Injection tuning of Optical Parametric Oscillators

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Abstract- In this paper, I tried to provide a mathematical model to describe the time dynamics of injection tuning of the optical parametric oscillators. This model can account for all modes under the gain profile of the oscillator. It is shown that the injected mode and the fastest growing mode are the main participants in the time dynamics of injection tuning of the OPO's. A set of prescriptive criteria is developed for successful injection tuning.

Recently, much attention has been paid to the construction of tunable lasers, particularly to the optical parametric oscillator (OPO), whose emission frequency can be varied continuously. But it seems difficult to control the spectral output of the OPO, considering many modes can be stimulated by the pumping source. One method of accurately controlling the spectral output of an OPO is by "injection tuning". This technique makes it possible to control the output frequency of a high-powered pulsed singly-resonant OPO by injecting into the oscillator cavity low-powered radiation from a frequency controlled source, such as LED.

The experimental situation is as follows: a nonlinear optical crystal is placed within an optical resonator. A "pump" field at ω_p is then fed into the resonator. It is easy to understand that, above a threshold pumping pump intensity, oscillation is set up simultaneously at N pairs of frequencies ω_{n1} and ω_{n2}. The generalized equations for the amplitudes: a_p, a_{2n-1}, and a_{2n} for the pump mode, the nth signal mode, and the nth idler mode, respectively, are written as follows:

$$\frac{da_p}{dt} = -i\omega_p a_p - \frac{\omega_p}{2Q_p} a_p - \sum_{n=1}^{N} K_n a_{2n} a_{2n-1} + i\lambda_p \exp(-i\omega_p t) \tag{1}$$

$$\frac{da_{2n-1}}{dt} = -i\omega_{2n-1} a_{2n-1} - \frac{\omega_{2n-1}}{2Q_s} a_{2n-1} + K_n a_{2n}^{X} a_p \tag{2}$$

$$\frac{da_{2n}}{dt} = -i\omega_{2n} a_{2n} - \frac{\omega_{2n}}{2Q_i} a_{2n} + K_n a_{2n-1}^{X} a_p \tag{3}$$

where ω_{2n-1} and ω_{2n} are the frequencies for the nth signal and idler modes satisfying: $\omega_p = \omega_{2n-1} + \omega_{2n}$, Q_s and Q_i are the signal and the idler cavity-Q factors, taken to be the same for all N modes. The coupling constant K_n for the nth mode is given by $K_n = \frac{1}{2}\sqrt{\omega_p \omega_{2n} \omega_{2n-1}} \frac{d}{\varepsilon^{3/2} V^{1/2}} F(\Delta k_n L)$, with $F(\Delta k_n L) = \sin(\frac{1}{2}\Delta k_n L) / \frac{1}{2}\Delta k_n L$, where $\Delta k_n = k_p - k_{2n-1} - k_{2n}$.

For numerical solutions, it is easier to handle the equations in a dimensionless form. Thus, the coupling constants were normalized to that ($K_{\max}$) of the mode with the maximum gain, so that $\eta_n = K_n / K_{\max}$ represents the normalized coupling constant of the nth mode. The mode amplitudes were all normalized to the threshold value for the pump A_{pth}, which is the pump mode amplitude at the threshold of growth for the fastest growing mode. The time scale was normalized to the decay time for the pump $T = \dfrac{1}{\gamma_p} = \dfrac{2Q_p}{\omega_p}$. And we use $\beta = \lambda_p / \lambda_{pth}$ to represent the pump driving term normalized to the threshold driving magnitude λ_{pth}. Then we can transform the equations (1), (2), (3) into dimensionless forms and solve them to get the final solutions in the form of dimensionless parameters.

