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6. Polarization in Interferometry

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Abstract. This lecture covers the calibration and imaging of interferometric measurements of polarized radiation. The topics included are: basic concepts of polarized radiation, the instrumental response to polarized emission, calibration to remove the corrupting effects of the atmosphere and the instrument, and imaging in polarized light. Both circularly and linearly polarized feeds are discussed.

1. Introduction

Most of the non-thermal processes that produce radio frequency emission in astronomical sources are at least partially polarized. In addition, magnetized plasma along the line of sight to the source can further modify the polarization state of the emission. Thus, polarized emission is an important astrophysical diagnostic of the physical conditions in and in front of radio sources.

For several decades, radio interferometers have routinely measured the polarization of the emission (Morris et al. 1964; Conway and Kronberg 1969, Fomalont and Wright 1974; Thompson, Moran and Swenson 1986; Cotton 1993). As will be shown later, interferometers always measure some aspect of the polarized emission. There are a variety of effects that can either corrupt the measurements of polarized emission or complicate its interpretation. Many of these effects are the results of the atmosphere and the instrument itself and may be determined and corrected.

The details of polarization measurements depend strongly on the type of detector used for the radiation. The major types of detectors, or "feeds", couple either orthogonal circularly or linearly polarized emission into the telescope electronics. The type of feed used in an interferometer profoundly affects the calibration procedures.

The following sections will present some basic concepts of polarized electromagnetic radiation, describe the response of an element and an interferometer to polarized emission, discuss various potential corruptions of the polarized signals and their cures, and finally discuss the imaging of polarized emission. This discussion will attempt to be fairly general and will not include the details of the procedures in any particular software implementation.

1.1. Polarization Concepts

Electromagnetic radiation can be thought of as propagating sets of oscillating electric and magnetic vectors. As radio telescopes are sensitive only to the electric field, the following discussion will ignore the magnetic field. Figure 6-1a illustrates the concept of light as an oscillating electric field; the curved line represents the trace of the tip of the electric vector. For a given wave, this oscillation can take a variety of forms, several of which are illustrated in Figures 6-1b–d, which show the motion of the tip of the electric vector as viewed along the direction of propagation of the wave. The polarization state of the wave is simply the shape of the trace of the tip of the electric vector in this projection.

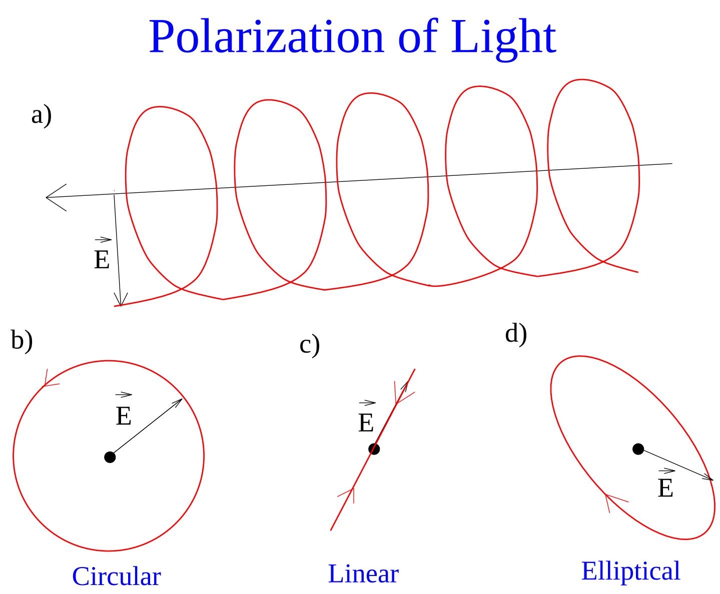

Figure 6-1. a) Illustrates the concept of electro-magnetic radiation as a propagating, oscillating electric vector.
b) End on view of the trace of the tip of the electric vector for a circularly polarized wave.
c) Like b) but for a linearly polarized wave.
d) Like b) but for an elliptically polarized wave.

Figure 6-1b illustrates circular polarization in which the E vector traces a circle; the two possible directions are called right and left circular polarizations. The E vector shown in Figure 6-1c is confined to a line and is referred to as linear polarization; the orientation of the line is arbitrary. Finally, combinations of circular and linear polarization are possible, giving rise to elliptical polarization as illustrated in 6-1d. For a detailed discussion of the polarization properties of light, see Born and Wolf (1975).

Radio sources are generally incoherent emitters of radiation — which complicates the issue of polarization, as the many different wave packets making up the light from the source may have independent polarization states. Thus, the polarization of the emission becomes a statistical measurement. If there are equal numbers of waves in a given polarization state and in the orthogonal polarization state, then the light is said to be unpolarized. Here right and left circular are "orthogonal", as are linear polarizations with angles separated by 90°. Thus, a source with "no circular polarization" actually has equal mounts of right and left circular polarization. A source is said to be partially polarized if any polarization is more common than its orthogonal state. A further concept is that of the total intensity which is the sum of all polarization states. The polarization of a source is described as the fraction of the total intensity of the excess of the given state over the orthogonal state. For linear polarization, the orientation on the sky of the dominant polarization state is called the "polarization angle".

