Nonlinear Optics (PHYS 555)
Spring 2005

Midterm Exam I, Open Text-Book Only, Closed Notes
Time: 1.5-2 hour

+SOLUTION

NAME ....
last *first*

Grade

Please staple and return this page with your exam.

Problem 1. SHG: A laser diode operating at a wavelength of 980 nm is to be frequency doubled to generate blue light using a lithium niobate ($LiNbO_3$) crystal. During this SHG experiment, the temperature of the crystal can be adjusted from 23 ^oC to 200 ^oC using a special oven.

The temperature dependent refractive index for $LiNbO_3$ is approximated here as:

$$n_{o,e}(\lambda,T) \approx n_{o,e}(\lambda) + \gamma_{o,e}(T - 23^o)$$

where $n_{o,e}(\lambda)$ are given in Figure-1 (next page), $\gamma_o \approx +1.0\times10^{-5}$ and $\gamma_e \approx +6.7\times10^{-5}$.

(a) Keeping the temperature fixed at room temperature ($T=23^o$), find the type-I phase-matching condition θ_m.

(12 points)

(b) Estimate the maximum useful length of the crystal if a beam spot size of 100 μm is used.

(8 points)

(c) What is the maximum d_{eff} that can be achieved in part (a)?
[You may *use equations 1.5.30 and table 1.5.3 in Boyd, 2nd ed.*]

(10 points)

(d) Find the type-I phase matching condition (θ_m and T_m) that eliminates walk-off and optimizes the efficiency for a long crystal.

(12 points)

(e) What is the temperature range ΔT around T_m for which SHG remains efficient? Take L=5 cm.

(8 points)

(f) The group indices for both axis are given in Figure-2 (Temperature dependence is negligible). Estimate the shortest pulsewidth at 980 nm that can be doubled without distortion in a 5 cm long crystal?

(10 points)

(g) Describe a method to utilize the d_{33} in $LiNbO_3$ for the above experiment. Describe (quantitatively) the geometrical structure needed to achieve phase matching. In the low depletion limit, what will be the gain in conversion efficacy compared to part (d)?

(15 points)

Figure-1

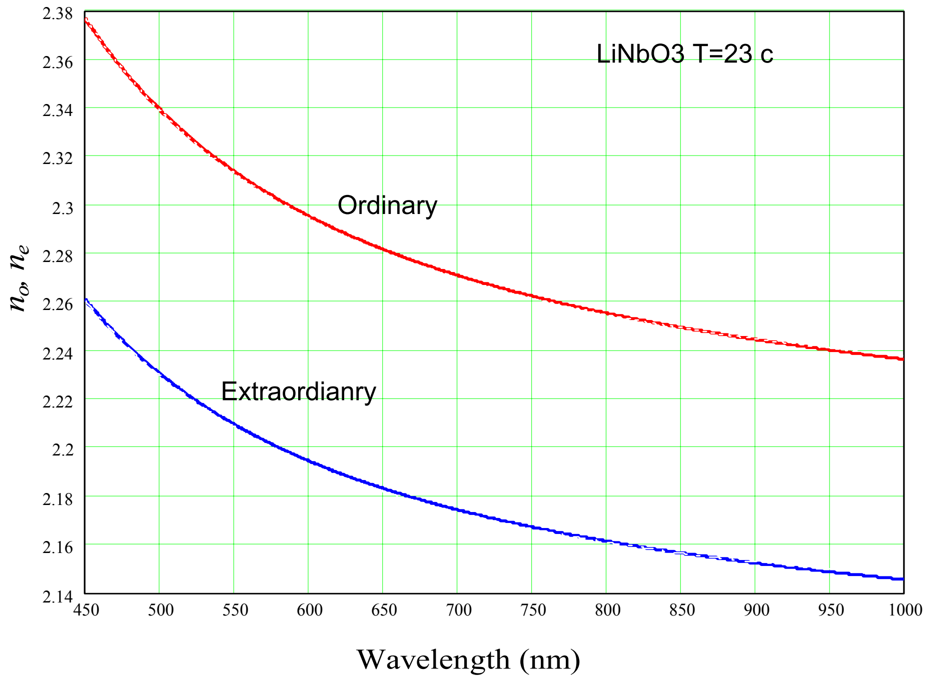
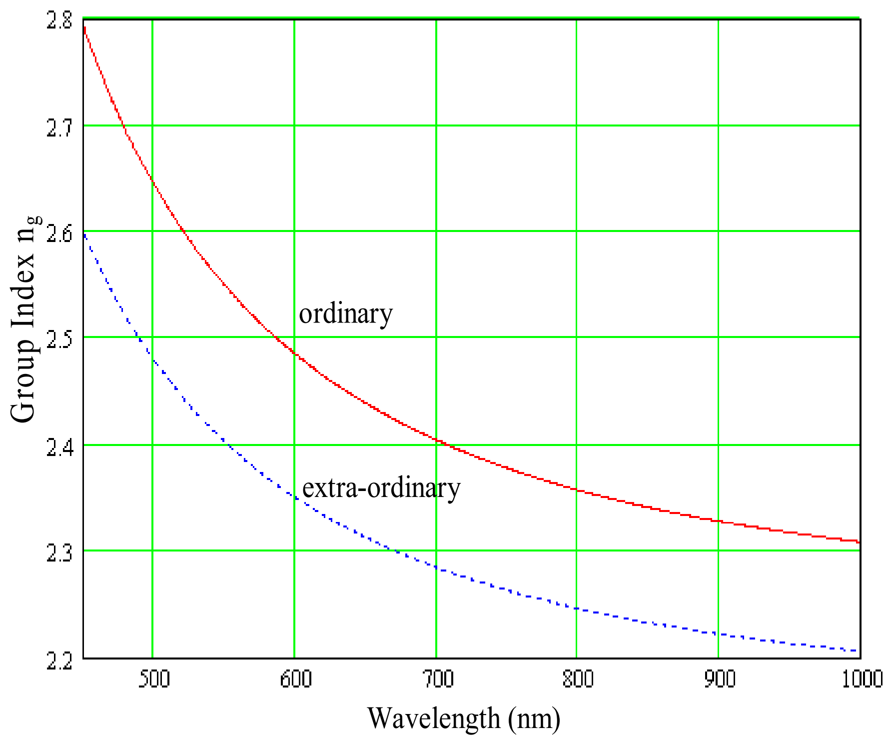


