Cross Correlators

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Outline

- The correlation function
- What is a correlator?
- Simple correlators
- Sampling and quantization
- Spectral line correlators
- The EVLA correlator in detail

This lecture is complementary to Chapter 4 of ASP 180 and is based on a lecture by Walter Brisken
The [old] VLBA Correlator

G. Taylor, Astr 423 at UNM
For continuous functions, $f$ and $g$, the cross-correlation is defined as:

$$(f \ast g)(t) \overset{\text{def}}{=} \int_{-\infty}^{\infty} f^*(\tau) \, g(t + \tau) \, d\tau,$$

where $f^*$ denotes the complex conjugate of $f$. 
The Correlation Function

\[ C_{ij}(\tau) = \langle v_i(t)v_j(t + \tau) \rangle_T \]

- If \( i = j \) it is an auto-correlation (AC). Otherwise it is a cross-correlation (CC).
- Useful for
  - Determining timescales (CC and AC)
  - Motion detection (2-D CC)
  - Optical character recognition (2-D CC)
  - Pulsar timing / template matching (CC)
What is a Correlator?

A correlator is a hardware or software device that combines sampled voltage time series from one or more antennas to produce sets of complex visibilities, $V_{ij}$.

- Visibilities are in general a function of
  - Frequency
  - Antenna pair
  - Time
- They are used for
  - Imaging
  - Spectroscopy / polarimetry
  - Astrometry
A Real (valued) Cross Correlator

\[ C_{ij}(\tau) = \langle v_i(t)v_j(t + \tau) \rangle_T \]

\[ v_i(t) \quad \tau \quad v_j(t) \]

\[ \frac{1}{T} \int_0^T (\cdot) dt \]

\[ C_{ij} \]
What astronomers really want is the complex visibility

$$V_{ij} = \langle E_i(t) E_j^*(t + \tau) \rangle$$

where the real part of $E_i(t)$ is the voltage measured by antenna $i$.

So what is the imaginary part of $E_i(t)$?

It is the same as the real part but with each frequency component phase lagged by 90 degrees.

$$E_i(t) = v_i(t) + \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{v_i(t')}{{t - t'}} \, dt'$$
\[ V_{ij} = \langle v_i(t)v_j(t + \tau) \rangle + i \langle \mathcal{H}[v_i(t)]v_j(t + \tau) \rangle \]
• \( \{v_i(t)\} \) are real-valued time series sampled at “uniform” intervals, \( \Delta t \).

• The sampling theorem allows this to accurately reconstruct a bandwidth of \( \Delta \nu = \frac{1}{2\Delta t} \).

• Sampling involves quantization of the signal
  - Quantization noise
  - Strong signals become non-linear
  - Sampling theorem violated!
Quantization Noise

Quantization efficiency

<table>
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<tr>
<th>N levels</th>
<th>$\eta_Q(f = 1)$</th>
<th>$\eta_Q(f = 2)$</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>0.64</td>
<td>0.74</td>
</tr>
<tr>
<td>3</td>
<td>0.81</td>
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<tr>
<td>4</td>
<td>0.88</td>
<td>0.94</td>
</tr>
<tr>
<td>$\infty$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

(For normal-distributed $v$)
Automatic Gain Control (AGC)

- Normally prior to sampling the amplitude level of each time series is adjusted so that quantization noise is minimized.
- This occurs on timescales very long compared to a sample interval.
- The magnitude of the amplitude is stored so that the true amplitudes can be reconstructed after correlation.
The Correlation Coefficient

- The correlation coefficient, $\rho_{ij}$ measures the likeness of two time series in an amplitude independent manner:

$$
\rho_{ij} = \frac{|V_{ij}|}{\sqrt{V_{ii} V_{jj}}}
$$

- Normally the correlation coefficient is much less than 1

- Because of AGC, the correlator actually measures the correlation coefficient. The visibility amplitude is restored by dividing by the AGC gain.
Van Vleck Correction

