Effective Aperture of a Large Pseudorandom Low-Frequency Dipole Array

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1 Introduction

The Long Wavelength Array (LWA) is a new aperture synthesis radio telescope, now in the design phase, that will operate at frequencies from about 20 MHz to about 80 MHz [1]. It will comprised of many "stations" distributed over the state of New Mexico. Each station will be an array of hundreds of dipole-type antennas distributed over a roughly circular area having a diameter of about 100 m, which is a lower bound determined by scientific requirements for the width of the station beam. In order for LWA to be calibrated as an imaging system, the effective aperture A_e of a station beam should be greater than ~ 4000 m^2 at 38 MHz [2]. On the other hand, the cost of a station scales almost linearly with the number of dipoles, so there is a strong motivation to minimize the number of dipoles in a station. To address these objectives, it has been proposed to distribute a minimum number of elements over the required station footprint in a pseudorandom geometry.

This paper describes some preliminary estimates of A_e for such an array. This is a non-trivial problem because the antennas are strongly coupled and interact strongly with the ground. To bound the scope of this preliminary investigation, the antennas are modeled as thin straight half-wave (nearly resonant) dipoles, and we restrict our attention to the co-polarized fields in the principal planes. First, we consider results for a single element in isolation. Next, we consider the results for the entire array, which are compared to the results for the single element and also to the physical aperture of the station.

2 Problem Statement

The antennas are modeled as thin straight half-wave dipoles, in contrast to any of the various types of broadband dipoles currently being considered as a candidates for the LWA [3]. The reason for this is that thin straight dipoles are simple to model, lead to relatively rapid computations, and offer better opportunities for comparison to theoretical results. Each dipole is constructed from perfectly-conducting material of circular cross section having a radius of 0.05 mm. The coordinate system is such that the dipoles lie in the $z = \lambda/4$ plane and the surface of the ground lies in the $z = 0$ plane. Two types of ground are considered: perfectly conducting ground, which can be achieved to a good approximation using a ground screen; and a realistic (somewhat lossy) ground having relative permittivity $\epsilon_r = 13$ and conductivity $\sigma = 5$ mS/m. The latter is not necessarily representative of the conditions at the proposed sites for LWA stations, however this is a commonly used "typical" value. (The actual values for LWA candidate sites have not yet been determined.)

The goal is to determine A_e for a pseudorandom planar array of these dipoles at 38 MHz, which corresponds to an important band for radio astronomy. A_e is defined as the ratio of the power successfully received by the array to the incident power density. Here we are interested in the *optimum* A_e, which is achieved when the power collected individually by the elements of the array is coherently combined through beamforming. A_e depends on quality of the match offered by impedance Z_L of the load attached to the terminals of the antenna elements. We will consider several possible values of Z_L, including the likely value (due to hardware considerations) of $100 + j0\ \Omega$ [4].

3 Single Stand Results

A single pair of orthogonally-polarized dipoles is referred to as a "stand." To illustrate the analysis and to provide some useful reference cases, we first consider the problem of a single stand. A number of scenarios will be considered, but in each case we are interested in the collecting area of the co-polarized dipole. In terms of the coordinate system used in this paper, this dipole is parallel to x axis. To limit the scope of the problem, we will consider only three directions: $\{\theta = 0, \phi = 0\}$ (zenith and broadside), $\{45^\circ, 0\}$ (in the E-plane), and $\{45^\circ, 90^\circ\}$ (in the H-plane). Three methods were considered:

(1) A "direct" method, in which A_e is calculated as power delivered to the loads in response to plane wave illumination. We refer to the collecting area determined in this way as A_e^d.

(2) An "indirect" method, based on reciprocity. The load on the dipole is replaced with a voltage source and the gain G_t is calculated. The collecting area in the case of a matched load is then $A_e = G_t\lambda^2/(4\pi)$. In the case of a mismatched load, this result is multiplied by a factor of $1-|\Gamma|^2$, where Γ is the reflection coefficient between the antenna and the load. The collecting area determined in this way is referred to as A_e^i.

(3) An emperical method, given by the equation:

$$A_e^e = (0.13\lambda^2)(3.4)(1 - |\Gamma|^2)(\cos^\alpha \theta)L_g \ , \tag{1}$$

where $0.13\lambda^2$ is the theoretical broadside A_e for a matched half-wave dipole in free space, the factor of 3.4 accounts for the presence of a perfectly conducting ground [5], $\cos^\alpha \theta$ accounts for pattern, and L_g accounts for ground loss. $L_g = 1$ for perfect electrical conducting (PEC) ground and $L_g = 0.6562$ for the "realistic" ground specified above, as determined from transmit-mode calculations. α was determined in the same way, and is taken to be 3 in the E-plane and 0.5 in the H-plane.

To determine the impedance of a matched load, the *in situ* antenna terminal impedance Z_A was calculated for each of the two ground conditions of interest. The results were $Z_A = Z_A^{p1} = 92.59 + j76.19\ \Omega$ for perfectly-conducting ground and $Z_A = Z_A^{r1} = 86.05 + j60.70\ \Omega$ for the realistic (lossy) ground. The matched loads are thus $(Z_A^{p1})^*$ and $(Z_A^{r1})^*$ respectively.

