Polarization Processing at LWA Stations

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Contents

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1 Introduction

The current version of the LWA station architecture [1] specifies that the beamforming units (BFUs) should accept the raw (essentially linear) polarizations from each antenna stand and output calibrated orthogonal circular polarizations. Because we expect the polarization of each antenna to be different due to mutual coupling, we wish to convert to calibrated circular polarizations prior to combining. This document describes the relevant theory and a possible scheme for accomplishing this conversion.

2 Theory

Consider a monochromatic plane wave incident on an antenna. The incident electric field is given with complete generality by:

$$\mathcal{E}(t) = \hat{\mathbf{a}}\mathcal{E}_a(t) + \hat{\mathbf{b}}\mathcal{E}_b(t) \text{ , where} \tag{1}$$

$$\mathcal{E}_a(t) = \mathcal{E}_{a0}\cos(\omega t + \theta) \text{ ,} \tag{2}$$

$$\mathcal{E}_b(t) = \mathcal{E}_{b0}\cos(\omega t + \theta + \delta) \text{ ,} \tag{3}$$

with $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ being any two orthogonal unit vectors in the plane transverse to the direction of propagation; and $\mathcal{E}_{a0}$, $\mathcal{E}_{b0}$, and δ being real-valued constants which parameterize the polarization of the wave. We can express $\mathcal{E}$ in phasor notation as $\mathbf{E}$ using the relationship

$$\mathcal{E}(t) = \text{Re}\left\{\mathbf{E}e^{j\omega t}\right\} \text{ , where} \tag{4}$$

$$\mathbf{E} = \hat{\mathbf{a}}E_{a0} + \hat{\mathbf{b}}E_{b0}e^{j\delta} \text{ , with} \tag{5}$$

$$E_{a0} = \mathcal{E}_{a0}e^{j\theta} \text{ and} \tag{6}$$

$$E_{b0} = \mathcal{E}_{b0}e^{j\theta} \text{ .} \tag{7}$$

In this formulation, circular polarization is defined as $E_{a0} = E_{b0}$ and $\delta = \pm\pi/2$, where the "+" and "−" signs correspond to left-hand circular polarization (LHCP) and right-hand circular polarization (RHCP) respectively. Thus we have:

$$\mathbf{E} = E_0\left(\hat{\mathbf{a}} + j\hat{\mathbf{b}}\right) \text{ for LHCP and} \tag{8}$$

$$\mathbf{E} = E_0\left(\hat{\mathbf{a}} - j\hat{\mathbf{b}}\right) \text{ for RHCP,} \tag{9}$$

where $E_0 = E_{a0} = E_{b0}$. It can be verified that the above expressions for LHCP and RHCP are orthogonal by computing the inner product between them and noting that the result is zero. Thus, any incident wave – regardless of polarization – can be represented as the sum of one LHCP wave and one RHCP wave. Let these waves be $\mathbf{E}_L$ and $\mathbf{E}_R$, respectively. Then we have

$$\mathbf{E}_L = E_L\left(\hat{\mathbf{a}} + j\hat{\mathbf{b}}\right) \text{ where } E_L = \mathbf{E}\cdot\frac{1}{2}\left(\hat{\mathbf{a}} - j\hat{\mathbf{b}}\right) \text{ ; and} \tag{10}$$

$$\mathbf{E}_R = E_R\left(\hat{\mathbf{a}} - j\hat{\mathbf{b}}\right) \text{ where } E_R = \mathbf{E}\cdot\frac{1}{2}\left(\hat{\mathbf{a}} + j\hat{\mathbf{b}}\right) \text{ .} \tag{11}$$

Finally, we note that

$$\mathbf{E} = \mathbf{E}_L + \mathbf{E}_R = \hat{\mathbf{a}}(E_L + E_R) + \hat{\mathbf{b}}j(E_L - E_R) \text{ or simply} \tag{12}$$

$$\mathbf{E} = \hat{\mathbf{a}}E_{a0} + \hat{\mathbf{b}}E_{b0} \text{ , where} \tag{13}$$

$$E_{a0} = E_L + E_R \text{ , and} \tag{14}$$

$$E_{b0} = j(E_L - E_R) \ . \tag{15}$$

When this wave arrives at the antenna stand, it generates voltages across the antenna terminals equal to

$$V_c = \mathbf{E} \cdot \mathbf{l}_c(\hat{\mathbf{r}}) \text{ and} \tag{16}$$

$$V_d = \mathbf{E} \cdot \mathbf{l}_d(\hat{\mathbf{r}}) \ , \tag{17}$$

where V_c and V_d are the voltages associated with the two antennas respectively, and $\mathbf{l}_c$ and $\mathbf{l}_d$ are the vector effective lengths (VELs) associated with these antennas. The parameter $\hat{\mathbf{r}}$ is the vector pointing in the direction from which the wave arrives at the antenna stand. The VELs are characteristics of the antennas (essentially, a unified compact representation of the gain and polarization patterns) and can be determined from theoretical analysis, computer modeling, or direct measurement. It should also be noted that this formulation is valid independently of the presence or absence of mutual coupling. In the presence of mutual coupling, however, it is important that the values of $\mathbf{l}_c$ and $\mathbf{l}_d$ used here are those determined in the presence of the same mutual coupling.

