LWA Beamforming Design Concept

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Contents

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1 Introduction

This document presents a design concept for LWA beamforming. Context for this work can be found in the Station Architecture document [1], currently in Version 0.6. Also relevant is LWA Memo 101 [2], which describes the sample rates and formats assumed here. LWA beamforming occurs in the beamforming unit (BFU) subsystems, with each BFU responsible for accepting raw polarizations from all antennas in the station, and outputting a single full-bandwidth (10-88 MHz sky frequency) beam in two calibrated orthogonal circular polarizations. The beamforming will be essentially of the "delay and sum" variety, where the signal from each antenna is delayed by an appropriate amount and then summed to generate the desired beam. Conversion of the raw, nominally-linear antenna polarizations into calibrated orthogonal circular polarizations occurs just prior to summing. The theory for accomplishing the latter is described in [3]. The design concept is summarized in Section 4.

2 Integer Sample Period Delay

As described in [1] and [2], the DIG subsystem will include a FIFO intended (primarily) to equalize delays due to unequal cable lengths. Let us assume that the maximum excess length of a cable is equal to $0.5D$, where D is the station diameter. Assuming a typical group velocity of $0.67c$, where c is the speed of light in free space, the maximum excess delay to be equalized in the DIG subsystem is about (2.5 ns/m)D. Assuming $D = 100$ m, this gives 250 ns. Given a ADC sample rate of 196 MSPS, the sample period is 5.1 ns and therefore a FIFO of length 50 is required to equalize excess cable-induced delays. Also, note that the equalization may include errors of magnitude up to the sample period, 5.1 ns.

The delay portion of the BFU must introduce the additional delay required required for beamforming, and must take into account the quantization error remaining from the DIG's attempt to equalize the cable delays. The scheme for doing this consists of a FIFO to introduce the coarse delay followed by a FIR filter to introduce the remaining (fine) correction. It is desirable to partition the delay processing in this way, as opposed to simply using a FIR filter without a preceding FIFO, as this minimizes the required length of the FIR filter. Per [1] and [2], the sample rate within the BFU is 98 MSPS with a corresponding sample period $T \approx 10.2$ ns.

We first determine the required length of the BFU FIFOs. The minimum and maximum possible geometrical delays for beamforming assuming the cable delays are equalized are zero and $D/c \approx 333$ ns, respectively. 333 ns corresponds to a FIFO of length 33, and the maximum residual error due to this operation will be 10.2 ns.

At this point, the total maximum excess delay error will be equal to the sum of the excess delay errors from introduced by the DIG and BFU FIFOs, which is 5.1 ns + 10.2 ns = 15.3 ns. First, note we can reduce the maximum error to 5.1 ns by increasing the length of BFU FIFO by one, to 34. Thus, the delay-generating FIRs of the BFU are required to equalize a delay anywhere in the range 0 to 5.1 ns.

Before continuing to a discussion of the design of this FIR filter, it is perhaps appropriate to justify the need for equalization of delay errors of this magnitude. 5.1 ns corresponds to a transversed distance of 1.53 m in free space, which corresponds to 45% and 5% of a wavelength at 88 MHz and 10 MHz, respectively. This corresponds to phase errors equal to 162° and 18°, respectively. Although a proper requirement for these errors is not available, it is the judgement of the author that these errors are unacceptable. For example, such errors can be expected to significantly increase sidelobe levels. For the purposes of this document, it is assumed that the maximum acceptable phase error at any given frequency is 10°, and that error magnitudes $\leq 1°$ are preferred. One way to justify a goal of 1° is from the perspective of RFI mitigation: a 1° error in phase limits null depth to about 30 dB. Another (admittedly weaker) justification is based on experience that sensing and correction of errors to precision better than 1° in phase (or to better than 1% for any parameter) is quite difficult at low signal-to-noise ratios. Thus attempts to achieve accuracy better than this may not be productive, since (for example) calibration may not be sufficiently accurate to exploit this accuracy.

3 Fractional Sample Period Delay

We established in the previous section the requirement for a FIR filter running at the BFU sample rate of 98 MSPS, capable of generating a continuously-variable delay in the range zero to 5.1 ns with phase accuracy of 1°. This performance is to be achieved over the (sky) frequency range 10–88 MHz, which corresponds to ±39 MHz in the I/Q baseband representation used within the BFU [2].

