And we can solve for the radius:

\[ R_S = \left( \frac{3N}{4\pi \alpha} \right)^{1/3} n_e^{-2/3} \]

NB, \( n_e \) in cgs units cm\(^{-3} \)

\( R_S \) is the \textit{Strömgren radius}.
The sphere is called \textit{Strömgren sphere}.
Not always this simple
$N$ comes from knowing spectral type of star, with a model atmosphere to predict the UV flux output.

The expression on the previous slide assumes:

1 star - in reality there may be many

all electrons are from hydrogen

all ionizing photons are absorbed by hydrogen

no ionizing photons escape - but probably $\leq 50\%$ do - WIM

Real HII regions are messy!
Example: The Rosette Nebula
In the center is a star cluster, NGC2244. Hα
Example

O6 star with $T_{\text{eff}} = 45,000 \text{ K}$
$L = 1.3 \times 10^5 \text{ L}_\odot$

What is the Number of ionizing photons?
What is the radius of the Stromgren Sphere?
Example

O6 star with \( T_{\text{eff}} = 45,000 \, \text{K} \)
\[ L = 1.3 \times 10^5 \, \text{L}_\odot \]

Assuming a blackbody spectrum, we use Wien's law:

\[ \lambda_{\text{max}} = \frac{0.29 \, \text{cmK}}{45,000 \, \text{K}} = 644 \, \text{Å} \]

To simplify, assume all ionizing photons from star have \( \lambda = \lambda_{\text{max}} \), and the star only emits ionizing photons.

Then, photon energy
\[ E = \frac{\hbar c}{\lambda_{\text{max}}} = 19 \, \text{eV} \]

and thus
\[ N = \frac{L}{E} = \frac{3 \times 10^{50} \, \text{eV}s^{-1}}{19 \, \text{eV}} = 1.6 \times 10^{49} \, \text{s}^{-1} \]
For H at temperatures and densities of HII regions, we use \( \alpha = 3 \times 10^{-13} \text{ cm}^3\text{s}^{-1} \). Assuming also \( n_e = 1000 \text{ cm}^{-3} \), then:

\[
R_s = \left( \frac{3 \times 1.6 \times 10^{49}}{4\pi 3 \times 10^{-13}} \right)^{1/3} (10^3)^{-2/3} = 2.33 \times 10^{18} \text{ cm} = 0.8 \text{ pc}
\]
Orion Nebula:

HST. Well studied region with ongoing star formation.
Orion Nebula:

VLA. Well studied region with ongoing star formation.

Felli et al. 1993
Orion Nebula in IR showing Becklin-Neugebauer object

Orion Trapezium, IR K-Band, 5.5’ x 5.5’ FOV

The Orion radio zoo - PIGS, DEERS and FOXES
Schematic diagram: Balance of ionization and recombination sets up HII region.

- UV light from hot O star
- Ionized H gas (HII region)
- Neutral H gas (HI region)
- Hot gas
- Cool gas

A few light years ~ 1 pc
Orion Nebula: Case study of star formation. A model of the association of the Orion Nebula, its molecular cloud and infrared sources.

- Orion Nebula
- Ionized gas
- Trapezium cluster
- Infrared cluster
- Less dense molecular cloud
- Dense molecular cloud
- Dust
- LOS

2 light years = 0.6 pc
Strömgren spheres: properties

<table>
<thead>
<tr>
<th>Spectral type</th>
<th>$M_v$</th>
<th>$T_\ast$ (°K)</th>
<th>Log $Q(H^0)$ (photons/sec)</th>
<th>Log $N_e N_p r_1^3$ ($N$ in cm$^{-3}$; $r_1$ in pc)</th>
<th>$r_1$ (pc)</th>
<th>($N_e = N_p = 1$ cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O5</td>
<td>− 5.6</td>
<td>48,000</td>
<td>49.67</td>
<td>6.07</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>O6</td>
<td>− 5.5</td>
<td>40,000</td>
<td>49.23</td>
<td>5.63</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>O7</td>
<td>− 5.4</td>
<td>35,000</td>
<td>48.84</td>
<td>5.24</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>O8</td>
<td>− 5.2</td>
<td>33,500</td>
<td>48.60</td>
<td>5.00</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>O9</td>
<td>− 4.8</td>
<td>32,000</td>
<td>48.24</td>
<td>4.64</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>O9.5</td>
<td>− 4.6</td>
<td>31,000</td>
<td>47.95</td>
<td>4.35</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>B0</td>
<td>− 4.4</td>
<td>30,000</td>
<td>47.67</td>
<td>4.07</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>B0.5</td>
<td>− 4.2</td>
<td>26,200</td>
<td>46.83</td>
<td>3.23</td>
<td>12</td>
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</tr>
</tbody>
</table>

