Estimates of the Classical Confusion Limit for the LWA

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1 Introduction

Classical confusion must be taken into account for LWA configuration design. The LWA will have a concentrated core of stations to provide good uv-coverage of the short baselines. However, within a given radius of the array center, it would be wasteful to build collecting area beyond what is needed to reach the confusion limit at the resolution defined by that radius. In this memo, I estimate the confusion limit as a function of resolution for 74 MHz. I then convert this relation into a plot of station number as a function of array radius. It should be noted that classical confusion gets worse for lower frequencies.

2 Classical confusion limit at 74 MHz

The classical confusion limit is reached when the density of sources brighter than the RMS noise becomes high enough within the area of a synthesized beam. Generally, a rule of thumb is that the confusion limit occurs when the source density reaches an average of one source every 10 synthesized beam areas, but the theory is not rigorous. For now, we will set this aside in the variable m defined such that classical confusion occurs when there is on average on source in every m beam areas. This gives:

$$1.13\,\theta^2\,m\,N(s) = 1 \tag{1}$$

where $N(s)$ is the number density of sources above the flux density level s, and θ is the FWHM of the synthesized beam, or the resolution. Let us also assume a power law form for N(s):

$$N(s) = A\,s^{\beta} \tag{2}$$

Combining Equations 1 and 2, and setting s equal to the classical confusion limit (σ_c) gives:

$$\sigma_c = (1.13\,m\,A)^{-1/\beta}\,\theta^{-2/\beta} \tag{3}$$

Therefore, knowing m, A and β, we can determine the classical confusion limit at 74 MHz as a function of image resolution. To estimate these parameters, we use the results of our ongoing VLSS sky survey (*http://lwa.nrl.navy.mil/VLSS*) at 74 MHz. Currently half complete, we have detected over 32,000 sources down to a flux density limits of about 0.4 Jy/beam. Our source counts are accurate down to about twice that level, and we find $\beta = -1.30$ and $A = 1.14$ for units of flux density and beam size in Janskys and degrees respectively. This fit is meant to describe the fainter region of the source counts function, as shown in Figure 1. It is unlikely that this simple power law function completely describes the source count function all the way down to mJy-level sensitivities, however as this observational regime has never been probed before, it serves as a useful estimation.

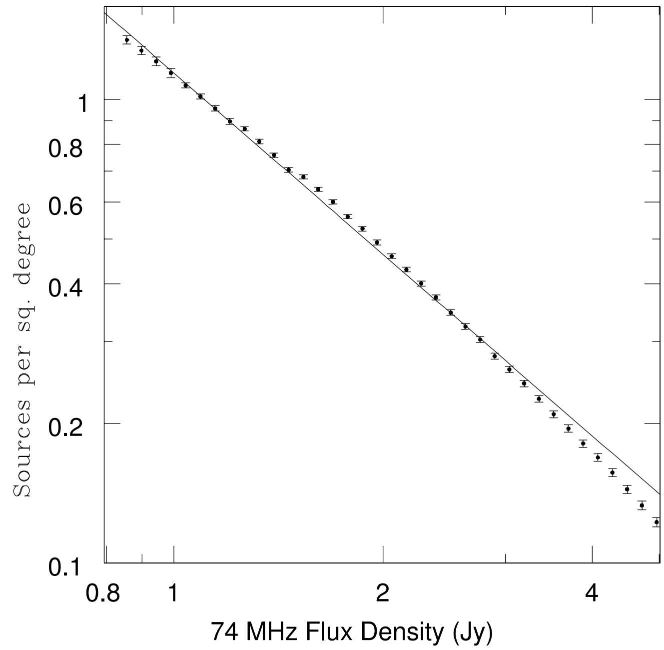

Figure 1: Source counts from the beginning stages of the VLSS, with our linear fit to the fainter end plotted.

That leaves m as the remaining unknown. To estimate m, we can consider our own experience with the classical confusion limit of the 74 MHz VLA in its smaller configurations. D-configuration data is unreliable because the RFI is so bad. But in the C-configuration, RFI is controllable, and yet we have found it impossible to get below an RMS noise level of 100 mJy/beam, regardless of the integration time. This is despite the fact that we can get much lower RMS noise with the larger A- and B- configurations in only a few hours. Given the 74 MHz C-configuration resolution of 200″, and taking 100 mJy/beam as the confusion limit gives $m = 12.9$. This suggests a somewhat higher value of the classical confusion limit than one would get for the canonical value of $m = 10$. While it is possible that it is higher due to the addition of sidelobe confusion as well, the difference only amounts to a noise level increase of 20%, so for this memo we will use the $m = 12.9$ value to stay on the conservative side.

Plugging in $A = 1.14$, $\beta = -1.30$, and $m = 12.9$ into Equation 3 gives us a direct relation between the classical confusion noise and the image resolution:

$$\sigma_c = [\theta/1'']^{1.54}\, 29\, \mu\text{Jy/beam} \tag{4}$$

Based on this relation, Table 1 shows the classical confusion limit at 74 MHz for various resolutions.

3 Maximum useful station number versus array diameter

For an array of N stations, the rms noise level is given by:

Resolution ($''$)	Confusion Limit (σ_c) (mJy/beam at 74 MHz)
$2''$	0.08
$5''$	0.35
$10''$	1.0
$20''$	2.9
$40''$	8.5
$60''$	16
$120''$	46

Table 1: Classical confusion limit at 74 MHz for various synthesized beam sizes.

$$\sigma = \frac{T_{sys}}{A_{eff}\sqrt{N(N-1)(N_{IF}\,\Delta T\,\Delta\nu)}}\,58.9\text{mJy/beam} \tag{5}$$

where T_{sys} is the system temperature, A_{eff} is the effective collecting area of each station in m^2, N_{IF} is the number of IF's, ΔT is the total observation time in hours and $\Delta\nu$ is the bandwidth in MHz. At low frequency, T_sys is dominated by the sky temperature, which for 74 MHz is about 2000 K. We take the effective area for each station to be the number of dipoles (256) times a dipole's effective area, $\lambda^2/4$. Thus for a 1 MHz bandwidth, 1 hour observation, with 2 polarizations at 74 MHz, the sensitivity of the array will be:

$$\sigma_{LWA} = \frac{79\,\text{mJy/beam}}{\sqrt{N(N-1)}} \tag{6}$$

Combining Equations 4 and 6 and using the relation $\theta = \lambda/D$ where D is the array diameter, we can relate the number of elements N needed to reach the confusion limit of an array of diameter D as follows:

$$\sqrt{N(N-1)} = 0.088\,[D/1\text{km}]^{1.54} \tag{7}$$

Table 2 shows the number of stations needed to reach the confusion limit within various array diameters.

4 Conclusions

Unless their is a scientific need to reach the classical confusion limit in much less than an hour or in a bandwidth much less than 1 MHz (or both) then it would be a waste of collecting area to put more than the number of antennas indicated by Equation 7 (or Table 2) within any given diameter about the array center. As this will make it difficult to achieve the needed uv-coverage at short baseline lengths, more stations with fewer dipoles each would be the most efficient use of collecting area.

Array Diameter (km)	Number of Stations (256-elements)
5	1.7
10	3.6
20	9.4
30	17
50	37
100	106
200	308
400	895

Table 2: Number of 256-element stations needed to reach classical confusion limit at 74 MHz in 1 hour with 1 MHz bandwidth and 2 polarizations as a function of array diameter.