parametric signal that consists of a single line with a 30 dB extinction ratio and a 10-pm linewidth or that consists of multiple lines. The source reaches threshold for a pump power of 1.28W and saturates for pump powers in excess of ~1.6W. Fig. 3 shows their experimental configuration.
Fig. 3 Experimental configuration of the all-fiber parametric oscillator using holey fiber
References