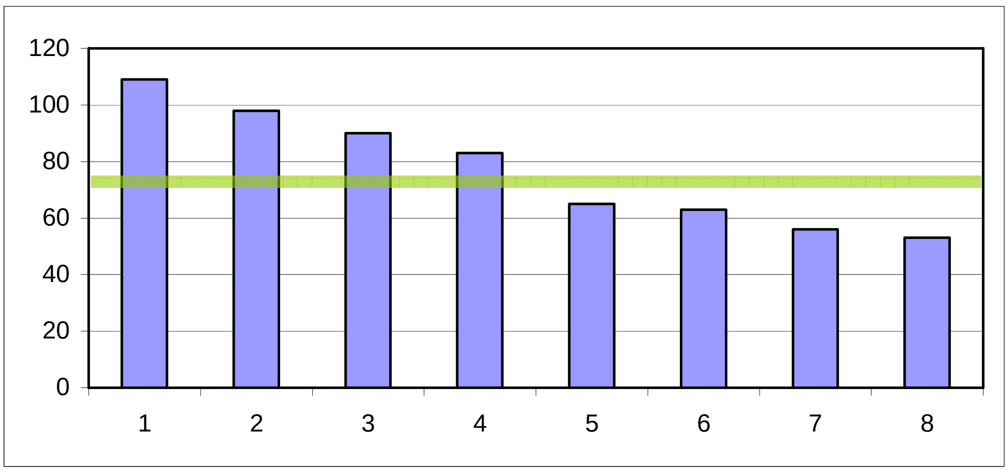
Nonlinear Optics (PHYC/ECE 568)
Fall 2008
Instructor: Prof. M. Sheik-Bahae

Midterm Exam, Open Text-Book, Time: 1.5 hour

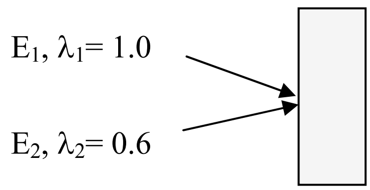
Solutions & Class Statistic

8 students
Average= 77 (72 discarding max & min)



1. NL Susceptibilities (45 points)

Two optical beams E_1 and E_2 with wavelengths of 1.0 and 0.6 μm respectively are incident on a nonlinear material.



(a) Assuming a $\chi^{(2)}$ nonlinearity, what new wavelengths can possibly be generated in this material? (8 pts)

The above nonlinear material is now replaced with a <u>centro-symmetric</u> material for the reaming part of this problem.

(b) What is the dominant nonlinear susceptibility? (2 pts)

(c) Assuming $\chi^{(3)}$ nonlinearity, what new wavelengths λ_j can possibly be generated in this material that <u>simultaneously</u> involve the interaction of both E_1 and E_2 beams? Write down the corresponding nonlinear polarization $P(\lambda_j)$ including their $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ terms (ignore Cartesian indices). (10 pts)

(d) If $|E_1|>>|E_2|$, identify the most dominant terms in part (c). (5 pts)

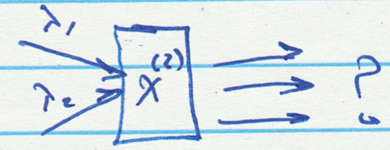
(e) Write down the nonlinear polarization terms associated with self- and cross phase modulation of each beam (identify $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ terms) (10 pts)

(f) Under what condition the simultaneous presence of both beams leads to a nonlinear attenuation (absorption) of both beam? Describe this process, the required energy resonance (use diagrams), and the nature of the complex susceptibility $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ (with respect to part e). (10 pts)

(g) Under what condition the simultaneous presence of both beams leads to a nonlinear attenuation (absorption) of one beam (which?) and gain in the other (which?)? Describe this process, the required energy resonance (use diagrams), and the nature of the complex susceptibility $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ (with respect to part e and f). (Bonus: 10 pts)

a) $\lambda_1 = 1\ \mu m$

$\lambda_2 = 0.6\ \mu m$



$2\omega_1 \Rightarrow \frac{\lambda_1}{2} = 0.5\ \mu m$

$2\omega_2 \Rightarrow \frac{\lambda_2}{2} = 0.3\ \mu m$

$\omega_3 = \omega_2 - \omega_1 \Rightarrow \lambda_3 = \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)^{-1} = \frac{0.6 \times 1}{1 - 0.6} = 1.5\ \mu m$