We now present the relevant theory. Consider a waveform $x(t)$ sampled in a manner satisfying the Nyquist criterion, resulting in a time series $x[n]$ where n indexes the samples. The original waveform is obtained using the *reconstruction formula* as follows:

$$x(t) = \sum_{n=-\infty}^{\infty} x[n] \ \mathrm{sinc}\left(\frac{t - nT}{T}\right) \tag{1}$$

where sinc(t) is defined as $(\sin(\pi t))/(\pi t)$. We can compute a version of the waveform which has been delayed by a time equal to τ from the original time series as follows:

$$x(t-\tau) = \sum_{n=-\infty}^{\infty} x[n] \ \mathrm{sinc}\left(\frac{t - \tau - nT}{T}\right) \tag{2}$$

If we sample the analog waveform $x(t-\tau)$ at the same sample rate, at times $t = kT$ where k indexes the samples, we obtain:

$$x_d[k] = \sum_{n=-\infty}^{\infty} x[n] \ \mathrm{sinc}\left(\frac{kT - \tau - nT}{T}\right) \tag{3}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \ \mathrm{sinc}\left([k-n] - \frac{\tau}{T}\right) \tag{4}$$

Let us define the time series $h[n]$ as follows:

$$h[n] = \mathrm{sinc}\left(n - \frac{\tau}{T}\right) \tag{5}$$

We can then write Equation 4 as follows:

$$x_d[k] = \sum_{n=-\infty}^{\infty} x[n] \ h[k-n] \tag{6}$$

This is recognized as a digital filter, with $h[n]$ being the impulse response of the filter. Reducing the limits on the sum from $\pm\infty$ to $\pm N$ results in a FIR filter having $2N+1$ taps, which is said to be of order $M = 2N$. Thus the "prototype" for the FIR design of interest in this document (shifting the result to obtain a calculation which is symmetric with respect to the coefficients of $h[n]$) is:

$$\sum_{n=-N}^{+N} x[n+k] \ h[-n] \tag{7}$$

where the free parameter to be selected is the the filter order M.

We now consider the relationship between M and the performance of the delay FIR. Figures 1–5 show the results for M ranging from 60 to 2, respectively. In each case, the delay being implemented is $\tau = \pi$ ns, which is ≈ 0.308 times the sample period and ≈ 0.616 times the maximum

