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LWA Memo No. 143

LWA Fine Delay Tracking

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The current baseline design for LWA beamforming, based on both the preliminary design [1] and the JPL proposal [2], includes an FIR filter to implement sub-sample delay adjustment in each antenna's signal path prior to combining. All of the pre-combining signal processing operates over the full 10-88 MHz bandwidth. Here we consider the requirements on the fine delay filter. This is important because it now appears that these FIR filters, if implemented as planned, will dominate the station's digital logic.

The required delay depends on the beam direction, and if all antennas are perfectly aligned for the selected direction they remain nearly aligned for nearby directions, leading to the concept of the "delay beam" as the set of directions within which the alignment is acceptable. The angular size of the delay beam is $\theta_d = ac/BL$, where B is the final bandwidth, L is the maximum projected baseline length, c is the speed of light, and a is a constant depending on the criterion for "acceptable." We choose a=2, corresponding to a null in the signal on the longest baseline (but not on shorter baselines). Whereas L generally varies with position angle as well as with beam direction, so does θ_d. L=100 m and B=8 MHz gives θ_d=86º. Note that θ_d is independent of center frequency, unlike the synthesized beamwidth (which varies from 17.5º to 2.0º at zenith over 10 to 88 MHz for the same array size [3]). At much smaller bandwidths, the delay beam covers the entire sky and no delay tracking is needed if the delays are aligned at zenith.

If the delays are not accurately aligned, the gain on one baseline is reduced by a factor of $\sin(\pi\Delta\tau B)/(\pi\Delta\tau B)$, where $\Delta\tau$ is the delay error on that baseline, for a white noise signal of bandwidth B. We adopt the requirement $\Delta\tau < 0.2/B$, since this keeps the maximum loss below 7%. The typical loss in the synthesized beam gain of a large array will be much less. To achieve this on every baseline, the departure of any antenna's delay from its ideal value should be less than $\Delta\tau/2 = 0.1/B$, which is 12.5 nsec for B=8 MHz or 1.28 nsec for B=78 MHz.

After combining, the bandwidth will be reduced to 8 MHz or less in the digital receivers (DRXs) [3]. Since the pre-combining sampling rate is 196 MHz, integer sample delay adjustment already provides a resolution of 5.1 nsec, suggesting that fine delay tracking with the FIR filter may not be necessary. However, there are some complications.

First, the 8 MHz maximum bandwidth is based only on limitations downstream: the aggregate bandwidth of all beams and polarizations would otherwise be too large for data transmission, recording, and/or correlator systems. As technology improves, the DRXs may be replaced by larger-bandwidth versions, or bypassed entirely to allow the downstream processing to see the full bandwidth. It is advantageous if the upstream beamforming system, which is much larger and more expensive than the DRXs, already supports the full bandwidth so that it need not also be replaced.

Second, even if the final bandwidth is limited to 8 MHz per DRX channel, there will be at least two DRX channels per beam and they can be tuned to opposite ends of the RF band, so the beamformer's delay tracking must be sufficiently accurate at all frequencies[2].

[2] Another possibility, suggested by Robert Navarro, is moving the DRXs to the beamformers, before combining. If we retain two DRXs per beam, this doubles the number of separate channels that must be carried across the combining dasiy-chain, but the total bandwidth per RF channel is reduced from 78 MHz to 16 MHz. It also means that the number of integer sample delays (FIFOs) must be doubled and the number of DRXs is multiplied by 256. Nevertheless, the

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Third, there is dispersion in the response of LWA antennas [4][5] as well as in the coaxial cables from each antenna to the processing shelter [6], so the required value of compensating delay might vary with frequency. It has been assumed that "dispersion can probably be effectively equalized ... using the same FIR filters used to implement fractional-sample delay" [4], but this will not be true if the FIR filters are not present. The integer-sample delay implemented by FIFO memories is independent of frequency, so it can provide no dispersion compensation.

Finally, it has also been suggested [5] that FIR filters be used to compensate for variation of polarization of the antennas with frequency. This is more complicated since it requires a 2x2 matrix of filters, rather than just one per polarization channel, but it is in principle possible to include the delay and polarization compensation in the same set of filters.

Although all of these functions (fine delay, dispersion, and polarization compensation) can be combined in the same set of filters, the filter complexity (number of taps) will be larger than that needed for the most complex function.

To proceed further, we must examine these issues quantitatively.

Dispersion

Variation of group delay with frequency is unimportant provided that it is the same for all antennas. Any correction need only standardize the dispersions by making them the same; it need not make the delay constant with frequency.