Figure-2

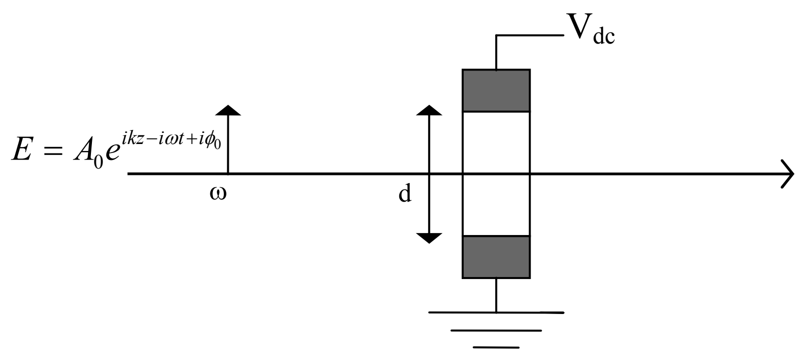


$v_g = c/n_g$

Walk-off angle for type-I: $\rho \approx \dfrac{n_o(2\omega) - n_e(2\omega)}{n_e(2\omega)} \sin(2\theta_m)$

Problem 2. Pockel's Effect:

A 2nd order nonlinear crystal with a known $\chi^{(2)}$, refractive index n_0 and a thickness L is used as an electro-optic modulator as shown below. Here a DC voltage (V_{dc}) is applied across two transverse electrodes (separated by d).



(a) Ignoring anisotropy and tensor properties, show that the phase of the transmitted electric field will be modulated according to:

$$\Delta\phi(V) = \kappa V_{dc}$$ What is κ? (20 points)

(b) Recall that from the CEO model, $\chi^{(2)}$ of a non-centrosymmetric crystal can be derived as:

$$\chi^{(2)}(\omega_3 = \omega_1 + \omega_2) = \frac{N(e^3/m^2)a}{D(\omega_1)D(\omega_2)D(\omega_1+\omega_2)}$$

Write κ in terms of the parameters in the right-hand-side of the above expression.