A single phase sensitive detector of electro-magnetic radiation can sample only a single polarization state. However, the signal is fully described in terms of two orthogonal polarization states so detectors coupled to two orthogonal polarization are sufficient to measure the polarization state of the signal.

The polarization of a partially polarized source can be described in terms of the Stokes parameters (Stokes, 1852) I, Q, U, and V. I is the total intensity, Q and U describe the linear polarization (Q points north and U is 45° towards east), and V is the circular polarization (positive is right circularly polarized). (Note: I is always positive but the other Stokes parameters can have either sign.) The polarization angle, or apparent orientation of the projected E vectors on the sky, measured from north towards east, is related to the Stokes parameters by

$$\Phi = \frac{1}{2}\tan^{-1}\frac{U}{Q} \tag{6-1}$$

The fractional linear polarization is

$$m = \frac{\sqrt{Q^2 + U^2}}{I}, \tag{6-2}$$

and the fractional circular polarization is

$$v = \frac{|V|}{I}. \tag{6-3}$$

In this lecture, the Stokes parameters will refer to visibility measures which are the same as the brightness distribution only for point sources. The Stokes parameter brightness distributions of extended sources are the Fourier transform of the Stokes parameter visibility function. For a more rigorous discussion of the Stokes parameter representation of partially polarized radiation, see equations 1-16, 1-17, 5-21, and 5-22, or Chandrasekhar (1960) and Kraus (1966).

1.2. Polarization Measurements Using an Interferometer

Polarization measurements are generally made using a pair of feeds on each interferometer element; usually these are sensitive to orthogonal circular or linear polarizations. (For technical reasons it is not possible to construct a feed sensitive to total intensity.) If polarization measurements are desired, then all four combinations of feeds on all baselines are cross-correlated. Even for total intensity measurements, both of the correlations of the parallel polarization feeds are required (although if the system uses circularly polarized feeds and is observing sources with no circular polarization, then one correlation of parallel feeds is sufficient). The relationship between the measured correlations and the Stokes parameters depends on the polarization type of the feeds and is the subject of Section 2.

1.3. Errors and Calibration

There are a number of effects that can modify or corrupt the polarized signals. These need to be evaluated and the derived corrections must be applied to the data before it can be used. Polarized signals can be modified in passage through the atmosphere. In particular, the ionosphere is a magnetized plasma capable of Faraday rotation, i.e. the rotation of the polarization angle of linear polarization.

The antenna feeds will not have an ideal response, which will cause an "instrumental" contribution to the measured polarized signal even for a completely unpolarized source. The phase relation of the signals detected by the two feeds of any given interferometer element can be disturbed both by ionospheric Faraday rotation and by the interferometer electronics. Interferometers using circularly polarized feeds do not directly measure the polarization angle of a source, so observations of a calibrator of known polarization angle are needed. Similarly, interferometers using linearly polarized feeds may have unknown orientation errors and may have trouble separating calibrator and instrumental polarization. This is especially true if the antennas have equatorial mounts, in which case, observations of a source of known polarization may therefore be required. The process of determining and correcting these errors in the data is known as calibration. The details of calibration depend on the feed polarization type and are the subject of Section 3.

2. Instrumental Response to a Polarized Signal

This section considers the response of a telescope to a polarized signal. A parameterized model of the response of each feed is needed in order to quantitatively characterize it. The parameters of this model can be determined and used to correct the data. There are several formalisms for describing the polarization response of radio telescopes; one is to describe the feed in terms of the polarization state to which it is sensitive, i.e. to the polarization that it would transmit if the antenna were used as a transmitter rather than a receiver. In this formalism, each feed is parameterized by the ellipticity and orientation of the polarization to which it is sensitive; see Fomalont and Wright (1974) or Cotton (1993) for a more detailed discussion.

This lecture will use an alternate description of the polarization response in which the feed is assumed perfectly coupled to the nominal polarization, with the addition of a complex factor times the orthogonal polarization. This is called the "Leakage" or "D–term" model. Leppänen (1995) discusses the equivalence of these two models of the feed response.

Schwab (1979) has pointed out that the response of an interferometer can be factorized into antenna based components which can be further factorized into discrete contributions arising from a number of effects. These individual contributions to the instrumental response are expressed in terms of "Jones matrices". A more detailed discussion of the Jones matrices can be found in Hamaker, Bregman and Sault (1996) and Sault, Hamaker and Bregman, (1996).

2.1. Feed Response

The response of antenna i with orthogonally polarized feeds p and q can be factorized into a number of physically distinct components using the Jones' matrix formalism:

$$J_i = G_i D_i P_i. \tag{6-4}$$

The first term, G_i, is usually called the "gain" and is

$$G_i = \begin{pmatrix} g_{ip} & 0 \\ 0 & g_{iq} \end{pmatrix}, \tag{6-5}$$

where g_{ip} and g_{iq} are complex gain factors for the two orthogonally polarized signals. This term represents the uncorrected effects of the atmosphere and electronics.

The second term, D_i, models imperfections in the feed polarization response. This term is given by:

$$D_i = \begin{pmatrix} 1 & d_{ip} \\ -d_{iq} & 1 \end{pmatrix}, \tag{6-6}$$

where d_{ip} and d_{iq} are complex "leakage" terms which represent the fraction of the orthogonally polarized signal "leaking" into a given feed.