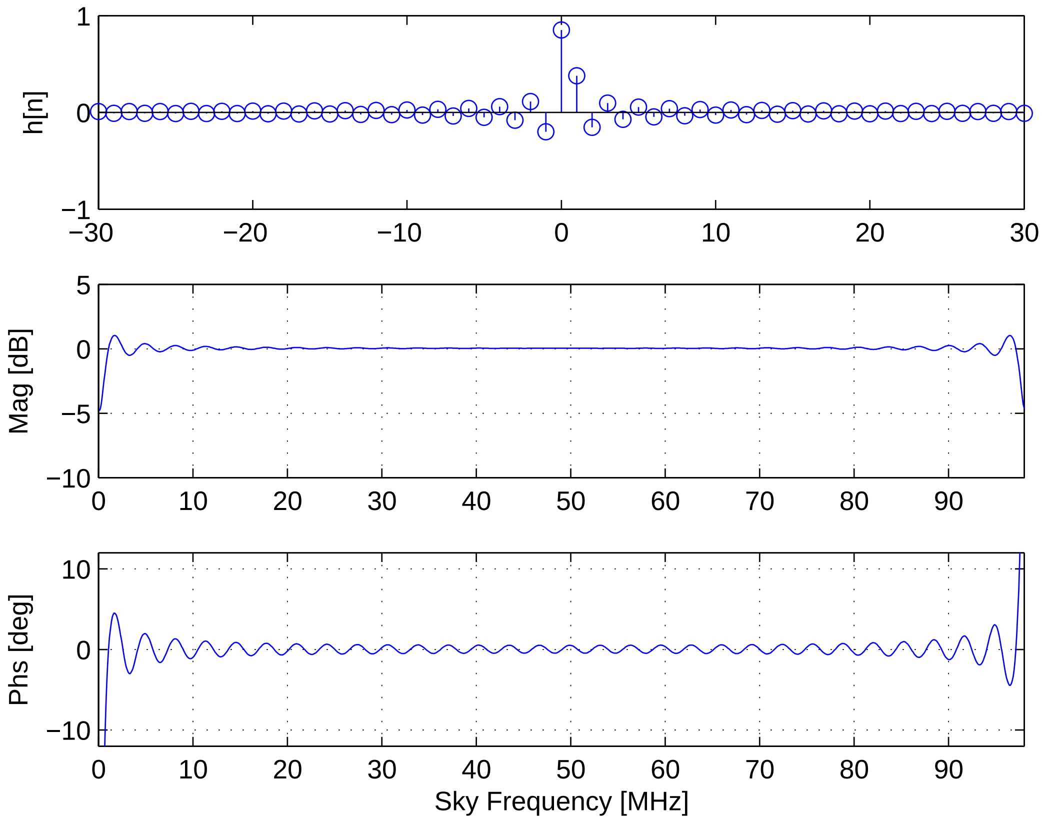

Figure 1: $M = 60$ Delay FIR.

delay required. Each figure consists of three panels: The top panel shows $h[n]$, whereas the middle and bottom panels show the magnitude and phase response, respectively, of the FIR. The phase response in each case has been modified to remove the unavoidable pipeline delay (equal to $MT/2$ and constant with respect to τ) and desired delay (τ), so that result shown in each case is deviation from the ideal response.

A summary of results is shown in Figure 6. We note that it requires a 61-tap ($M = 60$) FIR to achieve a phase accuracy of 1.0° over 10–88 MHz. Reducing the number of taps increases the maximum phase error, as expected. However, note that reducing M also reduces the rate of ripple in the phase response, which may be a consideration in subsequent processing. Further, even the smallest M's considered here yield maximum phase error less than 10°. The magnitude response appears to be quite reasonable in all cases except perhaps $M = 2$.

In conclusion, there appears to be no obvious optimal choice of M. The proposed selection strategy is therefore to select the largest M that will fit in largest FPGA that the budget will permit. FPGA power consumption may also influence this decision. It may be useful to conduct a study to determine the relationship between per-antenna phase error and resulting sidelobe levels, which may provide some further insight as to how to optimally select M.

Finally, it should be noted that for *any* M it is simple to modify $h[n]$ to make the phase error exactly zero at any one desired frequency. So, for example, if the principal frequency of observation is 38 MHz, then it can be arranged for the phase error associated with each delay to be exactly zero