NOTE: $T = 7500°$ K assumed for calculating $\alpha_B$. 
Astronomy 422

Lecture 4: The Interstellar Medium IV
Announcements:

Project topics are due on Tuesday (title + one paragraph)
Tools - ADS, NED

Key concepts:

Molecular clouds

Dust

The Initial Mass Function (IMF)
Mostly molecular hydrogen $\text{H}_2$, which is easily dissociated:

$$\text{H}_2 \rightarrow \text{H} + \text{H}$$

by photons with $E > 10.2 \text{ eV (}\lambda < 1216 \text{ Å)}$

This means that we only find $\text{H}_2$ in dense regions which are shielded from radiation.
Molecular clouds are
dense: \( n \sim 10^2-10^3 \text{ cm}^{-3} \)
cold: \( T \sim 10-50 \text{ K} \)

Recall from last semester that these are the conditions under which star formation occurs.

**How to observe molecular gas:**

\( \text{H}_2 \) does not have any electric dipole transition \( \Rightarrow \) weak emission. Instead we use the CO molecule as the most common *tracer*. 
Low energies of CO rotational levels => easily excited by CO-H$_2$ collisions, despite low temperatures.

Radiative de-excitation occurs via $\lambda$ ≈ mm photons.

*mm-wave astronomy*
Masses

If molecular clouds are gravitationally bound and in equilibrium, the Virial Theorem states:

$$-2 < KE > = < PE >$$

Per molecule:

$$KE \sim \frac{1}{2} m_{H_2} v_{rms}^2 \quad PE \sim -\frac{GMm_{H_2}}{R}$$

Then

$$M = M_{vir} \sim \frac{Rv_{rms}^2}{G}$$

If spectral line is broadened by motion of molecules, get $v_{rms}$ from linewidth.
We get $R$ from mapping the extent of the cloud (must know its distance). From this we will find the virial mass $M_{\text{vir}}$.

Observation: $L_{\text{CO}} \propto M_{\text{vir}}$

Thus, if clouds are virial, $L_{\text{CO}}$ is a good tracer of H$_2$ mass.

From this we derive conversion factor $X$ from CO to H$_2$ mass.

Scoville et al 1987
The CO-to-H$_2$ Conversion Factor

Ratio of virial mass to CO luminosity vs. metallicity.

- $X_{\text{CO}}$ for virialized clouds.

No strong trend.

SMC completely compatible with MW clouds and Solomon (0.8) slope...

- i.e., $M_{\text{vir}}/L_{\text{CO}} \sim L_{\text{CO}}^{-0.2}$

Bolatto et al. 2008
**Luminosity-Virial Mass Relation**

(One version of other independent Larson’s Law)

We find $M_{\text{vir}} \sim L_{\text{CO}}$ ...
- Solomon ($MW$): $M_{\text{vir}} \sim L_{\text{CO}}^{0.8}$

CO-to-H$_2$ factor roughly as $MW$ if GMCs are virialized...

SMC falls on Galactic line.

Other scaling relations:
- $L_{\text{CO}}$ vs. $R$
- $L_{\text{CO}}$ vs. $\sigma$
- $M_{\text{vir}}$ vs. $R$

follow from these two.

Bolatto et al. 2008
Results:

Most molecular gas is in *Giant Molecular Clouds* (GMCs):

- A few hundred known in the Milky Way
- \( M \approx 10^3 - 10^7 \, M_\odot \)
- sizes 5-100 pc.