$\omega_4 = \omega_2 + \omega_1 \Rightarrow \lambda_4 = \left(\frac{1}{\lambda_2} + \frac{1}{\lambda_1}\right)^{-1} = \frac{0.6 \times 1}{1 + 0.6} = 0.375\ \mu m$

b) $\chi^{(3)}$

c) $\omega_3 = 2\omega_1 + \omega_2 \qquad \lambda_3 = \left(\frac{2}{\lambda_1} + \frac{1}{\lambda_2}\right)^{-1} = \frac{0.5 \times 0.6}{0.5 + 0.6} = 0.273\ \mu m$

$\omega_4 = 2\omega_2 + \omega_1 \qquad \lambda_4 = \left(\frac{2}{\lambda_2} + \frac{1}{\lambda_1}\right)^{-1} = \frac{0.3 \times 1}{1 + 0.3} = 0.231\ \mu m$

$\omega_5 = 2\omega_2 - \omega_1 \qquad \lambda_5 = \left(\frac{2}{\lambda_2} - \frac{1}{\lambda_1}\right)^{-1} = \frac{0.3 \times 1}{1 - 0.3} = 0.428\ \mu m$

$\omega_6 = 2\omega_1 - \omega_2 \qquad \lambda_5 = \left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)^{-1} = \frac{0.5 \times 0.6}{0.6 - 0.5} = 3.0\ \mu m$

$P(\omega_3) = 3\epsilon_0 \chi^{(3)}(\omega_3; \omega_1, \omega_1, \omega_2) E_1^2 E_2$

$P(\omega_4) = 3\epsilon_0 \chi^{(3)}(\omega_4; \omega_2, \omega_2, \omega_1) E_2^2 E_1$

$P(\omega_5) = 3\epsilon_0 \chi^{(3)}(\omega_5; \omega_2, \omega_2, -\omega_1) E_2^2 E_1^*$

$P(\omega_6) = 3\epsilon_0 \chi^{(3)}(\omega_6; \omega_1, \omega_1, -\omega_2) E_1^2 E_2^*$

d) $|E_1| >> |E_2| \qquad \omega_3 = (2\omega_1 + \omega_2)$ and $\omega_6 = (2\omega_1 - \omega_2)$

are dominant

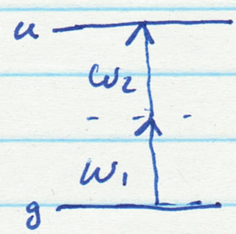
e) SPM & XPM

$$P^{(3)}(\omega_1) = 3\epsilon_0 \Big[\underbrace{\chi^{(3)}(\omega_1; \omega_1, \omega_1, -\omega_1)|E_1|^2 E_1}_{\text{SPM}} + \underbrace{\underset{4}{2}\chi^{(3)}(\omega_1, \omega_1, \omega_2, -\omega_2)|E_2|^2 E_1}_{\text{XPM.}} \Big\}.$$

Similarly.

$$P^{(3)}(\omega_2) = 3\epsilon_0 \Big[\chi^{(3)}(\omega_2; \omega_2, \omega_2, -\omega_2)|E_2|^2 \overset{E_1}{\cancel{E_2}} + 2\chi^{(3)}(\omega_2; \omega_2, \omega_1, -\omega_1)|E_1|^2 E_2 \Big]$$

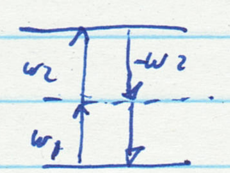
f) Nonlinear absorption in both beams occur under nondegenerate "two-photon absorption"



when $\omega_1 + \omega_2 = \omega_{gu}$ resonance condition

This corresponds to Imaginary part of

$$\chi^{(3)}(\omega_1; \omega_1, \omega_2, -\omega_2) \quad \text{or}$$
$$\chi^{(3)}(\omega_2; \omega_2, \omega_1, -\omega_1)$$

