

Estimated Collecting Area Needed for LWA Calibration

Aaron Cohen¹

`aaron.cohen@nrl.navy.mil`

August 30, 2006

ABSTRACT

One crucial parameter for the design of the LWA is the number of antennas that should be built per station. There are many considerations that go into this, including cost, and electronic capabilities, sidelobe levels and of course the scientific goals that require various amounts of collecting area. In this memo, this issue is examined from the perspective of how much collecting area is needed for full-field ionospheric calibration to be possible. At present, there are far too many unknowns about calibrating the LWA to determine this precisely. However, extrapolating from the experience of the VLA 74 MHz system allows a very rough estimate that, if not definitive, can at least provide some guidance to the current design process and facilitate discussion. The rough estimate produced in this memo is that the minimum necessary number of antennas per station is 176, assuming a total of 52 stations in the final LWA.

1. Introduction

The single most challenging issue for LWA imaging will be ionospheric calibration. Currently there is no clear consensus on the method that will be used to remove these distortions, but most agree that any successful method must use information gathered from calibrator sources visible within the field of view of the observation. These calibrator sources must be detected within a very short time interval – short enough that the ionospheric distortions do not change significantly. The purpose of this memo is to estimate the total number of antennas needed per station to have enough collecting area to detect enough sources for calibration within the required time interval. This requires information about an observational regime far from any that have yet been explored. Therefore, this memo will use the experiences of the 74 MHz VLA system to extrapolate to the regime in which LWA will observe. This is necessary to make an educated guess as to the needs of the LWA, but it must be considered to be only a very rough estimate. Early stages of the LWA will allow for probing of this observational regime for the first time, and therefore allow for refinement of this estimate.

¹Naval Research Laboratory, Code 7213, Washington, DC 20375

2. Estimating the Number of Calibrators in a FOV

2.1. Source Density at Low Frequency

First, it is necessary to determine how many sources the LWA is likely to detect within a field of view. To estimate this, I will begin with the sky density of sources found with the VLSS and other 74 MHz surveys:

$$N(s) = A s^\beta \quad (1)$$

where $N(s)$ is the number of sources per square degree with flux density above s . This function was fit to the available 74 MHz data to give values of $A = 1.14$ and $\beta = -1.30$ for s given in Janskys (Jy). This is accurate for values of s down to about 0.4 Jy/beam but, for this memo, I will assume that it holds for smaller flux values. As more frequencies than just 74 MHz are of interest, this formula can be adjusted to other wavelengths assuming that for a given source, $s \propto \lambda^{-\alpha}$, where α is the typical spectral index of a low frequency source, which is taken to be $\alpha = -0.7$. This results in the following expression for the source density:

$$N(s) = A s^\beta (\lambda/4m)^{\alpha\beta} = 1.14 s^{-1.30} (\lambda/4m)^{0.91} \quad (2)$$

Now this should be multiplied by the effective area of the field of view, given by (in square degrees):

$$A_{FOV} = 1.13 (180/\pi)^2 (\lambda/D)^2 \quad (3)$$

where D is the station diameter, which is 100m. Combining Equations 2 and 3 gives the total number of calibrators above flux density s in a full field of view:

$$N_{calibrators} = 6.77 s^{-1.30} (\lambda/4m)^{2.91} \quad (4)$$

2.2. LWA Sensitivity

For an interferometric array of N elements, the rms noise level in an image is given by:

$$\sigma = \frac{2 k_B T_{sys}}{\eta_s A_{eff} \sqrt{N(N-1)(N_{IF} \Delta T \Delta \nu)}} \quad (5)$$

where k_B is Boltzmann's constant, T_{sys} is the system temperature, η_s is the system efficiency, A_{eff} is the effective collecting area of each station, N_{IF} is the number of IF's, ΔT is the total observation time and $\Delta \nu$ is the bandwidth.