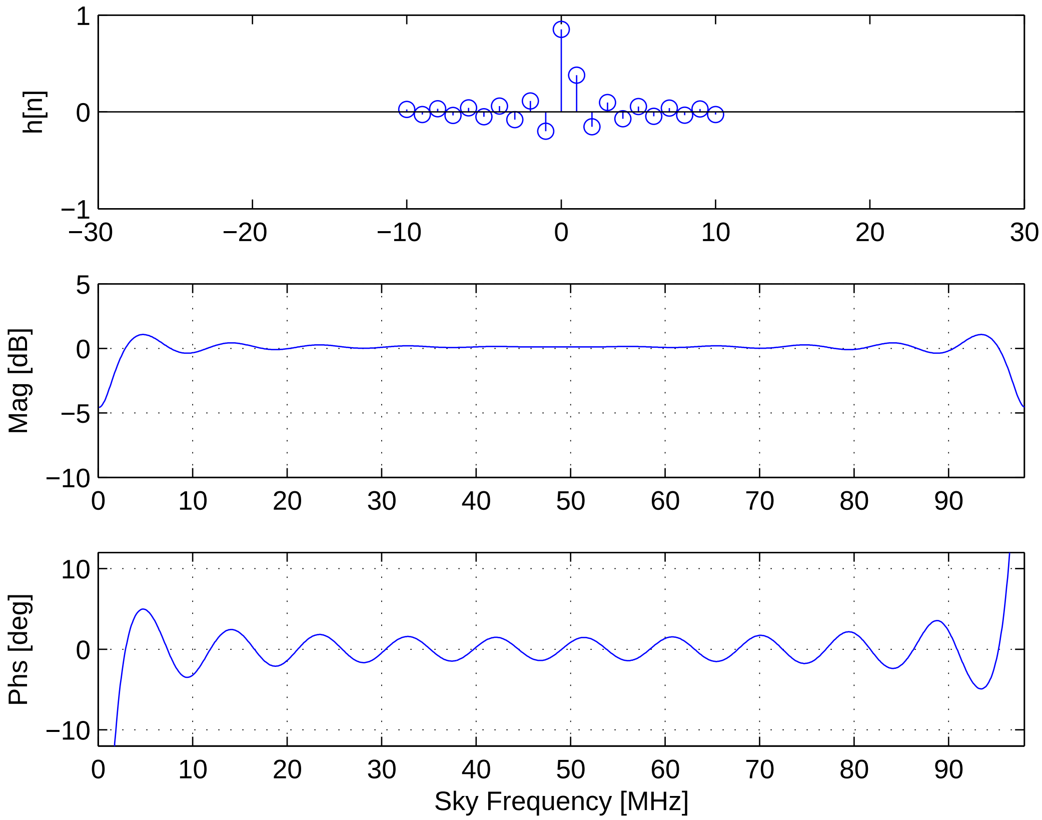

Figure 2: $M = 20$ Delay FIR.

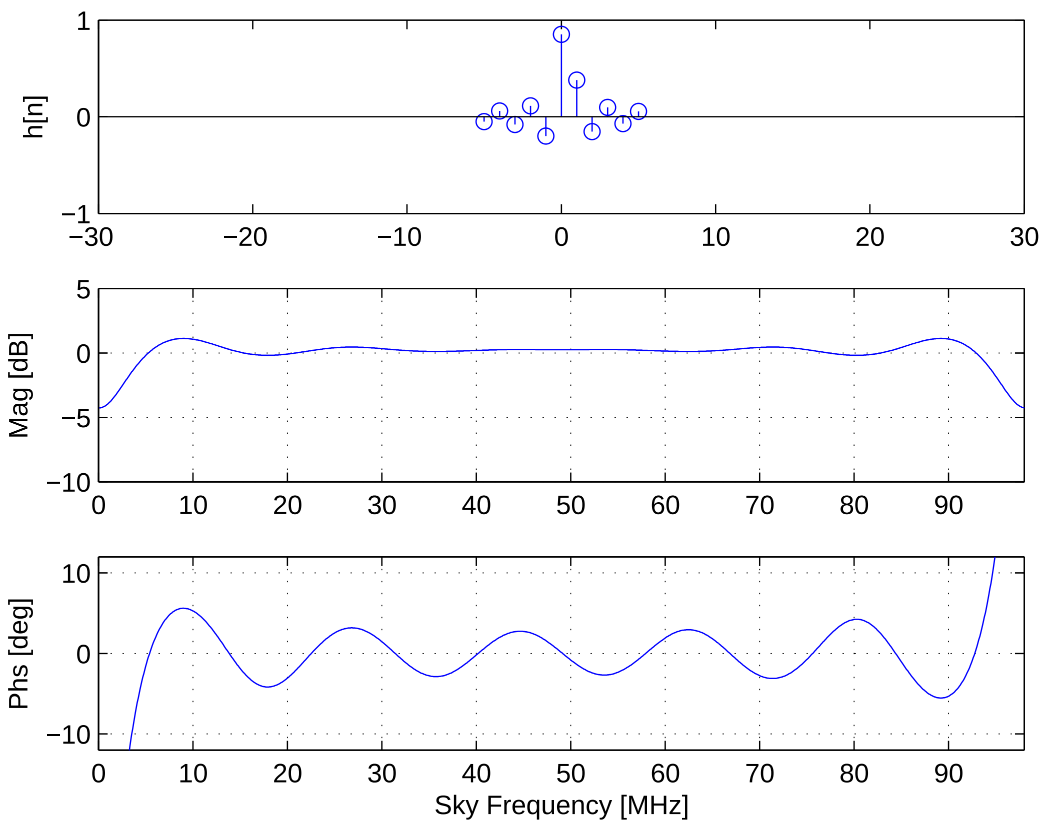

Figure 3: $M = 10$ Delay FIR.

