Initial Mass Function for Stars
By observing the relative numbers of various masses of stars, we can deduce something about the cloud fragmentation process.

The *initial mass function* (IMF) describes the relative numbers of each stellar mass. Defined for stars in the Solar neighborhood by Salpeter (1955):

$$\xi(M) = \xi_0 M^{-2.35}$$

$M =$ mass in solar units.

Thus, the number of stars that form with masses between $M$ and $\Delta M$:

$$\xi(M) \Delta M$$

Total number of stars formed with masses $M_1$ and $M_2$:

$$N = \int_{M_1}^{M_2} \xi(M) dM = \xi_0 \int_{M_1}^{M_2} M^{-2.35} dM = \frac{\xi_0}{1.35} \left[ M_1^{-1.35} - M_2^{-1.35} \right]$$
Similarly, we can work out the total mass in stars born within that given mass range:

\[ M_{\text{tot}} = \int_{M_1}^{M_2} M \xi(M) dM \]

Properties of the Salpeter IMF:

- most of the stars, by number, are low mass stars
- most of the mass in stars reside in low mass stars
- following a burst of star formation, most of the luminosity comes from high mass stars.

The Salpeter IMF fails at low masses, since extrapolating to very low masses means total mass \( \rightarrow \infty \)

Observations implies Salpeter IMF valid for \( M > 0.5 \, M_\odot \), and that it flattens at lower masses.
Example:

Consider a cloud with a total mass of $1000M_\odot$.

How many $10M_\odot$ stars are formed if it follows the Salpeter IMF and forms stars over a range from 1 to $50M_\odot$?
Astronomy 422

Lecture 5: The Milky Way Galaxy I
Key concepts:

Morphology

Stellar number density

ISM distribution
Structure of the Milky Way is hard to determine from where we are sitting
The Milky Way Galaxy
Take a Giant Step Outside the Milky Way

Artist's Conception

Example (not to scale)
Take a Giant Step Outside the Milky Way

Artist's Conception

- Galactic halo
- Galactic disk
- Gas and dust
- Open cluster
- Globular clusters
- O, B stars
- Emission nebula
- Galactic center
- Sun

Example (not to scale)
Shapley (~1915) measured distances to globular clusters from RR Lyraes, and estimated:

- Diameter of Milky Way to ~ 100 kpc.
- Sun-to-center distance \( R_0 \approx 15 \) kpc. This was about two times too large, due to a poor luminosity calculation. Modern value \( R_0 \approx 8 \) kpc.
Shapley reasoned globular clusters were so massive that they would trace the Galactic gravitational potential: with spherical distribution centered on MW.

- Star counts (Kapteyn 1922, still used today): For stars of some fixed $L$ or $M_V$, count them in random directions.
  - Distribution of stars with different apparent brightness gives distribution in space.
Simple example: Class of stars with luminosity $L$. For star at a distance $r_0$:

$$f_0 = \frac{L}{4\pi r_0^2}$$

Stars at $r<r_0$ will have $f>f_0$ and vice versa.

Suppose density, $n(L)$, independent of $r$. Number of stars of luminosity $L$ with $f>f_0$ is:

$$N_L(f > f_0) = n(L) \frac{4\pi r_0^3}{3} = \frac{n(L)L^{3/2}}{3(4\pi)^{1/2}} f_0^{-3/2}$$

If power $>-3/2$, $n(L)$ decreases with $r$.

If power $<-3/2$, $n(L)$ increases with $r$. 
Example 1: Uniform distribution gives

\[ N_L(f > f_0) \propto f_0^{-3/2} \]

Example 2: If \( n(L) \propto r \), the number \( N \) in a volume is

\[ n \frac{4\pi r_0^3}{3} \propto r_0^4 \]

Since \( r_0^4 \propto f_0^{-2} \), it grows faster than \( f_0^{-3/2} \)
Actual star counts are less than predicted by a uniform distribution.

\[
\log N \\
(f > f_0)
\]

\[\propto f_0^{-3/2}\]

Star counts in the direction of the disk, and perpendicular to it (blue and red lines respectively).
The Kapteyn Universe
The conclusions are that the stellar density is NOT uniform:
It decreases as a function of distance from the Solar system.
It decreases more slowly in the plane of the Milky Way, and faster perpendicular to it.
Kapteyn used star counts to find more details about the Milky Way shape:

• ~ 8 kpc across
• ~ 2 kpc thick
• ~ 650 pc distance Sun-to-Center

Compare modern values: ~30-50 kpc across, and $R_0$~8kpc.

Q: Why so far off?