- At low correlation, quantization *increases correlation*
- Quantization causes predictable non-linearity at high correlation $V_{ij}$
- Correction must be applied to the real and imaginary parts of separately
  - Thus the visibility phase is affected as well as the amplitude

![Graph showing Van Vleck Correction](image-url)
The Delay Model

- $\tau$ is the difference between the geometric delays of antenna $j$ and antenna $i$. It can be + or -.
- The *delay center* moves across the sky
  - $\tau$ is changing constantly
- Fringes at the delay center are stopped.
  - Long time integrations can be done
  - Wide bandwidths can be used
- Simple delay models incorporate:
  - Antenna locations
  - Source position
  - Earth orientation
- VLBI delay models must include much more!
Pulsar Gating

- Pulsars emit regular pulses with small duty cycle
- Period in range 1 ms to 8 s; \( \Delta t \ll P_{\text{pulsar}} < T \)
- Blanking during off-pulse improves sensitivity
- Propagation delay is frequency dependent
Spectral Line Correlators

- Chop up bandwidth for
  - Calibration
    - Bandpass calibration
    - Fringe fitting
  - Spectroscopy
  - Wide-field imaging
  - (It's all Spectral Line these days)

- Conceptual version
  - Build analog filter bank
  - Attach a complex correlator to each filter
Practical Spectral Line Correlators

- Use a single filter / sampler
  - Easier to calibrate
  - Practical, up to a point

- The FX architecture
  - F : Replace filterbank with digital Fourier transform
  - X : Use a complex-correlator for each frequency channel
  - Then integrate

- The XF architecture
  - X : Measure correlation function at many lags
  - Integrate
  - F : Fourier transform

- Other architectures possible
The FX correlator
FX Correlators

- Spectrum is available **before integration**
  - Can apply fractional sample delay per channel
  - Can apply pulsar gate per channel
- Most of the digital parts run $N$ times slower than the sample rate
- Fewer computations (compared to XF)
FX Spectral Response

- FX Correlators derive spectra from truncated time series

\[ v(\nu) = \mathcal{F} \left[ v(t) \cdot \mathcal{F} \left( \frac{t}{N \Delta t} \right) \right] \]
\[ = \mathcal{F} \left[ v(t) \right] \star \mathcal{F} \left[ \mathcal{F} \left( \frac{t}{N \Delta t} \right) \right] \]
\[ \propto \mathcal{F} \left[ v(t) \right] \star \text{sinc} \left( N \Delta t \nu \right) \]

- Results in convolved visibility spectrum

\[
V_{ij}(\nu) = \left\langle (\mathcal{F}[v_i(t)] \star \text{sinc}(N \Delta t \nu)) \left( \mathcal{F}[v_j(t)] \star \text{sinc}(N \Delta t \nu) \right)^* \right\rangle \\
= \left\langle \mathcal{F}[v_i(t)] \mathcal{F}[v_j(t)]^* \right\rangle \star \text{sinc}^2 (N \Delta t \nu)
\]
FX Spectral Response (2)

5% sidelobes
VLBA Multiply Accumulate (MAC) Card

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The XF Correlator

\[ v_i(t) - N \Delta t \]

\[ v_j(t) \]

\[ \tau - N \Delta t \]

\[ \Delta t \]

\[ \Delta t \]

\[ \Delta t \]

\[ \int dt \]

\[ \int dt \]

\[ \int dt \]

\[ \int dt \]

\[ \{V_{ij}(\nu_k) \mid k = 1 \ldots N\} \]
XF Spectral Response

- XF correlators measure lags over a finite delay range

\[ V_{ij}(\tau) = \langle v_i(t)v_j(t + \tau) \rangle \cdot \mathcal{F} \left( \frac{t}{N\Delta t} \right) \]

- Results in convolved visibility spectrum

\[ V_{ij}(\nu) = \mathcal{F} \left[ \langle v_i(t)v_j(t + \tau) \rangle \cdot \mathcal{F} \left( \frac{t}{N\Delta t} \right) \right] \]

\[ = \mathcal{F} \left[ \langle v_i(t)v_j(t + \tau) \rangle \right] \ast \text{sinc} (N\Delta t \nu) \]
XF Spectral Response (2)