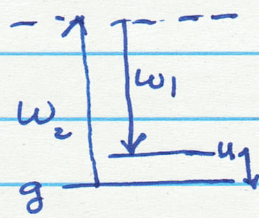


$$\frac{dI_1}{dz} = -\beta I_1 I_2$$

$$\frac{dI_2}{dz} = -\beta I_1 I_2$$

g) BONUS

This is a Raman-type process where $\omega_2 - \omega_1 = \omega_{gu}$ in which ω_2 is absorbed and ω_1 is amplified



$E_{ug} = \hbar\omega_2 - \hbar\omega_1$

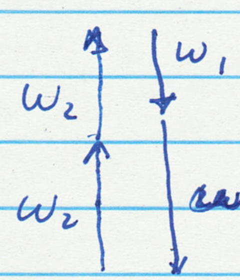
$\chi_I^{(3)}(\omega_1; \omega_2, \omega_2, -\omega_2)$ or $\chi_I^{(3)}(\omega_2; \omega_2, \omega_1, -\omega_1)$.

Imaginary part

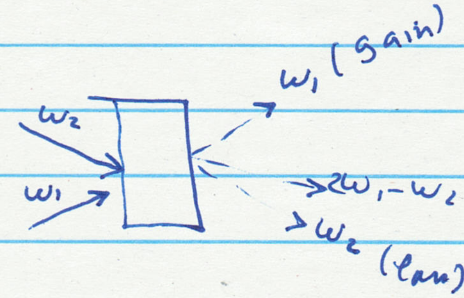
$$\frac{dI_2}{dz} = -\eta I_1 I_2 \qquad \eta \propto \mathrm{Im}\{\chi^{(3)}\}$$

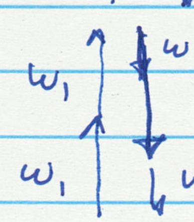
$$\frac{dI_1}{d\,dz} = +\eta I_1 I_2$$

② another process that may result in absorption of $\omega_{2,1}$ and gain in $\omega_{1,2}$ is the <u>4-photon</u> <u>parametric</u> process:

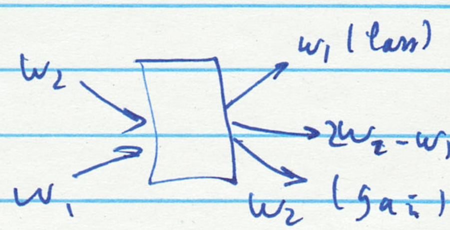


$\omega_3 = 2\omega_2 - \omega_1$.





$\omega_3 = 2\omega_1 - \omega_2$



2. SHG (25 points)

Consider GaAs crystal in a SHG process (see attached data sheet). The orientation of the input field at ω is given by $E(\omega) = E_0(\cos(\phi)\hat{x} + \sin(\phi)\hat{y})$ propagating along the z-axis (c-axis) of the crystal

(a) Obtain d_{eff} vs. ϕ and find optimum ϕ?

(b) For a fundamental wavelength of λ=2 μm, plot relative SHG signal vs. sample thickness L (for 0<L<2 mm). Identify the minimum thickness that gives the highest efficiency.

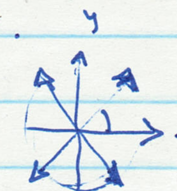
(c) Discuss the possible phase matching technique(s) for this crystal.

$$E(\omega) = E_0(\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}).$$

(a) $$d : \hat{e}_1 \hat{e}_2 = \begin{pmatrix} 0 & 0 & 0 & d_{36} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{36} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix} \begin{pmatrix} \cos^2\varphi \\ \sin^2\varphi \\ 0 \\ 0 \\ 0 \\ 2\sin\varphi\cos\varphi \end{pmatrix} =$$