The last factor in Equation 6-4 includes the effects of the rotation of an alt-az mounted antenna as seen by the source while the antenna tracks the source. This rotation, known as the parallactic angle, is given by

$$\chi = \tan^{-1}\left(\frac{\cos(\lambda)\sin(\mathrm{h})}{\sin(\lambda)\cos(\delta) - \cos(\lambda)\sin(\delta)\cos(\mathrm{h})}\right) \tag{6-7}$$

where δ is the source declination, λ is the latitude of the antenna, and h is the source hour angle. Antennas with equatorial mounts do not rotate and therefore have a constant parallactic angle (0). Parallactic angle has an effect on the measured signals which depends on the feed polarization type:
$P_i^{+} = \begin{pmatrix} \cos(\chi) & -\sin(\chi) \\ \sin(\chi) & \cos(\chi) \end{pmatrix}$ for linear or $P_i^{\odot} = \begin{pmatrix} e^{-j\chi} & 0 \\ 0 & e^{j\chi} \end{pmatrix}$ for circular feeds where $j = \sqrt{-1}$.

This assumes that the "X" (p) linear feed is oriented north–south when observing a source on the meridian and the "Y" (q) feed is rotated by 90° to the east from the "X" feed. If the feeds are rotated from this orientation, then the amount of this rotation needs to be added to χ. Since this orientation is a mechanical adjustment, its precise value may not be known and must be determined from calibration sources.

2.2. Interferometer Response

Jones/Mueller Matrices Hamaker, Bregman and Sault (1996) discuss the Mueller matrix which gives the interferometer response to polarized radiation, $v = (qq, qp, pq, qq)$. They show that the Mueller matrix is derived from the outer product of the individual antenna Jones' matrices:

$$v = (J_i \otimes J_k^*)\ S\ s \tag{6-8}$$

where s is the true Stokes visibility vector (i, q, u, v) and * indicates the complex conjugate. Matrix S is the coordinate transformation from the Stokes system to that of the correlations:
$S^{+} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & j \\ 0 & 0 & 1 & -j \\ 1 & -1 & 0 & 0 \end{pmatrix}$ for linear or $S^{\odot} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & j & 0 \\ 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$ for circular.

Linearized Response In the limit of a weakly polarized source and nearly perfect feeds, any higher order terms involving source or instrumental polarization can be ignored as well as the products of such terms. The linearized approximation for crossed linearly polarized feeds on baseline $i-k$ is:

$$\begin{aligned}
v_{pp} &= \tfrac{1}{2} g_{ip} g_{kp}^* (I + Q\ \cos 2\chi + U\ \sin 2\chi), \\
v_{pq} &= \tfrac{1}{2} g_{ip} g_{kq}^* ((d_{ip} - d_{kq}^*) I - Q\ \sin 2\chi + U\ \cos 2\chi + jV), \\
v_{qp} &= \tfrac{1}{2} g_{iq} g_{kp}^* ((d_{kp}^* - d_{iq}) I - Q\ \sin 2\chi + U\ \cos 2\chi - jV), \\
v_{qq} &= \tfrac{1}{2} g_{iq} g_{kq}^* (I - Q\ \cos 2\chi - U\ \sin 2\chi),
\end{aligned} \tag{6-9}$$

and for circularly polarized feeds:

$$\begin{aligned}
v_{pp} &= \tfrac{1}{2} g_{ip} g_{kp}^* (I + V), \\
v_{pq} &= \tfrac{1}{2} g_{ip} g_{kq}^* ((d_{ip} - d_{kq}^*) I + e^{-2j\chi}(Q + jU)), \\
v_{qp} &= \tfrac{1}{2} g_{iq} g_{kp}^* ((d_{kp}^* - d_{iq}) I + e^{2j\chi}(Q - jU)), \\
v_{qq} &= \tfrac{1}{2} g_{iq} g_{kq}^* (I - V).
\end{aligned} \tag{6-10}$$

3. Source and Instrumental Polarization

Equations 6-9 and 6-10 show that the instrumental contribution to the cross polarized interferometer response (v_{pq} and v_{qp}) is unaffected by parallactic angle, whereas the contribution from the source does depend on the parallactic angle. In the case of circularly polarized feeds, the interferometer response is the sum of two vectors and the orientation of one varies with parallactic angle (time). If the data from interferometers using circularly polarized feeds are corrected for the phase effects of the parallactic angle, as is done for VLBI, then the source contribution is constant and the instrumental polarization rotates with parallactic angle. For interferometers with linearly polarized feeds, the situation is more complex; a point source at the phase center contributes a function of Q, U and χ to the real part of all correlations.

For interferometers with alt-az mounts (e.g. VLA, VLBA, ATCA), observations of a calibrator source over a range of parallactic angles can be used to separate the effects of source and instrumental polarization. This is illustrated in Figure 6-2a, which shows the imaginary part of the v_{qp} correlation plotted against the real part for calibrator observations made in a number of scans over a range of parallactic angle using circular feeds. The data shown in this figure have been calibrated such that the source polarization has a constant contribution whereas the instrumental polarization varies with observing geometry. The varying instrumental contribution is clearly distinguished from the source contribution. The data shown in Figure 6-2b have been corrected for the effects of instrumental polarization.

