Dispersion by Antennas

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December 11, 2007

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1 Summary

Antennas are inherently dispersive. In this memo the extent of the dispersion is quantified using the example of a thin straight dipole in free space. It is shown that the group delay variation over the span 10–88 MHz is on the order of tens of nanoseconds for a dipole resonant at 38 MHz. This is sufficient to affect beamforming over the affected bandwidth. The dispersion is worst around resonance, and improves with increasing load impedance.

For LWA, the anticipated dispersion can probably be effectively equalized within the beamformer architecture described in LWA Memo 108 [1], using the same FIR filters used to implement fractional-sample delay. However, it may be difficult to perform the measurements necessary to determine precisely the proper equalization, especially if this is something that is likely to vary over time. Therefore, it would be wise to characterize the dispersive properties of LWA candidate antennas and perhaps consider selecting an antenna with minimum group delay variation.

2 Theory

Any antenna can be modeled as an electric circuit consisting of a voltage source V_A in series with the impedance of the antenna, Z_A. When the antenna is connected to a load Z_L (in our case, the input impedance of an active balun), the result is a simple series circuit. The current flowing through this circuit is

$$I_L = \frac{V_A}{Z_A + Z_L} \ . \tag{1}$$

This current is the quantity which is measured by the receiver.

In this model, the value of the voltage source is given by

$$V_A = \mathbf{E}^i \cdot \mathbf{l}_e \tag{2}$$

where $\mathbf{E}^i$ is the electric field incident on the antenna and $\mathbf{l}_e$ is the vector effective length (VEL) of the antenna. In general, VEL is dependent on frequency and the direction from which the electric field arrives. For thin straight dipoles, however, VEL is a real-valued quantity which is proportional to the length of the antenna and oriented in parallel with the antenna. If we assume the electric field is perfectly co-polarized with the antenna, we have that the VEL for a thin straight dipole is

$$V_A = E^i l_e \tag{3}$$

where E^i is the scalar value of the electric field incident on the antenna (i.e., no longer a vector quantity) and l_e is the real-valued scalar effective length of the antenna.

No simple expression exists for the impedance Z_A of a thin straight dipole (much less any more complicated dipole). However, Z_A can be computed approximately, albeit with excellent accuracy, using a model by Tang, Tieng, and Gunn (1993) [2].

The quantity of interest in this study is the group delay τ_g, which is given by

$$\tau_g = -\frac{d\phi}{d\omega} \tag{4}$$

where ϕ is the phase of the transfer function of the system of interest and ω is the angular frequency $2\pi\nu$ with ν being frequency (in Hz). Since a frequency-independent delay is the same as a phase response which is linear with respect to frequency and with negative slope, we see that in this (desirable) situation the group delay is a positive constant with respect to frequency. Dispersion occurs when the group delay is not constant with frequency, as this means individual frequency components experience phase shifts which are not consistent with a frequency-independent delay, resulting in distortion.

In the present problem, the system transfer function can be expressed as

$$\frac{I_L}{E^i} = \frac{l_e}{Z_A + Z_L} \ . \tag{5}$$

Since l_e is a real-valued constant under the conditions stated above, it plays no role in determining the group delay. In fact, we see the group delay in this case determined completely by Z_A and Z_L.

3 Example

In this study, we assume a straight dipole of length 4 m and radius 5 mm, which can be accurately modeled using the technique in [2]. The calculated antenna impedance Z_A is shown in Figure 1. The associated group delay is shown in Figure 2. When $Z_L = 100\Omega$, we see about 35 ns variation in the group delay over the frequency range 10-88 MHz. For comparison, the free-space propagation time across the diameter of a 100 m station is 333 ns. Therefore, the associated dispersion will significantly affect beamforming, especially delay-and-sum beamforming over large portions of this bandwidth. The impact of the dispersion is obviously less for smaller bandwidths in the sense that the antenna can be assumed to be approximately dispersionless (group delay approximately constant with frequency) over a sufficiently small bandwidth.

Also shown in Figure 2 is the result assuming an increased load impedance of 200Ω. Note that the effect is to significantly decrease the group delay in the region around the first resonance of the antenna, where the group delay variation is most onerous.