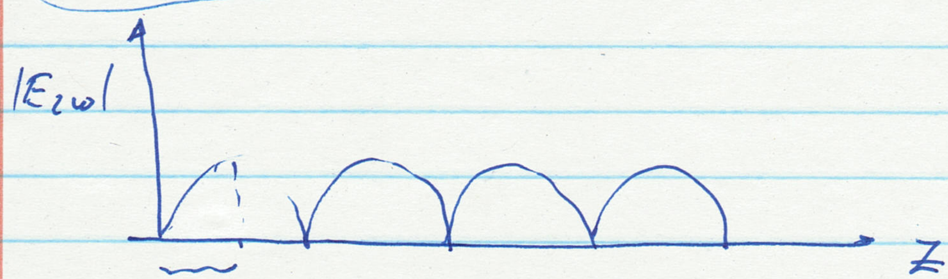
$$= (0,\ 0,\ 2 d_{36} \sin\varphi \cos\varphi).$$

(b) $$P_3(2\omega) = 2 d_{36} \sin\varphi \cos\varphi = d_{36} \sin 2\varphi.$$



max when $\varphi = \pm\frac{\pi}{4}$, $\pm\frac{3\pi}{4}$.

Note: In practice, this would not work as $\vec{P} = P(\omega)\hat{z}$ is longitudinal. But, we assume it does!!



$\ell'_c = \frac{\pi}{\Delta k}$.

$$\ell'_c = \frac{\pi}{\frac{4\pi}{\lambda}(n(2\omega) - n(\omega))} = \frac{\lambda}{4(n(2\omega) - n(\omega))} = \frac{2\ \mu m}{4(n(1\mu) - n(2\mu))}$$

$\approx 3.13\ \mu m$

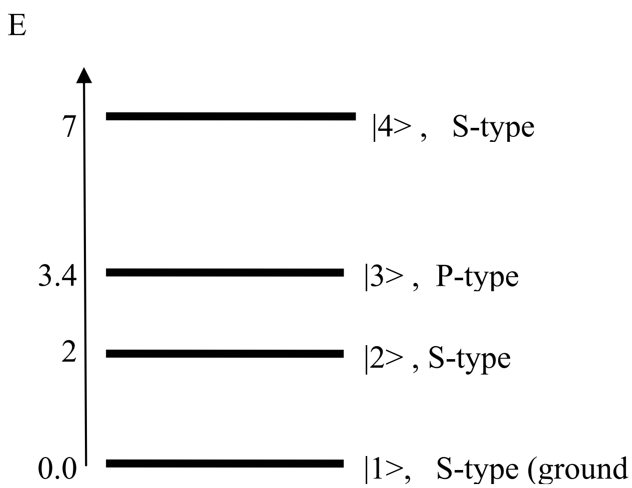
(c) Quasi phase matching

or Coil ℓ_c

3. NLO susceptibilities: resonances and selection rules (30 points)

A fictional molecule has the following energy levels. Draw the spectrum (for $0<\hbar\omega<8$ eV) for the **(a)** linear absorption coefficient α, **(b)** two-photon absorption (TPA) coefficient β, **(c)** SHG: $|\chi^{(2)}(2\omega;\omega,\omega)|$ and **(d)** THG: $|\chi^{(3)}(3\omega;\omega,\omega,\omega)|$.

Be quantitative in your x-axis. Assume a finite broadening in your drawings. Point out the resonances (diagrammatically) on your graph for each case and show the relative strengths if obvious. Note: no calculations needed for this problem.