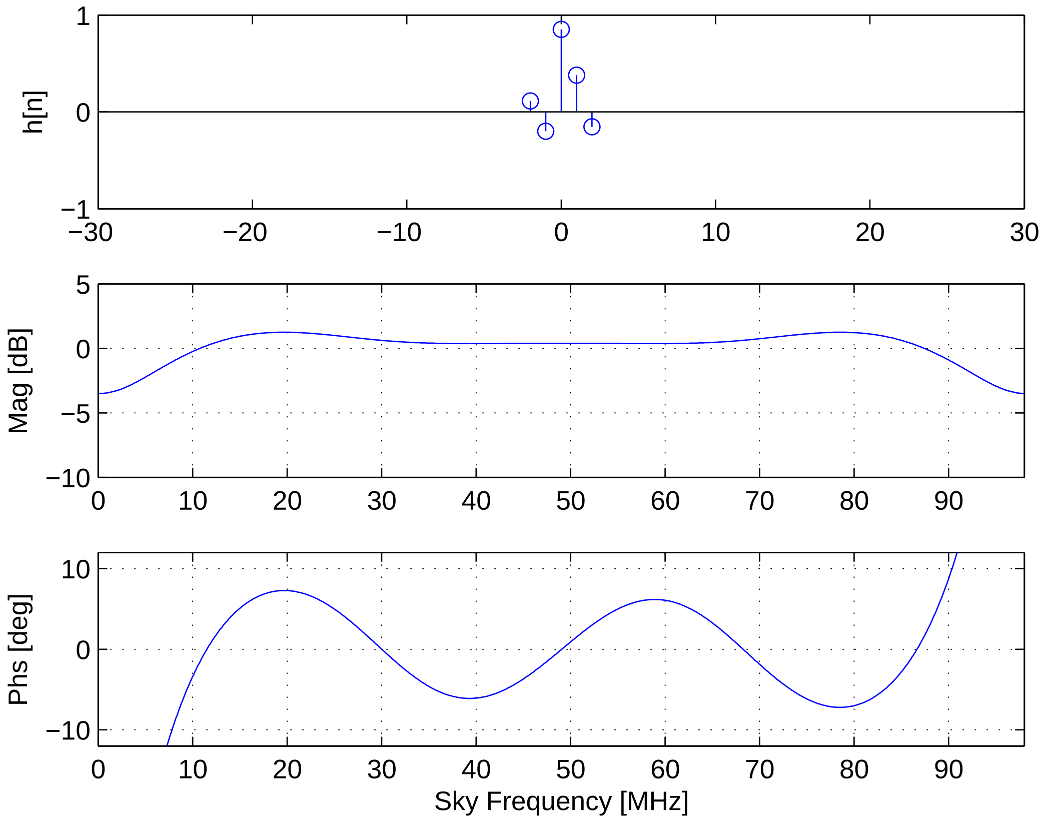

Figure 4: $M = 4$ Delay FIR.

