Nonlinear Optics – PHYC/ECE 568
Solitons

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Abstract: In this report generation of solitons and different types of it has been introduced. Wide applications of solitons in studying light and matter interaction in ultra-fast time scales and in very small dimensions has captured a lot of attention. FDTD numerical simulation has been implemented to verify the special features of soliton while propagating in a kerr-type nonlinear medium.

References and links

1. Tal Carmon, PhD thesis, Nonlinear optics and solitons.
2. F. Wise and P. Trapani, Spatiotemporal solitons, Optica and photonics news, Feb. 2002.
3. R. M. Joseph, and A. Taflove, Spatial soliton deflection mechanism indicated by FDTD maxwell's equation modeling, IEEE photonics tech. let., vol 6, No. 10, Oct 1994.
4. X. Liu, L. J. Qian, and F. W. Wise, Generation of optical spatiotemporal solitons, PRL., vol 82, No. 23, June 1999.

1. Introduction

Beams of light tend to diverge in space while propagating. This is not limited to light waves, but water waves and any other wave phenomena can experience such broadening called "diffraction". However, waves that keep their shape invariant while propagating do exist in the nature and are called "Solitons". Solitons were first illustrated in water waves in front of a boat by John Scott Russel in 1834. In optics solitons can be seen in the form of temporal solitons in fiber optics, spatial solitons, spatiotemporal solitons, and other various complex kind of it like discrete soliton and cavity solitons [1]. Temporal solitons are very important in fiber optic communication as they do not experience broadening caused by group velocity dispersion (GVD) while propagating in long distance. Spatiotemporal solitons (STS), are as a result of a balance in the interplay between diffraction, GVD, and nonlinear processes in high intensity light pulses. These particlelike states of light sometimes called "light bullets" are interesting for their applications in studying the light and matter interaction in ultra-fast time scales and very small dimensions. Fig. 1 compares a spatiotemporal soliton with a diffracting and dispersing pulse.
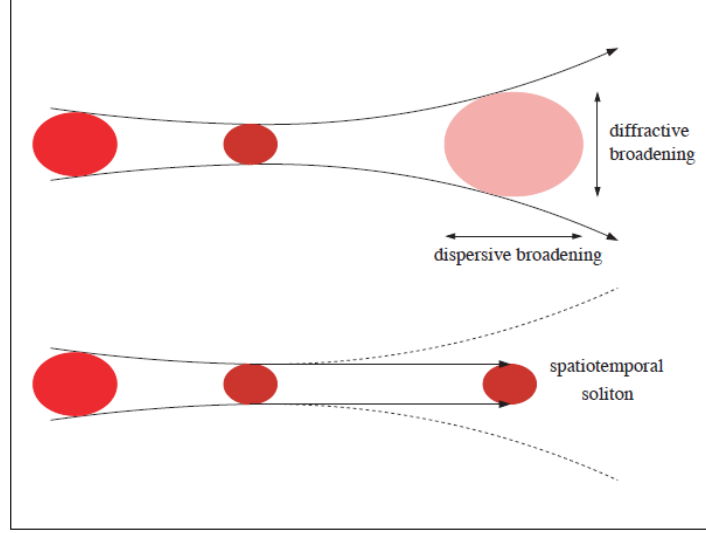
Fig. 1. Illustrating the energy density of a diffracting and dispersing pulse compared to a spatiotemporal soliton (STS) [2]

Starting form Maxwell equations assuming paraxial approximation and slowly varying envelope approximation (SVEA) one can derive the nonlinear Schrödinger equation for a kerr nonlinearity as follows below:

$$\frac{\partial}{\partial z}\psi(x,y,z) = [\frac{i}{2k}(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}) + \frac{ik\ n_2}{n_0}|\psi(x,y,z)|^2]\psi(x,y,z)$$

Where z is the propagation direction, k is the wave vector, and ψ is the slowly-varying amplitude of the electric field. One the best family of solutions are sech functions as follows below,

$$\psi = \sqrt{\psi_0}Sech[\frac{x}{W_0}]\exp[i\frac{z}{4z_0}]$$

Where, $W_0 = \sqrt{\frac{2n_0}{n_2}\frac{1}{k\psi_0}}$, and $Z_0 = \frac{kW_0^2}{2}$. It can be seen that the intensity of the propagating wave is independent to z. Stability of such solutions are also important in order to consider it as a soliton.