The results were computed using a NEC2-based moment method code, and are summarized in columns 4–6 of Table 1. The expected trends are all apparent; i.e., A_e suffers with increasing ground loss, increasing impedance mismatch, and increasing θ. Also we see that the direct and emperical methods give a reasonably consistent estimates.

4 Results for a Pseudorandom Array

Next, we consider a large array, shown in Figure 1, consisting of 256 stands identical to those described in the previous section. The layout used is pseudorandom, filling the required footprint with minimum allowed spacing of 4 m. The results for this array are shown in columns 7–9 of Table 1, where the total collecting area of the copolarized portion of the array, calculated using the "direct" method, has been divided by the number of stands to obtain the mean collecting area per dipole. Also shown is the percentage difference from the corresponding single-stand result from Table 1. The following conclusions can be made:

(1) A goal of 4000 $m^2 \sim 16$ m^2/stand is easily achieved for this array of 256 stands for broadside pointing, but not for very far off broadside. It may be possible to reach the goal over more of the sky by using more dipoles.

(2) In this study the effect of coupling is generally helpful in the E-plane, and generally unhelpful in the H-plane. Because a 256-stand array that fills the required station footprint cannot be simultaneously uniformly-spaced and free of grating lobes, it is difficult to apply traditional design techniques to improve this performance.

(3) The array collecting area cannot be reliably estimated from the single-stand results (not surprising given the strong coupling that must exist among the dipoles).

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References

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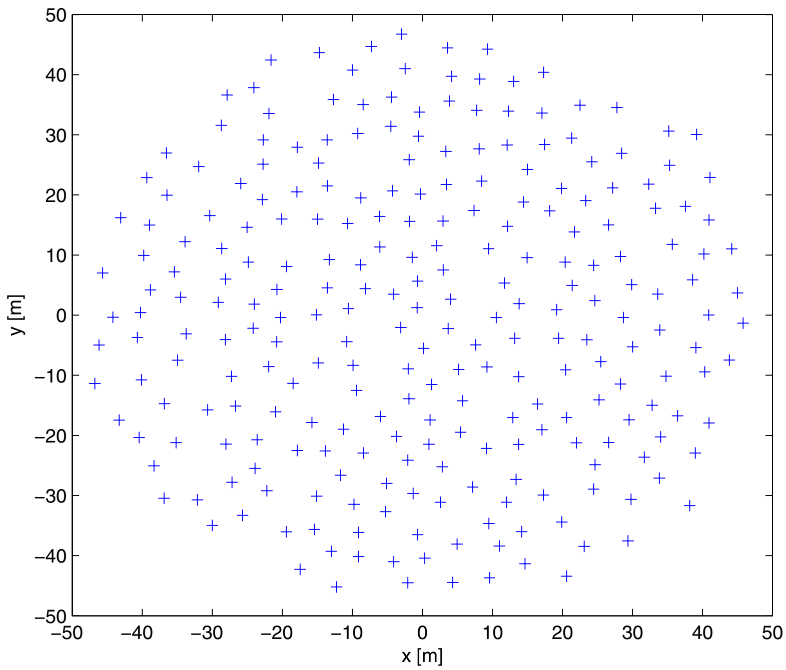

Figure 1: Pseudorandom array with 4 m minimum spacing. 256 stands, consisting of 256 co-polarized dipoles and 256 cross-polarized dipoles. "+" symbols indicate the actual orientation of dipoles, but are not to scale.

			Single Stand			Array		
Ground	Load	θ	A_e^e	A_e^d	A_e^i	$A_e^d/256$	$\Delta\%$	e_{ap}
PEC	$(Z_A^{p1})^*$	0°	27.55	29.52	27.76	25.22	−15%	0.82
		45°E	9.74	9.28	8.72	12.66	+36%	0.58
		45°H	23.16	22.31	23.73	16.07	−28%	0.74
PEC	100Ω	0°	23.78	25.49	23.97	25.96	+2%	0.84
		45°E	8.41	8.01	7.53	10.16	+27%	0.47
		45°H	20.00	19.26	20.49	14.25	−26%	0.66
Realistic	$(Z_A^{r1})^*$	0°	18.08	19.50	18.22	22.08	+13%	0.72
		45°E	6.39	5.55	5.21	7.47	+35%	0.34
		45°H	15.20	17.04	18.12	12.18	−28%	0.56
Realistic	100Ω	0°	16.24	17.41	15.73	19.08	+10%	0.62
		45°E	5.74	4.98	4.49	6.40	+28%	0.29
		45°H	13.66	14.71	15.65	11.56	−21%	0.53

Table 1: Effective aperture in m^2 for the single stand (no array), and the complete array. The array results are for the maximum gain beam pointed in the indicated direction, divided by the number of stands. The difference with respect to the single-stand result is indicated. e_{ap} is the aperture efficiency relative to the area of the projection of the station footprint in the indicated direction.