The accuracy of the instrumental polarization calibration can be limited by a number of effects, including variations of the instrumental polarization with observing geometry, instrumental phase fluctuations, ionospheric Faraday rotation and limited signal-to-noise ratio in the calibration data. Leppänen (1995) has a discussion of the effects of residual instrumental polarization errors on

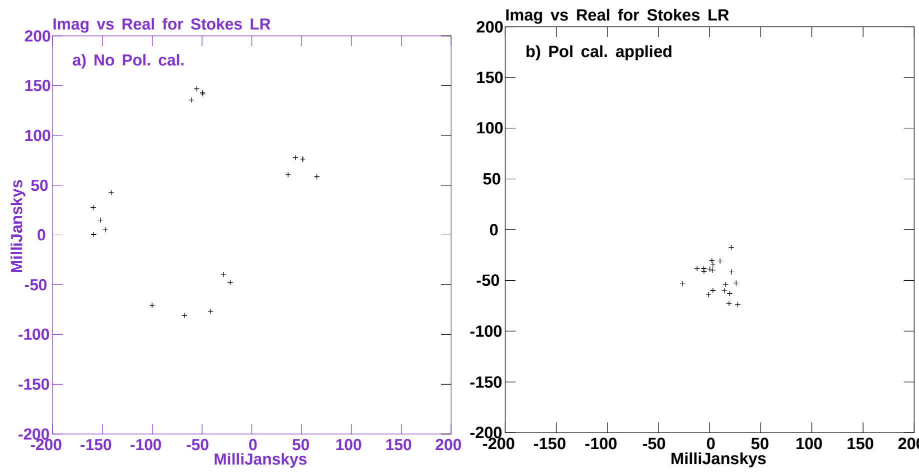

Figure 6-2. Plots of the imaginary versus real parts of the pq (left-right) correlation of the response to a weakly polarized source from a circular feed interferometer, with and without polarization calibration. The data have been rotated to remove the effects of parallactic angle, thus the source polarization is constant, and the instrumental contribution appears to rotate with time.
a) No correction for instrumental polarization, whose effects are seen to vary with parallactic angle.
b) The data shown in a) after the estimates of the instrumental polarization have been removed. Figure from Cotton (1993).

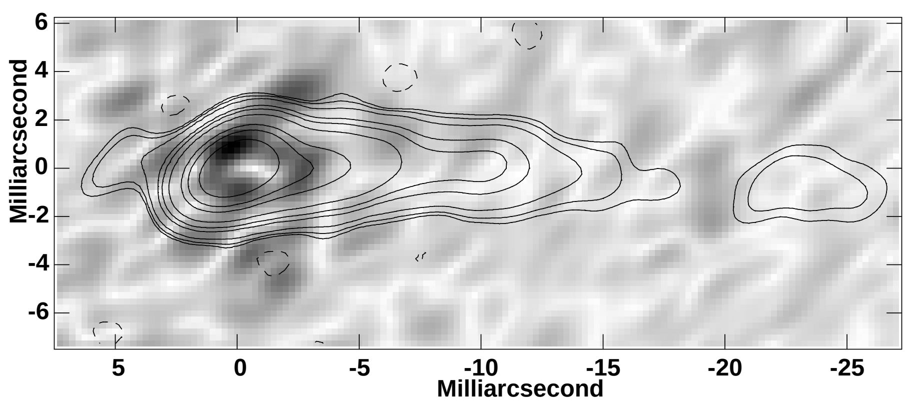

Figure 6-3. Example of an unpolarized source observed by the VLBA showing the effects of residual instrumental polarization. The contours show the total intensity of the source. The gray-scale shows the linearly polarized amplitude image after determining the instrumental polarization from the source itself. The peak polarized intensity in this image does not correspond to a feature in the total intensity (and therefore is unlikely to be real) and is about 0.5% of the peak in the total intensity.

linear polarization images. In the cases discussed by Leppänen, the source being imaged was used as its own calibrator and the result of residual instrumental polarization is the presence of weak polarized features mainly off the source. An example of this kind is shown in Figure 6-3, for the case of a strongly depolarized source. In the more general case of using a separate calibration source to determine the instrumental polarization, there will also appear to be spurious polarized emission where the total intensity is strong.

4. On-axis Calibration Strategies

This section concerns the calibration of the interferometer response at the center of the primary antenna pattern of the interferometer elements; calibration of the variations across the antenna pattern is covered in Section 5. Discussion of ionospheric Faraday rotation is deferred until Section 6.

Polarization calibration consists of several independent steps: determining and correcting instrumental polarization and calibrating the polarization angle for interferometers with circular feeds, or correcting for the mechanical alignment errors and the antenna gains errors due to calibrator polarization for antennas with linear feeds. Furthermore, polarization calibration and total intensity calibration are related and can be only partially separated.