The above solution can be a result of any wave phenomena in the nature following a nonlinear equation in a general format of,

$$\frac{d\psi}{dA} = (\widehat{D} + \widehat{N})\psi$$

Where $\widehat{D}$ is the differential operator describing the linear diffraction and $\widehat{N}$ is the nonlinear operator that governs the effects of nonlinear medium (self-phase modulation). When these two cancel each other we would have a soliton. Table 1 summarizes some examples of solitons

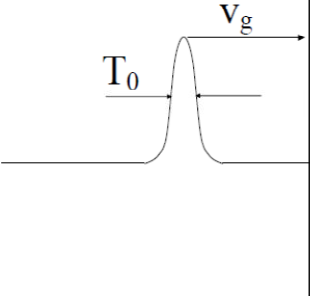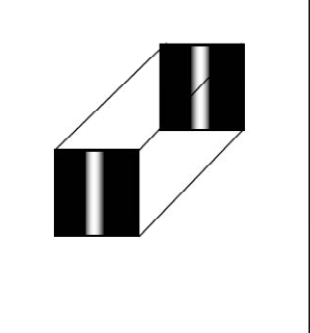governed by this equation: Optical spatial soliton, fiber solitons, surface waves in water, and superfluids;
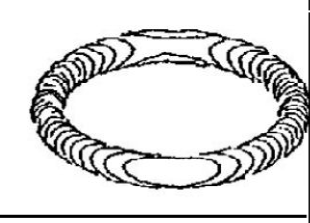
When the governing equation is $\frac{\partial \psi}{\partial A} = (\hat{D} + \hat{N})\psi$ where the					
	Propagation is in A= :	Dispersion $(\hat{D})$ is:	Nonlinearity $(\hat{N})$ is:	The solitary solution is $\psi =$	
Fiber	z	$\frac{-i\beta_2}{2}\frac{\partial^2}{\partial T^2}$	$i\gamma|\psi|^2$	$\sqrt{\psi_0}\,Sech[\frac{T}{T_0}]\exp[i(\frac{z}{2L_D})]$ $L_D = T_0^2/|\beta_2|$	
Optical spatial	z	$\frac{-i}{2k}\frac{\partial^2}{\partial x^2}$	$i\frac{k\Delta n_0}{n_0}|\psi|^2$	$\sqrt{\psi_0}\,Sech[\frac{x}{W_0}]\exp[i(\frac{z}{4z_0})]$ $Z_0 = \frac{kW_0^2}{2}$	
Water	t	$-\frac{\partial^3}{\partial x^3}$	$-\psi\frac{\partial}{\partial x}$	$3\eta\,Sech^2[\frac{1}{2}\sqrt{\eta}(x-\eta\,t)]$	
Superfluid	t	$i\nabla^2$	$-i(|\psi|^2-1)$	Vortex ring	

Fig. 2. Different examples of solitons [1]

2. FDTD Numerical Simulation

In order to study the propagation of fields in these nonlinear medium and to verify the specific features of solitons the electromagnetic wave equation have been implemented numerically. In particular, here I rederived the FDTD numerical simulation of the soliton investigated in [3].

In 1-D FDTD the Maxwell equation derivatives are replaced by central differencing and hence become as below,

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\frac{\partial E_z}{\partial x} \longrightarrow \quad H_y\Big|_{i+1/2}^{n+1/2} = H_y\Big|_{i+1/2}^{n-1/2} + \frac{\Delta t}{\mu_{i+1/2}\Delta x}\left(E_z\Big|_{i+1}^{n} - E_z\Big|_{i}^{n}\right)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\frac{\partial H_y}{\partial x} \longrightarrow \quad E_z\Big|_{i}^{n+1} = E_z\Big|_{i}^{n} + \frac{\Delta t}{\varepsilon_i\Delta x}\left(H_y\Big|_{i+1/2}^{n+1/2} - H_y\Big|_{i-1/2}^{n+1/2}\right)$$

