Creating Entangled Photons Via Nonlinear Processes

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I. BASICS OF ENTANGLEMENT

In 1935 Einstein and Schrödinger developed what they considered a paradox of quantum theory which Schrödinger named entanglement. Let two spin-1/2 particles be in the state

$$\frac{1}{\sqrt{2}}(|\uparrow\uparrow⟩ + |\downarrow\downarrow⟩)$$  \hspace{1cm} (1)

If we then separate the two particles and measure the first particle’s spin, then according to the Copenhagen interpretation of quantum mechanics, the wavefunction of the second particle instantly collapses into it’s corresponding state. Einstein referred to this ability of separated systems to instantly effect one another as spooky action at a distance.

30 years later John bell refined Einstein and Schrödinger’s initial work by noting that certain observations on the above state would give different outcomes from the quantum and classical case. A number of experiments followed, but they were plagued by the difficulty of producing large numbers of entangled particles, leading to attempts to develop better sources. To date the most widespread means of producing entangled particles is the process of spontaneous down conversion of light in non-linear crystals. [1, 2]

II. PARAMETRIC DOWN CONVERSION AS A SOURCE OF ENTANGLED PHOTONS

The polarization states of photons are an attractive system in which to create entangled states due to the availability of high quality polarization elements. If two thin nonlinear crystals are placed next to one another with their optic axes orthogonal as shown in figure 1a then a vertically polarized photon can down convert only in the first crystal, while a horizontally polarized photon can down convert only in the second crystal, both via type I phase matching. Phase matching considerations further require the down converted photons to be emitted in cones centered on the pump beam. If 45° light is incident on the crystals, it may down convert in either crystal, and if the spatial modes of the down converted light from the two crystals are identical then the state after the two crystals will be

$$\frac{1}{\sqrt{2}}(|HH⟩ + e^{i\phi}|VV⟩)$$ \hspace{1cm} (2)

![Diagram of the experiment](image)

FIG. 1: Initial transitions from ground state to the Rydberg levels.

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As shown in figure 1b the experimental source consists of an Ar⁺ laser at 351.1 nm with a diameter of 2 mm. After the laser a dispersive prism removes laser fluorescence and a polarizing beam splitter insures a pure input polarization. The half and quarter wave plates control the input polarization before the nonlinear crystal. After the nonlinear crystal, a combination half wave plate and polarizing beam splitter are used to analyze the output polarization. Apertures control the spatial mode of the output light and interference filters centered at 702 nm with a FWHM of 5 nm ensure only the desired frequency is measured. The nonlinear crystal was a 8.0x8.0x0.59mm BBO crystal cut so that the cone of degenerate frequency photons had a half opening angle of 3°.

If we measure in the ± 45° basis, we would expect the photons to be perfectly anti-correlated. As shown in Figure 2, if the polarization analyzer of path 1 is set to -45° and the analyzer of path 2 is rotated, the rate of coincident detection varies with the angle of the analyzer. When analyzer 2 is set to -45° almost no coincidences were detected while the coincidences were maximized for an angle of -45°, showing the expected anti-correlations. On the other hand, the total number of singles varied very little as analyzer 2 was changed. Fitting both the singles and coincidence data with a sinusoidal variation yields a visibility of < 3.4% for the singles and 99.6 ± 0.3% for the coincidences, once accidental coincidences were subtracted. This high visibility indicates the presence of highly entangled photons.

The relative phase \( \phi \) of the HH and VV states after the nonlinear crystal may be varied by varying the ellipticity of the input light. The polarization analyzers in both paths were set to 45° and coincidences were counted as the quarter wave plate before the crystal was rotated. Figure 2 shows the coincidence rate vs. quarterwave plate angle and demonstrates that the state can be varied from \( \frac{1}{\sqrt{2}} (|HH \rangle + |VV \rangle) \) to \( \frac{1}{\sqrt{2}} (|HH \rangle - |VV \rangle) \). A half wave plate in either path may be used to reach the other Bell states, \( \frac{1}{\sqrt{2}} (|HV \rangle \pm |VH \rangle) \).