Two graphs for two similar situations are shown below:

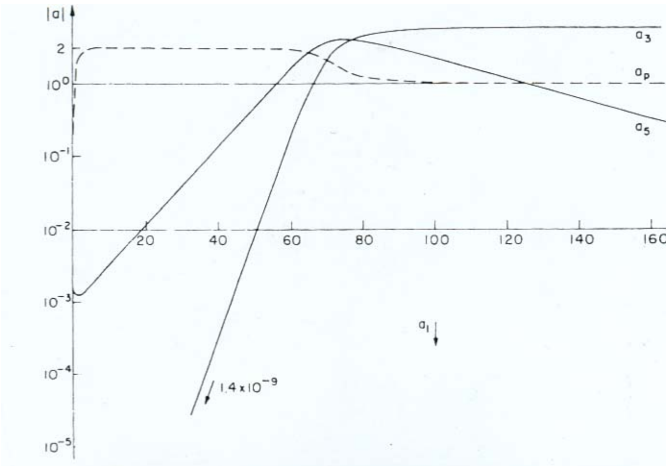

Fig. 3. OPO mode time dynamics (unsuccessful injection). Parame $\alpha_s = 0.055$, $\alpha_i = 1.0$, $\eta_1 = \eta_3 = 0.637$, $\eta_2 = 1.0$, $\beta = 2$, $N = 3$.

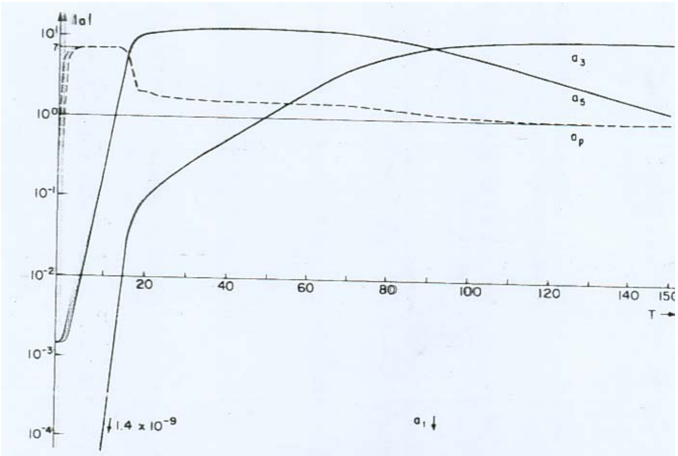

Fig. 4. OPO mode time dynamics (successful injection). Paramete same as Fig. 3 except $\beta = 7$.

In Fig.3, an N=3 case, the injection is on signal mode a_5, which along with signal mode a_1, is a non-maximum-gain mode. The maximum-gain signal-idler pair are labeled a_3 and a_4 and correspond to perfect phase-matching $[(k_p - k_3 - k_4)L = \Delta kL = 0]$ so that $\eta_2 = 1$. Here, we consider only 3 pairs $(a_1, a_2), (a_3, a_4), (a_5, a_6)$. With β =2, this results in a gain rate for signal a_5 which is insufficient to reach the quasi-steady state of successful injection. The reason is that there is no period of time in which we can expect the pure frequency ω_5 output. It is to be noted that the mode a_1, being neither the injected mode nor the maximum-gain mode, never rises enough above noise even to appear on the scale of Fig.3.

The "unsuccessful" injection for the oscillator parameters of Fig.3 can, however, be made "successful" by elevating the pump driving term β, which is a measure of the field strength of the pumping laser. Fig.4 shows the injection operation which is successfully achieved by increasing β to 7. In contrast to the previous case, higher pump drive here results in the injected mode a_5 rising rapidly to deplete the pump before the maximum-gain mode a_3 catches up and the final steady state is approached with $|a_p| = 1$. This prescription for achieving successful injection by increasing β was not, however, found effective in any case where injection was attempted on a mode having a relative coupling constant (η_n) less than about 0.6.