Here, a 2-D FDTD for the kerr type nonlinear medium was implemented. The resulting field overlay illustrated in Fig. 3 shows the spatial soliton propagating through the nonlinear medium.
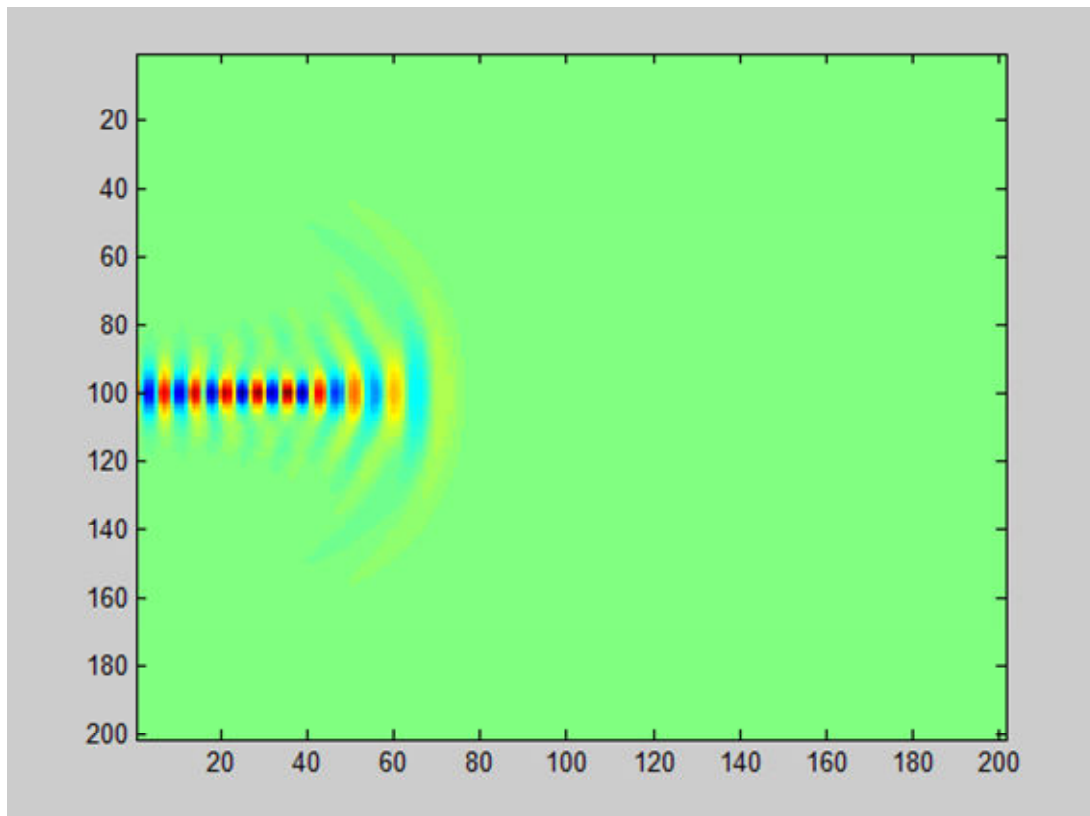

Fig. 3. Single spatial soliton propagation implemented with FDTD. This has been first done by [3]

This section would be detailed more in the presentation.

Appendix: FDTD field updates, MATLAB code

```matlab
% Source
% Propagating sinusoidal beam
n0=2.46;
n2=1.25e-18;
fc=4.31e14;
Ip=6.87e8;              % changed the amplitude (e9 to e8)!!!
FWHM=0.65e-6;

Lambda=c/fc;
dx=Lambda/10;                   % spatial step size
dy=dx;
T_step_bound=1/(c*sqrt(1/(dx^2)+1/(dy^2)));
dt=T_step_bound*1.00;

Ca=1;                           % Coefficients
Cb=dt;                          % for one medium
Da=1;
Db=dt/mu0;

% sweeping over time and spatially for
% propagating field components
for n=1:nmax

    for j=1:jmax+1
        for i=2:imax
            Hy(i,j)=Da*Hy(i,j)+Db*((Ez(i,j)-Ez(i-1,j))/dx);
        end
    end
    for j=2:jmax
        for i=1:imax+1
            Hx(i,j)=Da*Hx(i,j)+Db*(-(Ez(i,j)-Ez(i,j-1))/dy);
        end
    end
    for j=2:jmax-1
        for i=2:imax-1
            Dz(i,j)=Ca*Dz(i,j)+Cb*((Hy(i+1,j)-Hy(i,j))/dx-((Hx(i,j+1)-
Hx(i,j))/dy));
            % Source
            Envelope=sech(2.634*(j-jmax/2)*dy/FWHM);
            Ez(1,j)=Envelope*Ip*sin(2*pi*fc*n*dt);
        end
    end

…

end
```