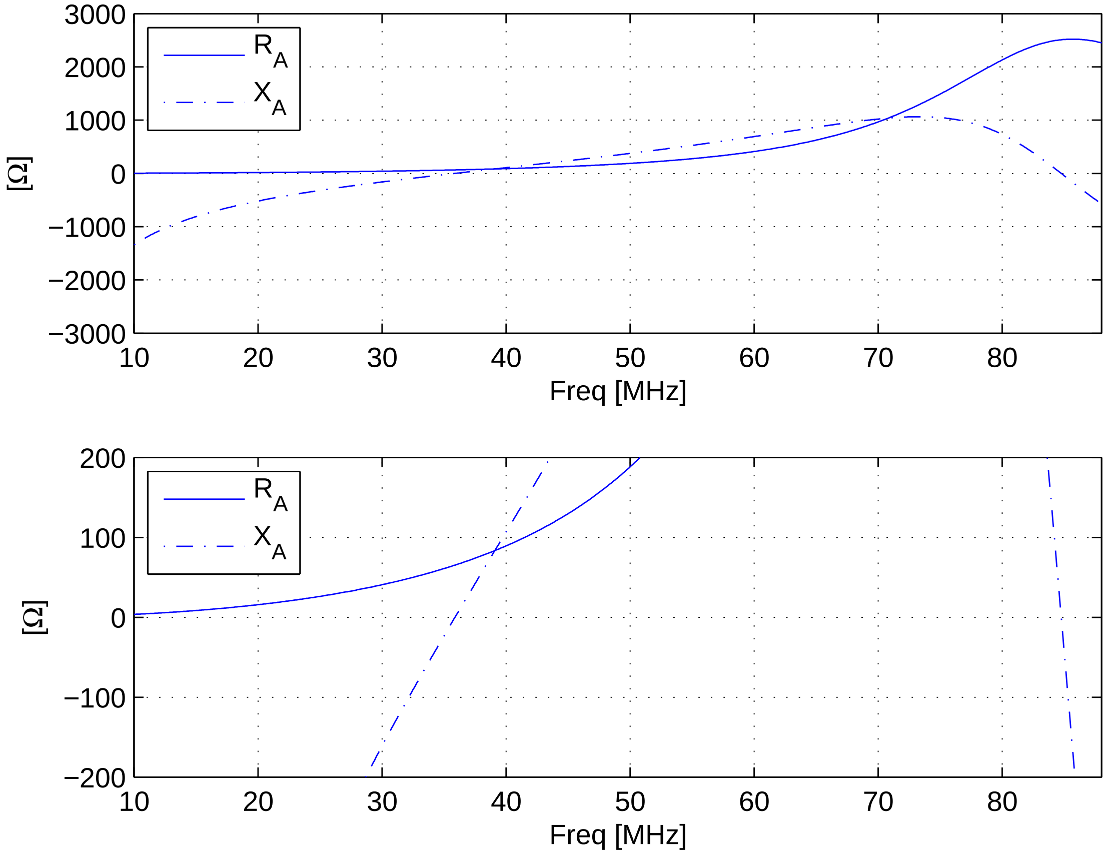

Figure 1: Impedance of a 4-meter-long straight dipole of radius 5 mm. The bottom panel is a close-up from the top panel.

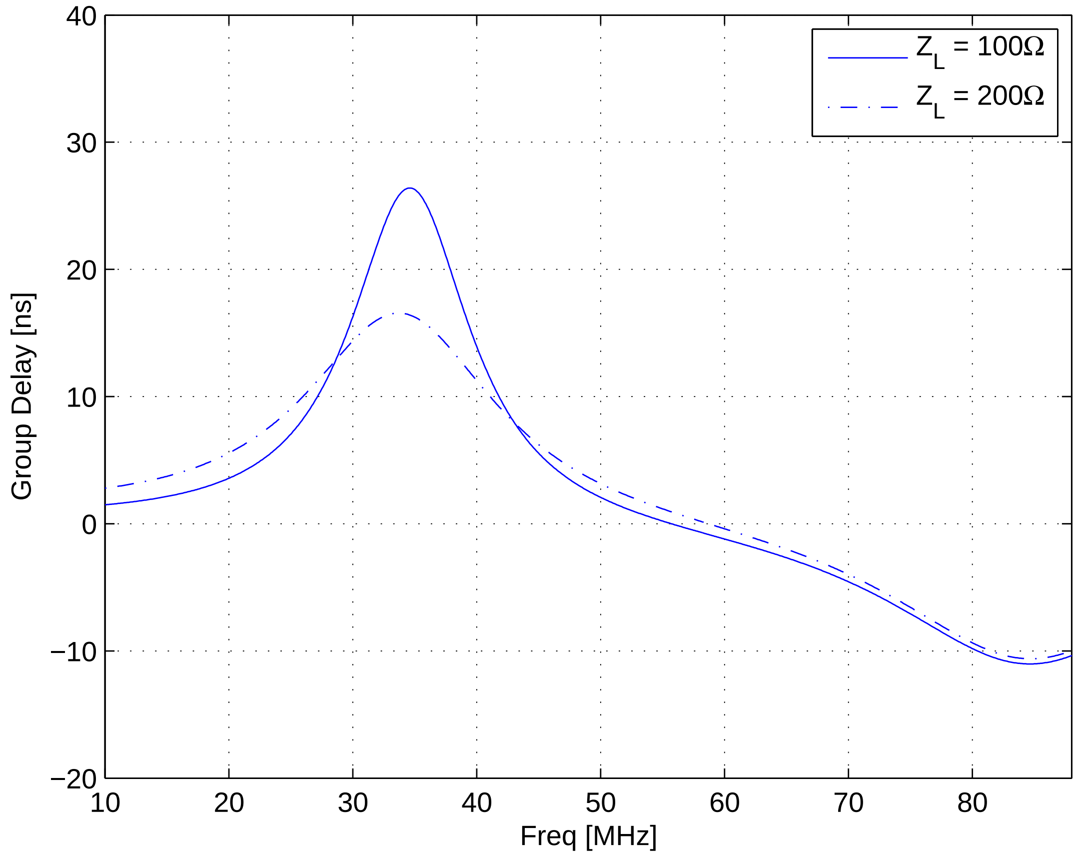

Figure 2: Group delay.

References

[1] S.W. Ellingson, "BFU & DP1 Preliminary Design," LWA Memo 108, November 4, 2007.

[2] T. Tang, Q. Tieng, and M. Gunn, "Equivalent Circuit of a Dipole Antenna Using Frequency-Independent Lumped Elements," *IEEE Trans. on Antennas & Propagation*, Vol. 41, No. 1, Jan 1993.