A: Didn't know about dust. Extinction mimics density falling with r.
What is a direct way to measure $R_0$?
Precise Distance to Galactic Center

Distance = 8 kpc

Orbital motion 6.37 mas/yr
Precise Distance to Galactic Center

Distance = 7.94 +/- 0.42 kpc

Eisenhauer et al. 2003
Modern view of the Milky Way:

- Thick disk
- Thin disk
- Central bulge
- Central bar
- Molecular ring outside bar
- Halo
- Gas and Dark Matter

We also know something about the age. Iron abundance in stellar atmosphere gives us *metallicity*.

\[
\left[ \frac{Fe}{H} \right] \equiv \log \left( \frac{N_{Fe}}{N_H} \right) - \log \left( \frac{N_{Fe}}{N_H} \right)_\odot
\]

From cluster MS turnoff points, older cluster => lower metallicity.
Use calibration from clusters for field stars.
Some Milky Way parameter values (Table 24.1 in C&O)

<table>
<thead>
<tr>
<th>Disks</th>
<th>Neutral Gas</th>
<th>Thin Disk</th>
<th>Thick Disk</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M (10^{10} M_\odot))</td>
<td>0.5(^a)</td>
<td>6</td>
<td>0.2 to 0.4</td>
</tr>
<tr>
<td>(L_B (10^{10} L_\odot))</td>
<td>—</td>
<td>1.8</td>
<td>0.02</td>
</tr>
<tr>
<td>(M/L_B (M_\odot/L_\odot))</td>
<td>—</td>
<td>3</td>
<td>—</td>
</tr>
<tr>
<td>Radius (kpc)</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Form</td>
<td>(e^{-z/h_z})</td>
<td>(e^{-z/h_z})</td>
<td>(e^{-z/h_z})</td>
</tr>
<tr>
<td>Scale height (kpc)</td>
<td>&lt; 0.1</td>
<td>0.35</td>
<td>1</td>
</tr>
<tr>
<td>(\sigma_w \ (\text{km s}^{-1}))</td>
<td>5</td>
<td>16</td>
<td>35</td>
</tr>
<tr>
<td>[Fe/H]</td>
<td>&gt; +0.1</td>
<td>-0.5 to +0.3</td>
<td>-2.2 to -0.5</td>
</tr>
<tr>
<td>Age (Gyr)</td>
<td>(\lesssim 10)</td>
<td>8(^c)</td>
<td>10(^d)</td>
</tr>
</tbody>
</table>

\(^a\) \(M_{\text{dust}}/M_{\text{gas}} \approx 0.007\).
\(^b\) The total luminosity of the Galaxy is \(L_{B,\text{tot}} = 2.3 \pm 0.6 \times 10^{10} L_\odot\), \(L_{\text{bol, tot}} = 3.6 \times 10^{10} L_\odot\) (\(\sim 30\%\) in IR).
\(^c\) Some open clusters associated with the thin disk may exceed 10 Gyr.
\(^d\) Major star formation in the thick disk may have occurred 7–8 Gyr ago.

• Disks are also exponential in \(r\)
• \(\sigma_w\) related to the thickness
Metallicity of bulge:
  - bursts of star formation?
  - accretion of satellites?
  - gas inflow?

<table>
<thead>
<tr>
<th>Spheroids</th>
<th>Central Bulge&lt;sup&gt;e&lt;/sup&gt;</th>
<th>Stellar Halo</th>
<th>Dark-Matter Halo</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ ($10^{10} , M_\odot$)</td>
<td>1</td>
<td>0.3</td>
<td>$190^{+360}_{-170}$</td>
</tr>
<tr>
<td>$L_B$ ($10^{10} , L_\odot$)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.3</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>$M/L_B$ ($M_\odot/L_\odot$)</td>
<td>3</td>
<td>$\sim 1$</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>Radius (kpc)</td>
<td>4</td>
<td>$&gt; 100$</td>
<td>$&gt; 230$</td>
</tr>
<tr>
<td>Form</td>
<td>boxy with bar</td>
<td>$r^{-3.5}$</td>
<td>$(r/a)^{-1} (1 + r/a)^{-2}$</td>
</tr>
<tr>
<td>Scale height (kpc)</td>
<td>0.1 to 0.5&lt;sup&gt;g&lt;/sup&gt;</td>
<td>3</td>
<td>170</td>
</tr>
<tr>
<td>$\sigma_{w}$ (km s&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>55 to 130&lt;sup&gt;h&lt;/sup&gt;</td>
<td>95</td>
<td>—</td>
</tr>
<tr>
<td>[Fe/H]</td>
<td>$-2$ to 0.5</td>
<td>$&lt;-5.4$ to $-0.5$</td>
<td>—</td>
</tr>
<tr>
<td>Age (Gyr)</td>
<td>$&lt; 0.2$ to 10</td>
<td>11 to 13</td>
<td>$\sim 13.5$</td>
</tr>
</tbody>
</table>

<sup>e</sup> The mass of the black hole in Sgr A* is $M_{bh} = 3.7 \pm 0.2 \times 10^6 \, M_\odot$.