22% sidelobes!
Hanning Smoothing

- Multiply lag spectrum by Hanning taper function

\[ H(\tau) = \frac{1}{2} \left( 1 + \cos \frac{\pi \tau}{N \Delta t} \right) \]

- This is equivalent to convolution of the spectrum by

\[ H(\nu) = \delta(\nu) - \frac{1}{2} \delta \left( \nu - \frac{1}{2N \Delta t} \right) - \frac{1}{2} \delta \left( \nu + \frac{1}{2N \Delta t} \right) \]

- Note that sensitivity and spectral resolution are reduced.
Hanning Smoothing (2)

2 chans wide

channel

amplitude

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VLA MAC Card

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Basic Correlator Stages for the LWA

1. Correlate LWA-1 beams with LWA-2 "mini" station
2. Digitize a single VLA dish and correlate with LWA-1
3. Feed LWA-1 signal into EVLA correlator
4. Correlate ~10 LWA Phase II stations (the "LWIA")
5. Correlate full LWA (up to 53 stations)
Some Potential Correlator Options

• Software (up to ~10 stations?)
  - SoftC (JPL)?
  - DifX software correlator?
  - Custom software correlator?

• Hardware (for 10+)
  - GPU-based correlator?
  - BEE2-based (FPGA) correlator?
JPL’s SoftC Correlator

- Software XF correlator, written by Steve Lowe at JPL.
- Available at no cost.
- Flexible: SoftC can correlate 1, 2, 4, and 8-bit sampled data, upper, lower or double (I/Q) sideband data, using either of two sample encoding schemes.
- Extensively tested, mission critical software.
- Mature: has been around several years.
- Input/Output:
  - Works on disk data files, not live data streams.
  - Requires small format translators to be written.
- Requires a computer cluster (e.g. Beowulf) to run on.
  - Recent single CPU can processes 8 lags of 1-bit sampled data at 10 MSamples/s, independent of sample rate.
BEE2-based Correlator

- BEE2: FPGA-based, scalable, modular, upgradeable signal processing system for radio astronomy developed at Berkeley
- BEE2&IBOB boards available now
- Being used for several projects
  - 32-station FX correlator for Backer & Bradley’s EOR telescope (PAPER)
  - 128 Mchannel SETI spectrometer
- LWA would need a custom interface board
- Low hardware cost ($20k/BEE2 +$1k/IBOB + Infiniband switch)
  - 8 antenna correlator done with 4 IBOBs and 1 BEE2
- Real effort is in the FPGA “software”

IBOB: Internet BreakOut Board

BEE2: Berkeley Emulation Engine
Strawman LWA Correlator Plan

- Correlate LWA-1 with a single other antenna placed few hundred meters away.
  - Use SoftC, DifX, or simpler software correlator
- Reformat a single VLA dish and correlate with LWA-1
  - Use software correlator
- Correlate first 9 (or so) LWA stations
  - Use software correlator on a cluster
- Correlate full LWA
  - Expand BEE2 (or BEE3 by then) correlator to multiple boards
The EVLA WIDAR Correlator

• XF architecture duplicated 64 times, or “FXF”
  - Four 2GHz basebands per polarization
  - Digital filterbank makes 16 subbands per baseband
  - 16,384 channels/baseline at full sensitivity
  - 4 million channels with less bandwidth!

• Initially will support 32 stations with plans for 48

• 2 stations at 25% bandwidth or 4 stations at 6.25% bandwidth can replace 1 station input

• Correlator efficiency is about 95%
  - Compare to 81% for VLA

• VLBI ready
Further Reading

- [http://www.nrao.edu/whatisra/mechanisms.shtml](http://www.nrao.edu/whatisra/mechanisms.shtml)
- [http://www.nrao.edu/whatisra/](http://www.nrao.edu/whatisra/)
- [www.nrao.edu](http://www.nrao.edu)

- Synthesis Imaging in Radio Astronomy
- ASP Vol 180, eds Taylor, Carilli & Perley