Replacing the circular irises with vertical slits effectively sampled a larger portion of the cone of entangled photons. Increasing the size of the aperture allowed a demonstrated coincidence rate of 21,000 s\(^{-1}\) while maintaining a visibility of 95%. Since the entire cone of emitted photons should consist of entangled photons, sampling it could give a coincidence rate of 1.5x10⁶ s\(^{-1}\).

Due to the dispersion of the crystal, the resulting phase between HH and VV of the down converted photons is frequency dependent. In order to describe the entangled photons by the same quantum state we must limit the bandwidth of collected light. In principle, an output bandwidth of 30 nm maintains a visibility of > 90%, giving a potential coincidence rate of 9x10⁶ s\(^{-1}\). [1]

### III. ENTANGLING MULTIPLE DEGREES OF FREEDOM

In addition to the entanglement in the polarization degree of freedom, the down converted photons are also entangled in their time of emission. Because of the relatively short time the crystal stays in any intermediate states during the process of down conversion, both down converted photons are emitted at nearly the same time. If the down converted
photon are then passed through separate interferometers, the entanglement between the two photons is manifested in an interference in the rate of coincidence detections.

Consider the experimental setup of figure 3. Because the down converted photons are either both horizontally polarized or both vertically polarized, they will either both follow the long path or both follow the short path of the interferometer. After passing through a polarization analyzer, the polarization information is erased, leaving the photons entangled in which path they took.

Instead of an extended interferometer, an optical path difference between the two polarizations may be introduced via the birefringence of a quartz rods and fine tuned with Pockels cells, as shown in the second half of figure 3. This has the advantage of greater stability in comparison to the extended interferometer. The polarization information may be erased by passing the entangled photons through a polarization analyzer at 45° [3, 4].

Due to the conservation of orbital angular momentum the spatial modes of the down converted photons are also entangled. If the incoming light has zero orbital angular momentum then the state of the field after the nonlinear crystal is, up to normalization,

\[(|l\rangle + \alpha |gg\rangle + |tr\rangle)\]  

(3)

Where \(|l\rangle, |G\rangle\) and \(|r\rangle\) are Laguerre-Gauss spatial modes with orbital angular momenta of \(-\hbar, 0\) and \(\hbar\), respectively. \(\alpha\) is determined by the details of the down conversion process. The spatial mode may be determined by a spatial mode analyzer as depicted in figure 4 [5]. The computer generated hologram diffracts the incoming light and alters the orbital angular momentum. Since the monomode fibers only accepts a gaussian mode, only light which is transformed by the hologram into the gaussian mode will be detected.

It is even possible to create photons that are entangled in every degree of freedom - polarization, energy-time, and
FIG. 5: Initial transitions from ground state to the Rydberg levels.

Spatial. The experimental setup is depicted in figure 5. As before a single photon from an Ar$^+$ laser at 351.1 nm down converts into two entangled photons via spontaneous parametric down conversion in one of two BBO crystals. The photons then pass into measurement stages which are capable of measuring each degree of freedom as described above. First the spatial degree of freedom is measured, followed by the energy-time and polarization degrees of freedom.

For each degree of freedom a series of measurements is taken whose expectation value $S$ is different for classical light which cannot be entangled and quantum entangled states. Theses Clauser-Horne-Shimony-Holt Bell inequalities provide a measure of the entanglement in each degree of freedom. Classically $S \leq 2$ while quantum mechanically $S \leq 2\sqrt{2}$. For each degree of freedom the measured value of $S$ was greater than the classical limit by at least 20 standard deviations. [6]

IV. CONCLUSION AND OUTLOOK

The process of spontaneous down conversion has been shown to create large numbers of polarization entangled pairs of photons. It has also been demonstrated to create photons which are entangled in the polarization, energy-time, and spatial mode degrees of freedom. Tests of their entanglement provide a demonstration of the fundamental difference between quantum and classical physics. The continued development of these sources, especially the ability to create more and more entangled pairs will prove useful in a number of potential applications.