(5 point)

Problem-1 (Solution)

1-(a)

At T=23

$n_o(\lambda_1)$=2.238, $n_e(\lambda_1)$=2.147, $n_o(\lambda_2)$=2.346, $n_e(\lambda 2)$=2.235

Condition for type-I SHG critical phase matching in negative BR crystal: $n_e(2\omega,\theta)=n_o(\omega)$.

$$\theta(T) := \operatorname{asin}\left[\sqrt{\frac{\left(\frac{1}{no(\lambda 1,T)}\right)^2-\left(\frac{1}{no(\lambda 2,T)}\right)^2}{\left(\frac{1}{ne(\lambda 2,T)}\right)^2-\left(\frac{1}{no(\lambda 2,T)}\right)^2}}\right]$$

θ_m(T=23C)=81

1-(b)

$$\rho(T) := \left(\frac{no(\lambda 2,T)-ne(\lambda 2,T)}{no(\lambda 1,T)}\right)\cdot\sin(2\cdot\theta(T))$$

ρ(T=23)=0.015

$L_a \approx w_0/\rho = 100*10^{-4}/.015=0.7$ cmr

Recall: HW#3 –Problems 3&4

1-(c)

$d_{eff}=d_{31}\sin(\theta)-d_{22}\cos(\theta)\sin(3\phi)=14*\sin(81)-7.4*\cos(81)*\sin(3\phi)$

We rotate the crystal azimuthally to set ϕ either to $-\pi/6$ or $+\pi/2$ to maximize d_{eff}=14.95 esu.

Recall: HW#2 –Problem 1

1-(d)

$\theta_m=90^o$ (non critical phase-matching, $L_a\to\infty$)

$n_o(\lambda_1)+\gamma_o(T_m-23)=n_e(\lambda_2)+\gamma_e(T_m-23)$

$T_m=23+(n_o(\lambda_1)-n_e(\lambda_2))/(\gamma_e-\gamma_o)=23+(2.238-2.235)/5.7\times10^{-5}$

$T_m\approx75$ C

Recall: HW#3 –Problems 3&4

1-(e)

$\Delta k=(4\pi/\lambda_1)\times[n_o(\lambda_1,T)-n_e(\lambda_2,T)]$

$\Delta(kL)=\pi=\Delta T\times[\partial(kL)/\partial T]_{T=Tm}=\Delta T\times(4\pi L/\lambda_1)\times(\gamma_o-\gamma_e)$

$\Delta T=|\lambda_1/2L(\gamma_o-\gamma_e)|=980\times10^{-7}/(10\times5.7\times10^{-5})=0.17$ C

1-(f) $\Delta t=L(\Delta(1/v_g))=L*(n_{g,e}(2\omega)-n_{g,o}(\omega))/c\approx30$ ps

Recall: HW#4 –Problem 1

1-(g) $E_1||E_2||$ c-axis (extra-ordinary) See HW#2. Then $d_{eff}=d_{33}$.

Quasi-Phase-Matching (QPM) using periodic polling is the answer.

Period: $\Lambda=2*l_c=\lambda_1/(n_e(\lambda_1,T)-n_e(\lambda_2,T))=0.980\mu m/|2.147-2.235|=11.14\ \mu m$

$d_{eff}=d_{33}*2/\pi$

Efficiency ratio= $(d_{33}*2/\pi)^2/d_{31}^2=(98*2/\pi*14)^2\approx20$

Problem-2 (Solution)

2.(a)

Recall: HW#1 –Problem 1

$P(\omega)=\chi^{(1)}E(\omega)+2\chi^{(2)}E(\omega)E(0)=(\chi^{(1)}+2\chi^{(2)}E(0))E(\omega)=\chi^{(1)}_{eff}E(\omega)$

$\chi^{(1)}_{eff}=\chi^{(1)}+2\chi^{(2)}E(0)=\chi^{(1)}+2\chi^{(2)}V_{dc}/d$

Index of refraction is defined as $n_0=(1+4\pi\chi^{(1)})^{1/2}$

$n(V)=(1+4\pi\chi_{eff}^{(1)})^{1/2}=(n_0^2+8\pi\chi^{(2)}V_{dc}/d)^{1/2}\approx n_0+4\pi\chi^{(2)}V_{dc}/n_0d=n_0+\Delta n(V)$

$\Delta\phi=2\pi L\Delta n/\lambda=8\pi^2\chi^{(2)}V_{dc}L/n_0\lambda d=\kappa V_{dc}$ Thus: $\boldsymbol{\kappa=8\pi^2\chi^{(2)}L/n_0\lambda d}$

2.(b)

$$\kappa=\frac{8\pi^2\chi^{(2)}(\omega=\omega+0)L}{n_0 d\lambda}=\frac{8\pi^2 N(e^3/m^2)aL}{n_0 d\lambda D(0)D(\omega)^2}$$

Exam Statistics:

Blue bars represent original score of each student.
The average was initially 64.
To raise the average to 74, each score was raised by 10 points (green bar).