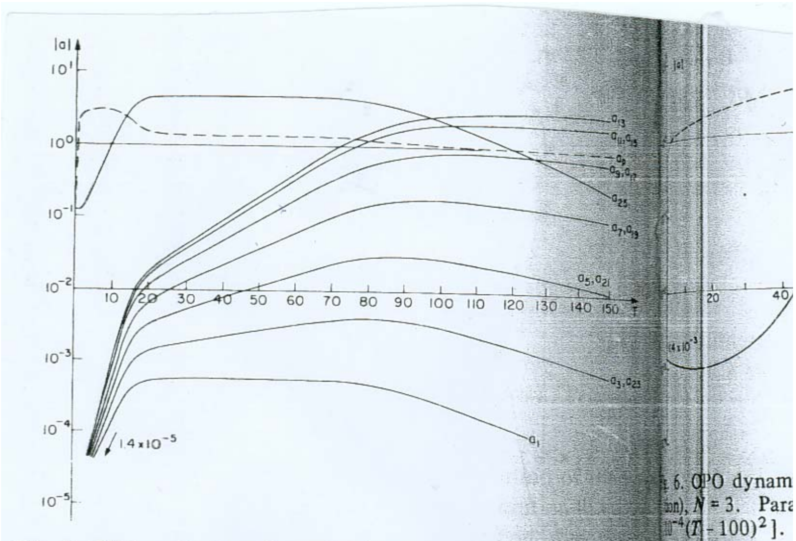

Fig. 5. OPO mode time dynamics (successful injection). Parameters: $\alpha_s = 0.1$, $\alpha_i = 1.0$, $\eta_1 = \eta_{13} = 0.73$, $\eta_2 = \eta_{12} = 0.80$, $\eta_3 = \eta_{11} = 0.87$, $\eta_4 = \eta_{10} = 0.92$, $\eta_5 = \eta_9 = 0.97$, $\eta_6 = \eta_8 = 0.99$, $\eta_7 = 1.00$, $\beta = 3$, $N = 13$.

In Fig. 5 is shown a more realistic multimode case with N=13.This is a case of successful injection, where the injected mode (a_{25}) has a relative coupling constant η_{13} =0.73 and a pump strength of β =3. The curves labeled with two modes represent modes whose frequencies are disposed symmetrically on either side of the maximum-gain mode, and thus these modes have identical gain constants. The injection tuning in Fig.5 are characterized by time intervals during which the various mode amplitudes are either constant or are nearly pure exponential growth. Here, we divide the dynamics for four time intervals $0 \le T \le T_1, T_1 \le T \le T_2, T_2 \le T \le T_3$, and $T_3 \le T < \infty$ .For Fig.5, these intervals are approximately $0 \le T \le 2, 2 \le T \le 15, 15 \le T \le 75$, and $75 \le T < \infty$ 。Let's look at these intervals in detail:

A. First interval $0 \le T \le T_1$

In this interval the pump rises from its initial value of 0 to its maximum value β . The pump amplitude is not large enough to boost the injected mode A_{2m-1}, which actually shows a small decay while all the other modes at noise level remain unaffected.

B. Second interval $T_1 \le T \le T_2$

In this interval A_p has reached its maximum value and is essentially undepleted at that value, since all the modes, including the injected mode A_{2m-1}, are too small to deplete the pump. Thus, in this period, A_{2m-1} is growing exponentially with T, while all the other modes are also being pumped up from the initial noise value.

C. Third interval $T_2 \le T \le T_3$

Once the injected mode A_{2m-1} rises enough, it starts depleting the pump and a quasi-steady state I reached. In this state, the injected mode is at its maximum value and the pump has a value corresponding to depletion only by the injected mode and its idler. Therefore, we can get an output of "pure" injected mode in this interval. During this period, A_{2l-1} and A_{2l} (the fastest growing and idler modes) are still too small to deplete the pump.

D. Fourth interval $T_3 \le T < \infty$

Here, the successful injection is essentially over, since the output is not a "pure" injected mode any more. The fastest growing mode gradually rises to its steady state while all the other modes begin their decline to insignificant values.

As we can see from the above examples the operating criteria to be achieved are:

1) The initial mode should saturate the pump before the maximum-gain mode does and

2) The gain of the maximum-gain mode, under pump saturation by the injected mode, must be small enough to leave the injected mode dominant for a useful period of time.

One parameter that stands out as a consistent predictor of success for injection tuning is $\eta = K_{inject} / K_{\max}$, the coupling constant of the injected mode relative to that of the highest gain mode. As a consequence of the results reported above and other similar cases, several injection criteria can be set down.

1) If η <0.6, then successful injection cannot be expected.

2) An increase in pump strength will not result in successful injection when $\eta \leq 0.6$

3) If η >0.6, an increase in pump strength can improve injection.

4) For low-pump strength ratios, coupling constants considerably higher than 0.6 are required for successful injection.

5) An increase in cavity Q or cavity reflectance appears to improve injection when the operation is already at least in the marginal success range.

References

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