Substitution of equations (13) through (15) into equations (16) and (17) yields:

$$V_c = (\hat{\mathbf{a}} \cdot \mathbf{l}_c)E_{a0} + (\hat{\mathbf{b}} \cdot \mathbf{l}_c)E_{b0} \text{ and} \tag{18}$$

$$V_d = (\hat{\mathbf{a}} \cdot \mathbf{l}_d)E_{a0} + (\hat{\mathbf{b}} \cdot \mathbf{l}_d)E_{b0} \ , \tag{19}$$

which are linear simultaneous equations which can be solved for E_{a0} and E_{b0} given V_c, V_d, and the known (*a priori*) antenna characteristics:

$$\begin{bmatrix} E_{a0} \\ E_{b0} \end{bmatrix} = \begin{bmatrix} (\hat{\mathbf{a}} \cdot \mathbf{l}_c) & (\hat{\mathbf{b}} \cdot \mathbf{l}_c) \\ (\hat{\mathbf{a}} \cdot \mathbf{l}_d) & (\hat{\mathbf{b}} \cdot \mathbf{l}_d) \end{bmatrix}^{-1} \begin{bmatrix} V_c \\ V_d \end{bmatrix} \tag{20}$$

Further, we note that equations (14) and (15) constitute linear simultaneous equations which can be solved for E_L and E_R given E_{a0} and E_{b0} from the previous calculation. Thus we find:

$$\begin{bmatrix} E_L \\ E_R \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}^{-1} \begin{bmatrix} (\hat{\mathbf{a}} \cdot \mathbf{l}_c) & (\hat{\mathbf{b}} \cdot \mathbf{l}_c) \\ (\hat{\mathbf{a}} \cdot \mathbf{l}_d) & (\hat{\mathbf{b}} \cdot \mathbf{l}_d) \end{bmatrix}^{-1} \begin{bmatrix} V_c \\ V_d \end{bmatrix} \tag{21}$$

Since the two 2×2 matrices in the equation above can be precomputed and combined into a single matrix, we find that a single 2×2 matrix multiply transforms the "raw" antenna voltages V_c and V_d into the incident electric field coefficients E_L and E_R associated with the LHCP and RHCP components, respectively, of the incident electric field.

3 Frequency-Dependent Antenna Polarization

One caveat applicable to the preceding derivation is that it is exact only at a single frequency. Thus, this method becomes inaccurate if applied over large fractional bandwidth. In terms of equation (21), the problem is that $\mathbf{l}_c$ and $\mathbf{l}_d$ become functions of frequency as well as direction ($\hat{\mathbf{r}}$).

A scheme for dealing with this is presented in Figure 1. In this figure, each block labeled "FIR" represents a finite impulse response (FIR) filter. To understand this scheme, first consider the monochromatic case. In this case, each one of the FIR blocks represents multiplication of the input by one of the elements of the 2×2 matrix implied in equation (21). To extend bandwidth, we compute these coefficients over the frequency range of interest, resulting in four complex-valued functions with frequency as the independent variable. Associated impulse responses can be obtained by application of the inverse Fourier transform, and the coefficients of the FIR filters are then simply the sampled impulse responses.

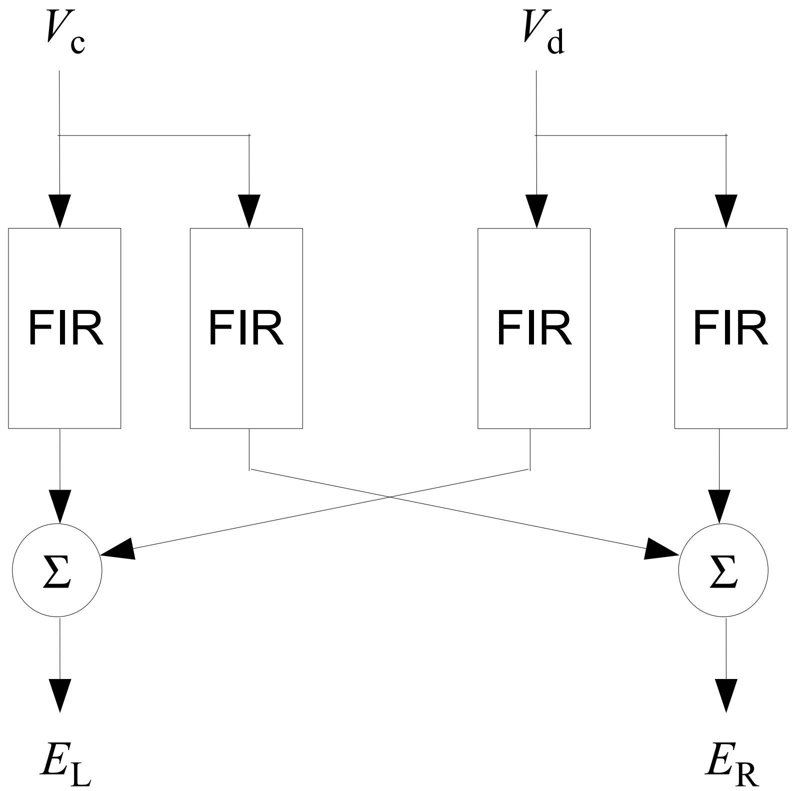

Figure 1: Scheme for polarization processing when antenna responses are frequency-dependent.

It should be noted that in this scheme both the input and coefficients will all be complex-valued in general, and thus all multiply/accumulate operations will be fully complex. This could potentially become quite computationally demanding if the polarization characteristics of the antennas vary rapidly with frequency, as this might then require large FIR filters for implementation.

References

[1] S. Ellingson, “LWA Station Architecture Ver 0.6,” October 9, 2007.