<sup>f</sup> $M = 5.4^{+0.2}_{-3.5} \times 10^{11} \, M_\odot$ within 50 kpc of the center.

<sup>g</sup> Bulge scale heights depend on age of stars: 100 pc for young stars, 500 pc for old stars.

<sup>h</sup> Dispersions increase from 55 km s<sup>-1</sup> at 5 pc to 130 km s<sup>-1</sup> at 200 pc.
Stellar populations

Pop I: small velocity dispersion, heavy metal absorption lines, confined to thin plane, young.

Pop II: large velocity dispersion, located further from plane.

Pop I mostly in the disk, less in bulge, not in halo.
Pop II can be found in all three places.

Visible mass of Galactic halo small compared to disk and bulge. Some arguments to believe that mass of halo is comparable to the rest, leading to a hypothetical Pop III stars (dim, low-metal stars).
Disk component

Stellar number density in the disk:

\[ n(Z, R) = n_0 \left( e^{-Z/Z_{\text{thin}}} + 0.085 e^{-Z/Z_{\text{thick}}} \right) e^{-R/h_R} \]

where

- **Z**: height above the plane
- **R**: distance from the Galactic Center
- **n_0**: 0.02 stars/pc\(^3\) (4.5 \leq M_V \leq 9.5, G to M stars only)
- **Z_{\text{thin}}**: \sim 350 pc
- **Z_{\text{thick}}**: \sim 1 kpc
- **h_R**: \geq 2.25 kpc

Radial properties hard because of dust.
Disk mass-to-light ratio gives an estimate of average stellar mass:

- Total mass of disk $6 \times 10^{10} \, M_\odot$
- Total luminosity of disk (stars) in B-band: $1.8 \times 10^{10} \, L_\odot$

Thus,

$$\frac{M}{L_B} \approx 3 \frac{M_\odot}{L_\odot}$$

Recall that $L \propto M^\alpha$, with $\alpha \sim 3-4$, and find a typical stellar mass $\sim 0.7 \, M_\odot$

Thus, a higher $M/L$ means lower stellar masses.
Spheroidal components

Central bulge:

~ 2 kpc in diameter, vertical scale height ~0.4 kpc.
Consists of spheroidal component and bar

Range of metallicity => wide range of stellar ages.
Galactic bulge, as observed by COBE (1.2 to 3.4 micrometers).
Halo:
Composed of globular clusters and *field stars*. Roughly spherical.

Globular clusters ~ 200 or so (?). Oldest stars in the Milky Way. Two populations:

1. Older, very spherical distribution in halo.
2. Somewhat younger, flattened distribution (like a thick disk).
Milky Way ISM distributions

- HI: density roughly constant with $R$, located outside a central hole of 3 kpc radius. Total extent $\sim 50$ kpc diameter.

  Vertically \[ n = n_0 e^{-Z/160\text{pc}} \quad n_0 \approx 1\text{cm}^{-3} \]

- H$_2$: CO indicates a \textit{molecular ring} centered at $R=5$ kpc. Vertically, there is a population of GMCs $\sim 100$ pc.

- HII regions: radial distribution like H$_2$, similar scale height.

- Overall: mass in HI $\sim$ mass in H$_2$ $\sim 2 \times 10^9 M_\odot$
  
  HII, WIM, HIM, dust much less, $\sim$ a few % of total mass.
HI observations of the LMC: (Kim et al). HI shells and super shells.
Scoville & Sanders
Spiral structure
Clear in HI, GMC, HII regions, OB stars, and young clusters. The exact shape is hard to infer from within the Galaxy.

Large scale
Open circles: giant HI clouds
Squares: GMCs
Triangles: optical HII regions
Small boxes: radio HII regions

Elmegreen 1985
Smaller scale:

Distribution of young Galactic clusters and HII regions.

Sun in center, near Orion-Cygnus Arm.
HI layer is 'warped' at radii beyond $R \sim 15$ kpc.
The origin of this warp is unclear: interactions? But isolated galaxies also exhibits warps. Bending modes? Halo changing shape as new material accreted?

Plan view showing z-heights of HI.
Kwee et al.
High velocity clouds ($v > 90$ km/s), up to $10^8$ M$_{\text{sun}}$ in total

Seen at 21 cm, with high velocities up to 500 km/s. Mixed metallicities.

Many partially ionized, and can contribute up to 1 M$_{\text{sun}}$/year (Lehner & Howk 2011)
However, many HVCs have subsolar metallicity suggesting a more primordial
Magellanic stream
21cm emission, about 180 deg across. Tidal debris tail. Gas falling into the Milky Way
Could be as much as 0.4 Msun/year (van Woerden et al. 2004)
**Coronal gas**
Observed in highly ionized lines, e.g. far-UV OIV (absorption).
Next time:

Milky Way kinematics
Galactic coordinate system
Rotation curves
LSR velocities

Read chapter 24.3