4.1. Interaction between Polarization and Total Intensity

Astronomical calibration of the total intensity is usually performed with compact extra-galactic radio sources; either separate calibration sources or self-calibration. These sources are thought to emit via the synchrotron process and generally have a few to ten percent linear polarization and less that a half percent circular polarization. Since the best calibration sources are physically small, they tend to be variable, usually on time scales of months to years but occasionally as short as days. The net effect is that the polarization of the calibration source is usually unknown. Equations 6-9 and 6-10 show that all possible correlations are at least partially sensitive to the unknown polarization of the source, correlations of parallel circular feeds are sensitive to total intensity and circular polarization, and correlations of parallel linear feeds are sensitive to total intensity and the linear polarization. If the terms neglected in Equations 6-9 and 6-10 are considered, all correlations are sensitive to all Stokes' parameters in the detected signal; this may become important for high-dynamic-range images.

It is desirable to separate the total intensity calibration from the polarization calibration to the greatest extent possible. This is useful for both measurements involving only total intensity and to simplify the calibration of polarization sensitive data. The degree to which this is possible depends on the polarization type of the feeds involved; these issues are discussed separately in the following sections.

4.2. Circular Feeds

Correlations between parallel circular feeds are sensitive to total intensity and the source circular polarization. Synchrotron emitting, compact extra-galactic sources are quite weakly circularly polarized, usually less that 0.1%. Calibration using such a source can assume it to have no circular polarization to quite high accuracy. This approximation allows the separation of the calibration of the p and q (right and left circular) gains from each other and from the instrumental polarization.

Measured interferometer phases are sensitive only to differences in gain phases, so the usual calibration procedure is to arbitrarily set the gain phase of a "reference" antenna to zero independently for the p and q systems of feeds. Note that this independent calibration of the p and q systems of gains does not constrain the phase relationship between the two systems but ensures that the $p - q$ phase difference is that of the reference antenna. For this reason, it is important to use the same reference antenna for both the p and q calibration at a given time, and to the extent possible use the same reference antenna for all times.

The dependence of gain on parallactic angle is not explicitly shown in Equation 6-10, but a rotation of the feed rotates the phase of the response to the source. In cases such as VLBI, where the antennas have very different parallactic angles, the calibration bookkeeping is simplified if the phase effects of the parallactic angle are corrected before further calibration.

Instrumental Polarization The spurious instrumental contribution to the measured correlations are given by the "D-terms" of Equation 6-6, and instrumental polarization calibration consists of determining these values and applying corrections to the measured correlations. If the array to be calibrated has sufficiently good feeds ("D-terms" of a few percent or less), calibration source is weakly polarized (up to a few percent), and very high dynamic range is not desired, then the linearized Equation 6-10 is adequate. If these conditions are not met, then a fully nonlinear model such as Equation 6-8 is called for.

Since the polarization of the calibrator is usually unknown it must be determined jointly with the instrumental polarization. As was discussed in Section 3, parallactic angle variation on an array of alt-az mounted antennas causes a relative rotation of the complex source and instrumental contributions to the measured correlations; measurements of a source of unknown polarization over a wide range of parallactic angle is sufficient for determining both the source and instrumental polarization (see Figure 6-2).

For a long synthesis, the phase calibration source usually has a sufficient range of parallactic angle to be used in this calibration. For a shorter observation, measurements of a calibration source as it goes through transit on at least one of the interferometer elements is necessary to obtain the required range of parallactic angle.

To calibrate an array of equatorial mount antennas, a source of known (including no) polarization is required. Even for an array of alt-az mounted antennas, observations at only a single parallactic angle are required for a calibrator of known polarization. There are a few very weakly polarized compact sources which can be assumed to be unpolarized for calibration purpose.

Polarization Angle As pointed out above, the usual calibration technique of independently calibrating the p and q systems will leave an unknown phase difference between them; in the case of VLBI measurements, there will also be a delay offset between the two systems.

A $p - q$ phase difference has an equivalent effect to a change of parallactic angle, in which case Equation 6-10 shows that a constant $p - q$ phase difference will cause a rotation of $Q + jU$. As Equation 6-1 shows, this results in a rotation of the apparent polarization angle of the source. In order to calibrate the $p - q$ phase difference, or equivalently the polarization angle, sufficiently sensitive measurements of a source of known polarization angle is required. Note that a constant $p - q$ phase difference will not corrupt the derived image other than to produce the wrong polarization angle: fractional linear polarization is unaffected.

Circular polarization Equation 6-10 shows that the parallel circular correlations pp and qq respond to the total intensity plus or minus the circular polarization. The effect of incorrectly assuming that the calibration source had no circular polarization is a systematic error in the amplitude of the derived gains. Thus, to accurately calibrate the gains, a calibrator of known circular polarization is needed.

A further potential complication for circular polarization is feeds offset from the optical axis of the antenna, such as are used in the VLA and VLBA. For these antennas, there is a so-called "beam squint" in which the beams in the two polarizations are not concentric on the sky. This imposes serious limitations on the use of these arrays for circular polarization measurements; this topic comes up again in Section 5.