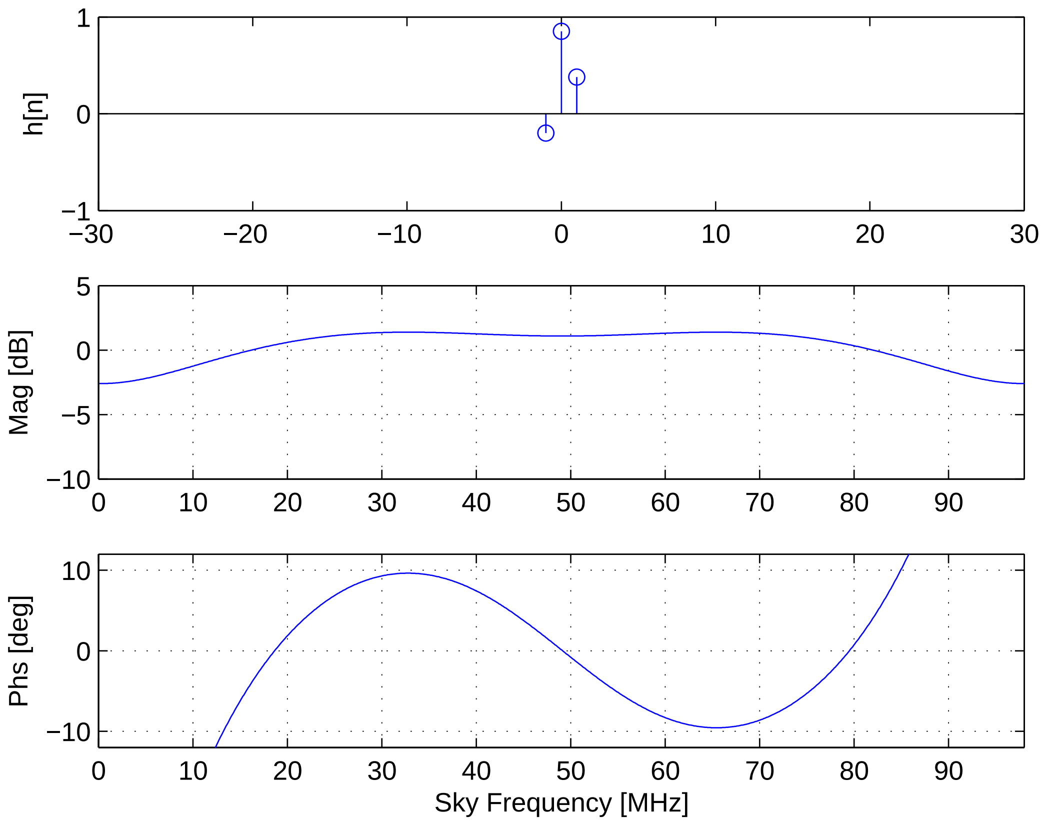

Figure 5: $M = 2$ Delay FIR.

M	N	Mag. [dB]	Phase [deg]	Coeff. Dyn. Range [dB]
60	30	0.2	1.0	40
40	20	0.3	1.4	36
20	10	0.4	2.5	30
10	5	1.1	4.3	25
4	2	1.3	7.3	18
2	1	1.3	9.6	13

Figure 6: Summary of Figures 1–5. "Mag." and "Phase" refer to the maximum deviations from ideal response in the sky frequency range 10–88 MHz. "Coeff. Dyn. Range" is coefficient dynamic range; specifically, the ratio of the maximum magnitude of a coefficient of $h[n]$ to the minimum magnitude of a coefficient of $h[n]$, intended to convey some notion of the number of bits required to accurately express the coefficients digitally.

at 38 MHz. The phase error will obviously increase with increasing separation from this nominal frequency, however.

4 Summary

The proposed strategy for LWA beamforming is summarized as follows:

- A delay FIFO of length 50 implemented in the DIG subsystem, intended primarily to equalize cable delays to an accuracy of 1 sample period at 196 MSPS.
- A delay FIFO of length 34 implemented at the input of each BFU, intended to introduce geometrical delays for delay-and-sum beam pointing to an accuracy of 1 sample period at 98 MSPS.
- A delay-inducing FIR filter following the BFU delay FIFO, intended to introduce a continuously-variable delay in the range 0–5.1 ns as a fine correction to the delay FIFO. It was demonstrated (by example, using $\tau \approx 0.308T$) that this FIR can be implemented with as few as 3 taps for phase accuracy of $\leq 10^\circ$ over the range of sky frequencies (10–88 MHz), and that 61 taps are required to improve accuracy to $\leq 1^\circ$.
- After delay, the result should be processed to convert the raw linear polarizations to calibrated circular polarizations. The method is described in LWA Memo 106 [3].
- Beamforming is accomplished by adding the results, yielding sample streams representing LHCP and RHCP beams.

References

[1] S. Ellingson, "LWA Station Architecture Ver 0.6," October 9, 2007.

[2] S. Ellingson, "ADC Sample Rate and Preliminary Design for a Full-RF ADC Post-Processor," LWA Memo 101, September 11, 2007.

[3] S. Ellingson, "Polarization Processing for an LWA Station," LWA Memo 106, October 29, 2007.