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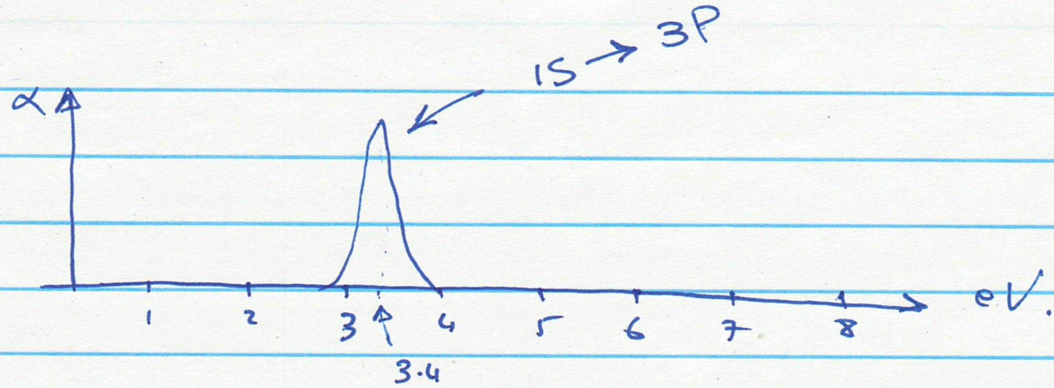
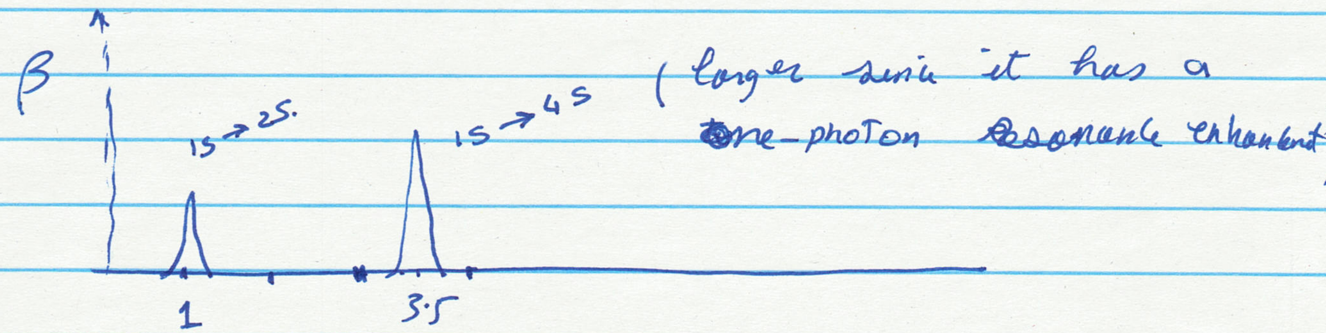
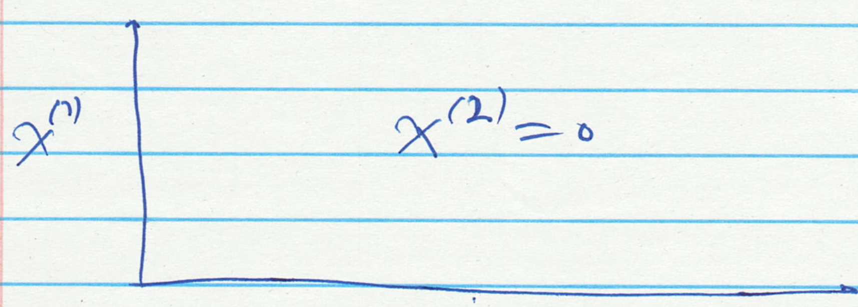
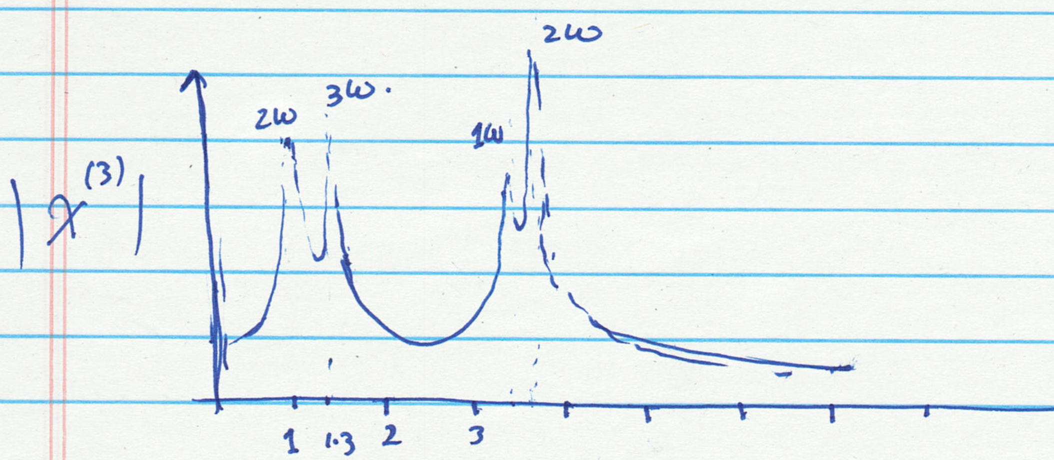
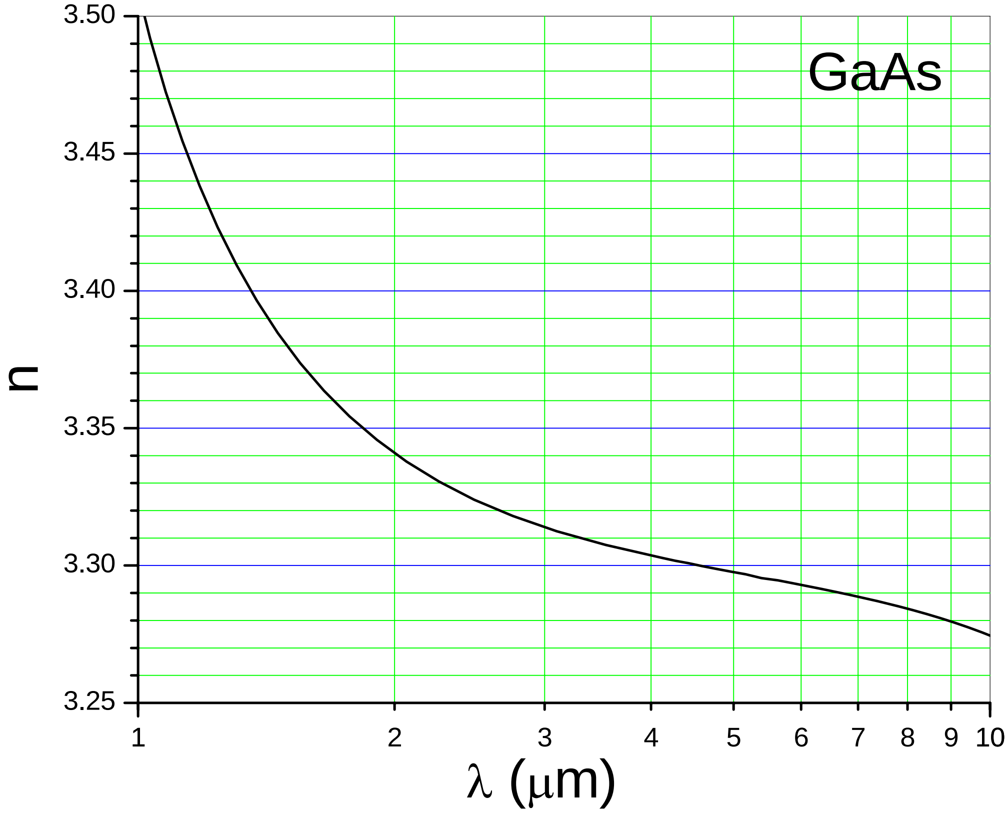

α
1s → 3P
eV.
1
2
3
4
5
6
7
8
3.4


β
1s → 2s.
1s → 4s
1
3.5
(larger since it has a one-photon resonance enhancement)


$\chi^{(1)}$
$\chi^{(2)} = 0$


$|\chi^{(3)}|$
2ω
3ω.
1ω
2ω
1
1.3
2
3

GaAs Data:
Refractive Index (isotropic):



From table 1.5.3 and Fig. 1.5.3 (in Boyd) d_{ij} for $\bar{4}3m$ is:

$$\begin{pmatrix} 0 & 0 & 0 & \bullet & 0 & 0 \\ 0 & 0 & 0 & 0 & \bullet & 0 \\ 0 & 0 & 0 & 0 & 0 & \bullet \end{pmatrix}$$ ($\bar{4}3m$ point group, with d_{36}=370 pm/V).