4.3. Linear Feeds

Correlations between parallel linearly polarized feeds are sensitive predominantly to total intensity plus a contribution from the linear polarization. The compact extra-galactic sources used for calibration purposes usually have a few to about ten percent linear polarization; in this case, the assumption of no polarization is not completely satisfactory. The effect of assuming no polarization for an unresolved calibrator and independently determining the p and q calibration is that the amplitudes of the p and q gains will be incorrect in the approximate ratio:

$$\frac{g_p}{g_q} \approx \frac{I + Q\ \cos 2\chi + U\ \sin 2\chi}{I - Q\ \cos 2\chi - U\ \sin 2\chi}, \tag{6-11}$$

which is a function of parallactic angle, hence time. If Stokes' I is formed from both the qq and pp correlations, then the effects of the erroneous assumption of no calibrator polarization cancel and the correct value of Stokes' I is obtained. Note, however, that this is not necessarily the case for self-calibration of resolved polarized sources; in this case, the data should be converted to Stokes' I before self-calibration.

The procedure described above is inadequate if polarization results are desired from the data being calibrated. A detailed description of the polarization calibration procedure used for the Australia Telescope (linear feeds and alt-az antenna mounts) is given by Sault, Killeen and Kesteven (1991).

The current discussion follows their procedure.

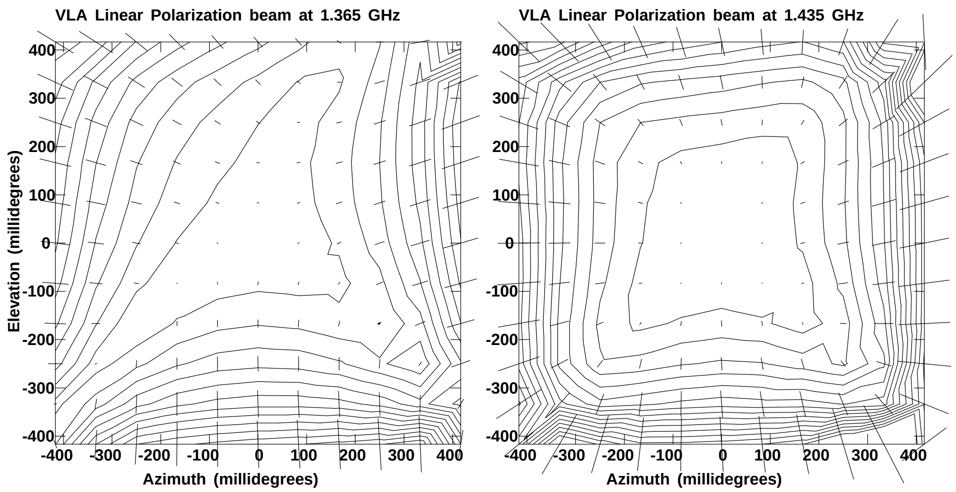

Figure 6-4. The average polarization response across the average VLA antenna pattern after correction for the on–axis instrumental polarization. a) The response at 1.365 GHz. b) The response at 1.435 GHz. Contours are shown every percent of instrumental polarization. The vectors have length proportional to the instrumental polarization and show the orientation of the E-vectors of the instrumental signals. Figure from Cotton (1994).

Calibrator and Instrumental Polarization Observations with an array using linearly polarized feeds need to include frequent measurements of an unresolved phase calibration source over a wide range of parallactic angle. If this is not possible during the normal course of the observations, then measurements of a calibrator source at a declination near the latitude of the array as it transits the meridian may suffice. The variable response to the calibrator's linear polarization allows separation of the source and instrumental contributions to the correlations. Sault, Killeen and Kesteven (1991) describe an iterative technique of alternately solving for antenna gains and then the source and instrumental polarization.

Feed Orientation/$p-q$ phase Measurements of a weakly polarized source, especially when its polarization is unknown, do not allow the determination of any errors in the assumed orientation of the feeds or of the $p-q$ phase difference. The determination of these parameters requires observations of a strongly polarized source of known polarization, such as 3C 138 or 3C 286.

5. Wide Field Polarization

The preceding discussion only relates only to the instrumental response at the center of the array element primary antenna pattern. In the general case, the instrumental polarization varies across the primary antenna pattern. Figure 6-4 shows this effect for the "average" VLA antenna at two nearby frequencies. In this case, there is significant off-axis instrumental polarization in regions of the antenna pattern which have sufficient gain to be useful for imaging extended

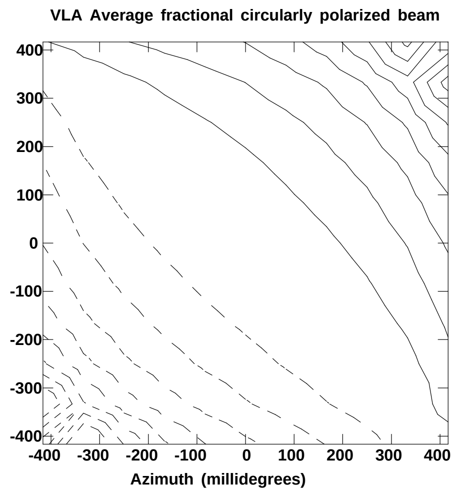

Figure 6-5. The spurious circularly polarized response of the average VLA antenna across its primary beam. Contours are shown every 10 percent of spurious instrumental circular polarization; negative contours are shown dashed. Figure from Cotton (1994).

sources or large fields of view. Figure 6-4 also shows that these effects are strong functions of observing frequency.