Associated with the spiral arms of the Milky Way (more on this later).

---

*Figure 1. Positions of superclouds in the I quadrant (sizes are proportional to masses) and within the Car arm in the IV quadrant (sizes are arbitrary), superimposed on the H I ridges and the local distribution of young clusters and associations (after Weaver, 1970). Two extreme possible positions for the bar are also shown. Within the I quadrant the superclouds for which the distance may be uncertain (far distances being adopted) are marked with a tick.*

Efremov 1988
From *Universe* (Freedman & Kaufman)
Dust
Even if you don't study the ISM, every astronomer needs to be aware of it. Dust scatters and absorbs light making stars appear dimmer and redder: *extinction* and *reddening*.

Extinction
Recall the distance modulus, e.g., in V-band:

\[ m_V - M_V = 5 \log d - 5 \]

- If \( m_V \) and \( M_V \) are known, we get the distance. Include dimming by dust, on the magnitude scale:

\[ m_\lambda = M_V + 5 \log d - 5 + a_\lambda \]

where \( a_\lambda \) = absorption at \( \lambda \) in magnitudes.
Example, $a_{\lambda} = 1 \Rightarrow$ dust along LOS dims object by a factor 2.5.

We can relate $a_{\lambda}$ to physical dust properties. Recall:

\[ I_\lambda = I_{0,\lambda} e^{-\tau_\lambda} \]

\[ m_1 - m_2 = -2.5 \log \frac{I_1}{I_2} \]

thus \[ a_{\lambda} = m_\lambda - m_{0,\lambda} = -2.5 \log e^{-\tau_\lambda} \]

\[ a_{\lambda} = 1.086 \tau_\lambda \]
Also recall:

\[ \tau_\lambda = \int_0^s n\sigma_\lambda \, ds \]

- \( n \) - number density of particles
- \( \sigma_\lambda \) - absorption and scattering cross section at \( \lambda \)
- \( s \) - path length

If \( \sigma_\lambda \) is constant along \( s \),

\[ \tau_\lambda = \sigma_\lambda \int_0^s n \, ds = \sigma_\lambda N_d \]

where \( N_d \) is the dust column density.
Reddening
Dust grains ~0.1\( \mu \)m in size. If \( a = \) grain radius, the *extinction coefficient* \( Q_\lambda \) is defined by:

\[
\sigma_\lambda = \pi a^2 Q_\lambda
\]

For \( \lambda \rightarrow 0 \), \( Q_\lambda \sim \) constant

For \( \lambda \sim a \) \( Q_\lambda \sim \frac{a}{\lambda} \)

For \( \lambda \gg a \) \( Q_\lambda \rightarrow 0 \),

Exact behavior depends on dust composition. Thus, dust grains are efficient at absorption and scattering wavelengths that are small compared to their size, inefficient at scattering longer wavelengths \( \Rightarrow \) *reddening*. 
Extinction versus wavelength
Take the spectrum of a star.
Absorption lines $\Rightarrow$ spectral type $\Rightarrow T_{\text{eff}}$ $\Rightarrow$ spectrum shape if no dust.
Difference compared to observed shape $\Rightarrow$ extinction versus $\lambda$. 

![Graph showing extinction versus wavelength]

- Expected spectrum without dust
- Observed spectrum

$I_\lambda$ vs. $\lambda$
Often measured in terms of color excess. For example, observed versus intrinsic color in B,V bands.

\[ E(B - V) \equiv (B - V) - (B - V)_0 \]

observed \hspace{1cm} intrinsic

Q: For non-zero dust column densities, will E(B-V) be positive or negative?
This shows the amount of dimming at $\lambda$ for an amount of dust that gives 1 mag of reddening in B-V, ie, amount of dust that makes B dimming 1 mag more than V dimming.

Why this way?

Because $E(B-V)$ commonly measured, then we can infer $a_{\lambda}$.

Example:

$A_V = 3.1 \ E(B-V)$

$A_{1100\text{Å}} = 11 \ E(B-V)$

Basic shape $\sim 1/\lambda$ (Mie).