At this point it is necessary to make reasonable estimates for some of the many unknowns. First, the system efficiency, η_s , relates to losses due to the correlator and electronics. For this memo, this will be taken to be the same value as for the VLA, which has $\eta_s = 0.78$. The effective area, A_{eff} , for each station is the number of antennas, N_a , times an antenna's effective area, λ^2/Ω . Here Ω is the total effective solid angle of the power pattern of a single antenna which is roughly 3 sr. This gives:

$$A_{eff} = N_a \lambda^2 / 3 \quad (6)$$

Note that this formula assumes little overlap between the collecting “regions” of antennas in the station, which is not true for the very lowest part of the planned 23-80 MHz frequency range. At low frequency, T_{sys} is dominated by the sky temperature, which for 74 MHz ($\lambda = 4m$) is about 2000 K and varies as $\lambda^{2.6}$. This gives:

$$T_{sys} = (2000K) (\lambda/4m)^{2.6} \quad (7)$$

The observation time, ΔT , that is relevant here is the solution interval for ionospheric calibration. Thus it should be the length of time over which the ionospheric changes are not significant. For the 74 MHz VLA in A-configuration, this is has generally been found to be about 1 minute. The LWA will have roughly 10 times the resolution, and therefore it is reasonable to assume that it will need to sample the calibrators 10 times as often which allows the estimate of:

$$\Delta T = 6s \quad (8)$$

The remaining variables are taken from the latest LWA design plans. These give $N = 52$, $N_{IF} = 2$ and $\Delta\nu = 4$ MHz. Plugging these values along with Equations 6 - 8 into Equation 5 gives:

$$\sigma = \frac{(\lambda/4m)^{0.6}}{N_a} 3.72\text{Jy}/\text{beam} \quad (9)$$

2.3. How Many Calibrators are Needed?

The results of Sections 2.1 and 2.2 can be used to relate the number of antennas needed to the number of calibrators that must be detected in an ionospheric solution interval. Combining Equations 4 and 9 and assuming that the calibrators need to be detected to at least the 5σ level gives the following relation:

$$N_{calibrators} = 0.151 (\lambda/4m)^{2.13} (N_a)^{1.3} \quad (10)$$

Thus, if the number of calibrators needed is known, the number of antennas per station, N_a , needed to detect this many calibrators can be calculated. Because the ionosphere has never been probed at the resolution that LWA will operate, there is no way to know this number until the LWA is built and can be experimented with. However, this can be estimated by again referring to the 74 MHz VLA experience. In A-configuration, field-based ionospheric calibration works when roughly 4-6 sources are detected within each solution interval. This calibration is not perfect however, and probably requires at least 10 sources to achieve an ideal level of calibration. It is reasonable to assume that this number scales with the number of synthesized beam elements in a field of view which in turn scales as the square of the ratio of the longest baseline to the station size. (Note that the LWA calibration method will be different from the 74 MHz VLA calibration method however, for this calculation, it is assumed that the sky density of calibrators will be roughly the same.) The LWA will have baselines roughly 10 times longer than for the VLA in A-configuration, but the LWA stations are 4 times larger than VLA dishes. The magnitude of ionospheric phase variations is proportional to λ , and so it makes sense to estimate that the number of calibrators per sky area needs to increase as λ^2 . Finally, not all sources will be compact enough to detect at such high resolution. Assuming roughly half of the sources will be resolved out, twice as many sources in the FOV are needed to detect enough calibrators. Thus the number of calibrators needed can be estimated as:

$$N_{calibrators} = 10 \times (10/4)^2 \times 2 \times (\lambda/4m)^2 = 125 (\lambda/4m)^2 \quad (11)$$

Combining this with Equation 10, gives the following relation:

$$N_a = 176 (\lambda/4m)^{-0.1} \quad (12)$$

Note the fact that the number of antennas is only weakly dependent on λ , only decreasing as a power of -0.1. Thus, while I estimate a need for 176 antennas per station at $\lambda = 4m$, for the lowest frequency with $\lambda = 15m$, this falls only to 154. The reason for this is that I have assumed that the collecting area increases as λ^2 , which is true only for the high end of the planned 23-80 MHz band in which LWA will operate.

3. Conclusions

I have concluded that based on extrapolations from our experience from the 74 MHz VLA, the LWA will require at least 176 antennas per station (in a 52 station array) to obtain enough useful calibrators to reliably remove ionospheric effects for the entire field of view. Given that this is only a rough estimate, it would seem very risky to allow the number of calibrators per station to go below the currently planned number of 256.