The pattern of off-axis polarization is fixed to the antenna and rotates on the sky with parallactic angle. This rotation of the instrumental polarization pattern on the sky will tend to average out its effects on the image derived from a long synthesis; however, any correction must be applied to the visibility data rather than to the resultant image. For snapshot images where there is not a significant variation of parallactic angle, or when using equatorial mounts, a correction for off-axis instrumental linear polarization can be applied to the final image. See Cotton (1994) for a detailed discussion of this technique as it applies to the VLA.

The spurious instrumental contribution to circular polarization may also vary across the antenna pattern, especially for off-axis circularly polarized feeds. An example of this which graphically illustrates the difficulties of circular polarization measurements with the VLA is shown in Figure 6-5.

6. Ionospheric Faraday Rotation

As a linearly polarized electromagnetic wave propagates through an magnetized plasma, its orientation rotates — an effect called Faraday rotation. An alternate way of describing this effect is that the speed of propagation of the right and left circularly polarized components of the radiation are different from each other in a magnetized plasma. The different velocities cause the relative phases of the right and left circular polarizations to change.

Faraday rotation of emission from the region around or in front of the source may be of interest to the astronomer, but Faraday rotation in the Earth's ionosphere seldom is. In addition, the ionospheric Faraday rotation varies with the ionization and recombination of the ionosphere and as the path through the Earth's magnetic field changes with observing geometry. The effect of Faraday rotation on the observed polarization angle is given by (Pacholczyk 1970):

$$\Delta\Phi = \frac{0.93 \times 10^6 \int N \; H_{||} \; ds}{(2\pi\nu)^2} \qquad (6\text{-}12)$$

where N is the electron density (cm^{-3}), $H_{||}$ is the component of magnetic field parallel to the direction of propagation (gauss), ν is the radio frequency (Hz) and s is distance along the line of sight (cm).

Since ionospheric Faraday rotation is time variable, it can corrupt polarization images from an extended synthesis or produce incorrect polarization angles for snapshot observations. The strong dependence on observing frequency means that it is most troublesome for low frequency observations, especially near maxima in solar activity when the ionosphere is the most active. There are rapid variations of the ionosphere near sunrise and sunset, or when the ejecta from a solar flare hits the earth. Any observations at frequencies below about 2 GHz may potentially be affected.

6.1. External Calibration

Corrections for ionospheric Faraday rotation may be determined from measurements of the total electron content (TEC) in the earth's atmosphere made using satellites (GPS these days) or estimated from the mean sunspot number (Chiu 1975). Using an assumed profile of the ionosphere and a model of the Earth's magnetic field, it is possible to estimate the Faraday rotation in a given direction.

Measurements of Faraday rotation toward cosmic sources may also be used to determine the TEC. However, since the Faraday rotation depends on the path through the ionosphere to the source, the observed Faraday rotation towards a calibrator cannot be directly applied to a target source unless the two are very close on the sky (the maximum separation depends on the amount of Faraday rotation).

Ionospheric Faraday corrections used for the VLA are derived from a dual frequency GPS receiver. The delay differences between the 1575 and 1227 MHz signals are used to determine the TEC along the line of sight to each satellite. These measurements are then fitted to a model of the ionosphere, which gives the zenith TEC at the location of the GPS receiver and gradients in latitude and longitude. This technique is discussed in Erickson et al. (1998).

6.2. Self Calibration

A variation on self calibration may be applied to determining the time varying relative Faraday rotation in the direction of the source, provided the source has sufficient polarized emission. A "reference" polarization image can be derived from data collected during a sufficiently short period that the ionospheric Faraday rotation is essentially constant. The visibility data can then be divided into time segments over which the Faraday rotation can be assumed to be constant; the Faraday rotation in each segment is determined by comparison with the "reference" polarization image. If there are strongly polarized, isolated features

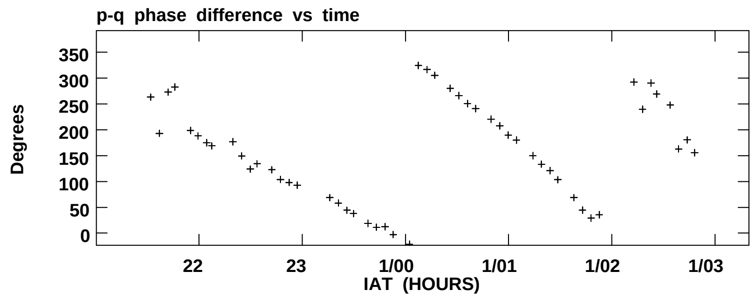

Figure 6-6. The $p - q$ phase difference derived for VLA observations of a source at 333 MHz. Several turns of phase variations are seen over the period of the observations. Figure from W. Cotton (1995)

in the source, this comparison may be done using images made from each time segment. If the polarized emission is more extended, then it may be necessary to determine the Faraday rotation using the Fourier transform of the "reference" polarization image and the visibility data in each time segment. Note that this procedure will remove the variable effects of the Faraday rotation but will not correct the polarization angles derived from the data. Fractional polarization and relative polarization angle will be preserved, but the absolute polarization angle will not. This is analogous to the loss of position information in the use of self calibration to calibrate the gain phases.

An example of the results of this procedure is shown in Figure 6-6 which shows the derived $p - q$ (R-L) phase difference as a function of time. These observations were made at a relatively low frequency and the Faraday rotation induced more than two full turns of $p - q$ phase during the course of the measurements. If uncorrected this would have very seriously corrupted the derived polarization image.

