We consider the nonlinearity of a quantum system with one single two-level atom and very high $Q$, single mode Fabry-Perot cavity strongly coupled together. The coupled state can be treated as a new state (we call it “dressed atom” state) with its own characteristic energy levels, which are very different from when the atom is not coupled to the cavity mode (bare atom state). Inharmonic energy level splitting due to strong coupling leads to photon blockade: the absorption of one photon of the system block the absorption of a second photon. Strong interactions between photons are taken one by one, thus hitting the limit of nonlinear optics.

1. Introduction

One can observe significant nonlinear effect only when the light field is very strong, due to the fact that material nonlinearity is very small. For example, to observe saturation, one need the number of photon be at the level of $10^{10}$! The question is, can we do nonlinear optics at single photons level? People observe nonlinear response of atoms–photon blockade, in the schematic of cavity enhanced quantum electrodynamics (cavity QED) at single photons and atoms level. In exact analog to Coulomb blockade, in which charge transport through metallic or semiconductor device occurs on an electron-by-electron basis, photon blockade [1] refers to the fact that absorption of a photon block the absorption of a second photon. Thus one can use only one photon and saturate the response of an atom!
Fig. 1. (a): Basic principle of cavity QED, one single cavity mode with characteristic frequency $\omega_{\text{cav}}$ plus one single two-level atom with characteristic frequency $\omega_{\text{at}}$ plus strong coupling between the two. (b): Energy levels of the system with no coupling and with strong coupling from Jaynes-Cummings model. The $n$ number denotes number of photons in the system. Inharmonic energy level splitting is predicted from this model. $\Omega_0$ is vacuum Rabi frequency [4] (c): Strong coupling: coupling $\gg$ dissipation is required such that the energy splitting $\Omega_0 \gg$ energy level broadening due to limited lifetime of cavity and atom $(\Gamma_{\text{cav}} + \Gamma_{\text{at}})/2$
2. Background Information [2] [3]

2.A. Cavity QED

Figure 1(a) shows the basic principle of cavity QED. It includes one single mode of a very high Q cavity, one single two-level atom and strong coupling between the two. The characteristic frequencies of the two are $\omega_{\text{cav}}, \omega_{\text{at}}$ respectively. In strong coupling regime, these two frequencies are very close to and interacting with each other such that one has to treat the cavity and atom as one whole new quantum system instead of treating them separately. Just as two coupled pendulums will produce new eigenfrequencies, coupling of a cavity mode and an atom leads to new eigen-energy levels. From semi-classical point of view (quantized atomic level and classical electromagnetic field), the energy level splitting is called vacuum Rabi frequency.

2.B. Strong coupling regime

Strong coupling regime means coupling $\gg$ dissipation. One can have a better picture by looking at the energy levels on Figure 1(c). $\Omega_0 = 2\mu_{eg}E/\hbar$ is energy level splitting (vacuum Rabi frequency) and it’s linear proportional to dipole coupling coefficient $\mu_{eg}$ and electric field strength $E$. Dissipation of the system leads to energy level broadening by $(\Gamma_{\text{cav}} + \Gamma_{\text{at}})/2$, in which $\Gamma_{\text{cav}}, \Gamma_{\text{at}}$ are the decaying rate of the cavity and atom respectively. If the dissipation is not so high (not strong coupling) that energy broadening is larger than energy separation, new energy levels are overlapping with each other, sharp resonant transition can no longer be observed. In this sense, strong coupling is essential in cavity QED, requiring high Q of a Fabry-Perot cavity as well as long lifetime of a two-level atom.

2.C. Jaynes-Cummings ladder

Jaynes-Cummings model [4] tried to solve the cavity QED problem from purely quantum mechanics point of view: quantized atomic energy level and quantized electromagnetic field (photon in stead of light wave). Energy level splitting is expected, as in the semi-classical model. However, the energy splitting is inharmonic, which is not expected before. Quantum number $n$ refers to the photon number of a specific eigenstate of the coupled system. As shown in Figure 1(b), the energy level separation equals $\sqrt{n}\Omega_0$. This inharmonic energy level splitting leads to photon blockade.

3. Experiments Observation

This experiment was finished in Caltech Kimble’s group and published on Nature 2005 [5].
Fig. 2. The atomic level structure used for implementation of the photon blockade effect and simple diagram of the experiment. (a): Atomic level diagram showing the lowest-energy states for a two-state atom of transition frequency $\omega_{at}$ coupled (with single-photon Rabi frequency $g_0 = \Omega_0/2$) to a mode of the Febry-Perot cavity of frequency $\omega_{cav}$, with $\omega_{at} = \omega_{cav} \equiv \omega_0$. Two (or more) photon absorption is suppressed for a probe field $\omega_p$ (arrows) tuned to excite transition $|0 \rightarrow |1, - >$, $\omega_p = \omega_0 - g_0$, leading to photon blockade. (b): Simple diagram of the experiment. (c): Experimental measurements of the intensity correlation function $g^{(2)}(\tau)$. $g^{(2)}(0) < g^{(2)}(\tau)$ indicates photon anti-bunching, $g^{(2)}(\tau)$ recovers to unity at $\tau = 45\,ns$, which is consistent with the lifetime for state $|1, - > = 48\,ns$.
3.A. Principle

As discussed in Section 2 and also shown in Figure 2(a), energy level of the system splits due to strong coupling of the atom and cavity mode. Instead of one photon resonant at $\omega_{at} = \omega_{cav} \equiv \omega_0$, it now resonant at frequency $\omega_p = \omega_0 \pm g_0$, and with two photon resonant at $2\omega_p = 2\omega_0 \pm \sqrt{2}g_0$.

Now if one tune the probe beam at resonant with $\omega_p = \omega_0 - g_0$, as the red arrow shows, absorption of one photon will block the absorption of a second photon because it’s out of two-photon resonance. The system can only accept the next photon until the first one leave the system due to decaying. This is called photon blockade effect. In analog to classical saturation effect, one can use one photon to saturate the absorption response the atom.

3.B. Results

Anti-bunching is expected due to photon blockade: if the detector registers one photon at some time $t$, it won’t register a second photon in the following time period corresponding to the lifetime of the system, because two (or more) photon is not accepted by the system. Results of the above experiment is shown in Figure 2(c) by measuring the intensity correlation function of the output. $g^{(2)}(0)$ being significantly smaller than $g^{(2)}(\tau)$ is a clear manifestation of photon anti-bunching. $g^{(2)}(\tau)$ rises to unity at a time $\tau = 45ns$ is consistent with a estimation of $\tau_- = 48ns$ based on the lifetime for the state $|1, - >$.

4. Summary

This report analyzes the basic principles of cavity QED and Jaynes-Cummings model in solving this problem, showing the inharmonic energy level splitting due to strong coupling between cavity mode and two-level atom. Jaynes-Cummings ladder leads to photon blockades effect, which has already demonstrated experimentally by Kimble, et al [5]. Photon blockade effect represent the strong interaction between photon and photon taken one by one, thus hitting the limit of nonlinear optics.
References