Bump at $1/\lambda \sim 4$ ($\sim 2000\text{Å}$) probably due to graphite grains. Other features suggest silicon compounds, PAHs.
What happens to absorbed starlight?
Grains heated to 10-100s of K => IR blackbody emission.
This provides way to estimate extinction. See paper by Schlegel, Finkbeiner and Davis (1998).
Basic idea - if you know the dust temperature from the IR color, then you can derive the dust column density from 100\(\mu\)m all-sky maps.

=> extinction calculated at any position in sky.

Very useful!
Finally: The Mass Function

Whenever you have a large number of objects with various masses, useful to describe the number as a function of mass, \( N(M) \), or size, \( N(R) \). Constrains theories of their origin. Useful for Kuiper Belt, asteroids, impact craters, Saturn’s ring particles, stars, gas clouds, galaxies.

Often have many small objects and a few large ones, which we can try to describe with a “power law” mass function:

\[
N(M) \propto M^{-\beta}
\]

Gives relative importance of large and small objects
For KBOs, can measure reflectivity of Solar radiation. Know distance from Sun by measuring orbit. From this and assumed albedo, can get radius of each. Find

\[ N(R) \propto R^{-4} \]

If they all have the same density then \( M/R^3 = \) constant, so \( M \propto R^3 \), so \( R^{-4} \propto M^{-4/3} \), and

\[ N(M) \propto M^{-4/3} \]

Now can ask, for example, how many are there of mass \( M_1 \) vs. \( 10xM_1 \)?

\[
\frac{N(M_1)}{N(10xM_1)} = \frac{M_1^{-4/3}}{(10xM_1)^{-4/3}} = 10^{4/3} = 21.5
\]
Can also ask: is most of the mass in larger KBOs or smaller ones? For example, how much mass in objects of mass $M_1$ vs. objects of mass $10xM_1$?

If you have $N$ objects of mass $M$, the total mass is $MxN$. So for $N=N(M)$, total mass is $MxN(M) \alpha MxM^{-4/3}$ or $M^{-1/3}$. So relative mass is

$$M_1^{-1/3}/(10xM_1)^{-1/3} = 10^{1/3} = 2.2$$

So more mass in lower mass objects. Recall $\beta$ was $-4/3$. Note if $\beta < -1$, more mass in higher mass objects.
Stellar Initial Mass Functions

mass fraction per dex

mass (m / M_{solar})
Initial Mass Function for Stars

By observing the relative numbers of various masses of stars, we can deduce something about the cloud fragmentation process.

The *initial mass function* (IMF) describes the relative numbers of each stellar mass. Defined for stars in the Solar neighborhood by Salpeter (1955):

\[
\xi(M) = \xi_0 M^{-2.35}
\]

\[M = \text{mass in solar units.}\]

Thus, the number of stars that form with masses between \(M\) and \(\Delta M\):

\[\xi(M) \Delta M\]

Total number of stars formed with masses \(M_1\) and \(M_2\):

\[
N = \int_{M_1}^{M_2} \xi(M) dM = \xi_0 \int_{M_1}^{M_2} M^{-2.35} dM = \frac{\xi_0}{1.35} \left[ M_1^{-1.35} - M_2^{-1.35} \right]
\]
Similarly, we can work out the total mass in stars born within that given mass range:

\[ M_{\text{tot}} = \int_{M_1}^{M_2} M \xi(M) dM \]

Properties of the Salpeter IMF:

- most of the stars, by number, are low mass stars
- most of the mass in stars reside in low mass stars
- following a burst of star formation, most of the luminosity comes from high mass stars.

The Salpeter IMF fails at low masses, since extrapolating to very low masses means total mass \( \rightarrow \infty \)

Observations implies Salpeter IMF valid for \( M > 0.5 \, M_\odot \), and that it flattens at lower masses.
Example:

Consider a cloud with a total mass of $1000M_\odot$. How many $10M_\odot$ stars are formed if it follows the Salpeter IMF and forms stars over a range from 1 to $50M_\odot$?
Next time:

- The structure of the Milky Way
- Stellar number density
- Age-metallicity relation

Read chapter 24.1-24.2