7. Imaging

Imaging of polarization data is quite similar to that of total intensity data, although there are a few differences that merit discussion. Once the calibration parameters are known, equation 6-8 or 6-9 or 6-10 can be inverted to derive the I, Q, U, and V visibilities, which can then be imaged and deconvolved using your favorite technique. One major difference between I and the polarized Stokes parameters is that I is always positive, whereas the other Stokes parameters may have negative values. This means that positivity may not be used as a constraint on the polarization deconvolution.

Conversion of the measured correlations to the Stokes' parameters generally requires all four (pp, qq, pq, and qp) correlations. There is a special case of observations using circularly polarized feeds in which some observations have only one of the pq or qp correlations. In this case, $Q + jU$ can be imaged and Q and U images recovered using a complex deconvolution. See Cotton (1993) for a more detailed discussion of this technique.

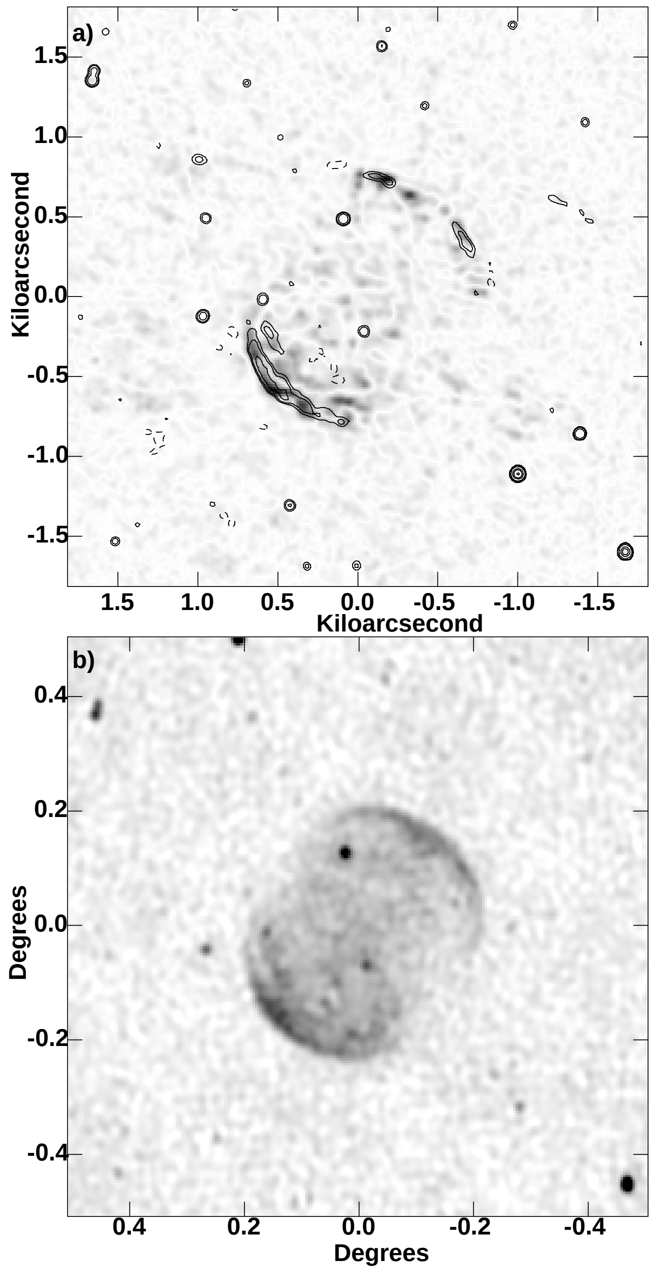

Figure 6-7. An example of the different spatial frequency filtering effects of interferometers on total intensity and linear polarization.
a) Total intensity contours superimposed on a gray scale representation of the linearly polarized emission. Polarized emission is visible in regions where total intensity is not. These images are from the NVSS survey (Condon et al. 1998) which does not well sample the spatial frequencies in this object.
b) The total intensity image of the same region of sky shown in a) but from the WENSS survey (Rengelink et al. 1997) which has approximately the same resolution but better sampling of the lower spatial frequencies. In this figure, there is visible total intensity emission in the regions of polarized emission of a).

In the true emission from a source, the total intensity cannot be less than the sum of all polarized components. This is not necessarily the case in images derived from interferometer arrays in which the sky has been subjected to a spatial frequency filtering. Structure on size scales that are strongly resolved by all interferometer baselines tend to be lost in the images derived from these measurements. The total intensity may be smooth on a size scale that is resolved by all interferometers, whereas the polarized emission may not be if there are variations in fractional polarization or polarization angle — perhaps due to variable Faraday rotation in the source — across this region. In this case, it is possible to have images which appear to exceed 100% polarization or to have polarized emission in regions where no polarized emission is visible.

An example of this effect is shown in Figure 6-7a, in which an extended source appears to have polarized emission where there is no total intensity. The total intensity image of the same region of the sky at similar resolution but made using better short baseline coverage is shown in Figure 6-7b. In Figure 6-7b, there is visible total intensity emission everywhere that polarized emission appears in Figure 6-7a.

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