Part of the dispersion is due to the cables [6], and this can be made essentially identical for all antennas by equalizing the cable lengths. Since the cable dispersion is small (estimated at 4.2 nsec change over 10 to 88 MHz for 150 m length), ±20% length variation gives ±0.84 nsec delay difference across the band. By setting the compensating delay for the band center, the maximum error is 0.42 nsec, which is negligible at full bandwidth.

There is also dispersion intrinsic to the antennas [4], and this could vary from stand to stand because of their interactions in the non-uniform array. There is evidence from simulations [7] that such effects are small, so it seems unlikely that any dispersion correction is needed. We will not consider it further in this memo.

Polarization

Similarly, variation of polarization with frequency is unimportant if it is the same for all antennas. Departure of polarization from a canonical form (like RCP/LCP) is also unimportant, contrary to implications in [5] and [7]. Transformation to a particular basis, if desired, can be implemented after the signals are combined into beams, saving a factor of b/n in processing effort, where b is the number of beams and n is the number of antennas. Polarization processing for each antenna stand, if needed at all, can be limited to standardizing the response.

The simulations in [7] also indicate that the interactions among stands have little effect on their polarization responses, but more realistic simulations (or measurements) may be needed to be sure about this. Any per-stand processing is costly because it requires a 2x2 matrix of filters. We neglect it here.

Fractional-sample delay

For now we consider only the fine delay tracking, and we ask how complex a digital filter must be to achieve adequate accuracy over the full 10 to 88 MHz bandwidth. Coarse delay of 5.1 nsec resolution is available with a FIFO at the 196 MHz sampling rate, but resolution ~1 nsec is needed. An FIR interpolation filter can be used to approximate the fractional-sample part of the delay, and the filter length for a given accuracy is independent of the delay resolution. The latter

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added pre-combining logic may be less than that required of the FIR filters, which could be eliminated. This idea needs more study, but it will not be further discussed in this memo.

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affects only the number of coefficient sets that must be stored and the frequency at which one set must be replaced with the next as the desired delay changes. For a given filter complexity, maximum error occurs when interpolating to the mid-point between samples, so we confine our analysis to that case.

Let $x(t)$ be the input signal, and let $x_n = x(n/f_s)$ be the nth input sample and y_n the nth output sample of the interpolating filter, where f_s is the sampling frequency. Then the ideal filter output is $z_n = x((n–R+0.5)/f_s)$, where R is a fixed integer delay needed to make the filter realizable. The mean-square interpolation error is then $\langle (y_n - z_n)^2 \rangle$, where the average is over all samples n. This can be calculated for simple forms of $x(t)$, and in particular we consider $x(t) = \cos(2\pi f t)$ for frequencies f in our 10 to 88 MHz band. This leads to several possible FIR filter designs [8], of which we consider two. First, for a given filter length L, we minimize the average interpolation error over the band of interest; call this the MMSE filter. This is sensible if the actual signal has an approximately flat spectrum; otherwise we could consider minimizing the weighted average. Second, we minimize the maximum interpolation error over the band of interest; call this the minimax filter.Methods for synthesizing the coefficients of both filter types are given in [9] and references cited therein (sections 4.3.6 and 4.3.9, respectively). MATLAB code for the MMSE case was written by the author, and for the minimax case it was downloaded from [10].

Results for MMSE filters of several lengths are plotted in Figure 1. This gives the interpolation error for a sinusoidal signal as a function of normalized frequency $F=2f/f_s$ (fraction of Nyquist). For the upper plot, error minimization was over the normalized band 0 to a, where a was chosen to make the error at F=0.9 equal to the peak over the lower frequencies. (F=0.9 corresponds to 88.2 MHz at f_s = 196 MHz). In the lower plot, a second "don't care" region at F<0.1 (f < 9.8 MHz) was added, but this made very little difference over the band of interest. An error magnitude of 0.1 may be interpreted as an additive error signal 20 dB below the desired one; similarly .031 corresponds to –30 dB error and .01 to –40 dB.

Similar results for minimax filters are plotted in Figure 2. Here the calculations are more complicated and the available code [10] allowed only a lowpass band (no lower don't care region). All minimizations were over F = 0 to 0.9. The minimax critereon leads to an equal-ripple error pattern, and it can be seen that this permits only a small reduction in the maximum error (at the upper end of the band) compared with the MMSE filter of the same length.

Acknowledgments

Thanks to Robert Navarro for pointing out the Valimaki and Laakso paper [8] and their filter synthesis software [10].

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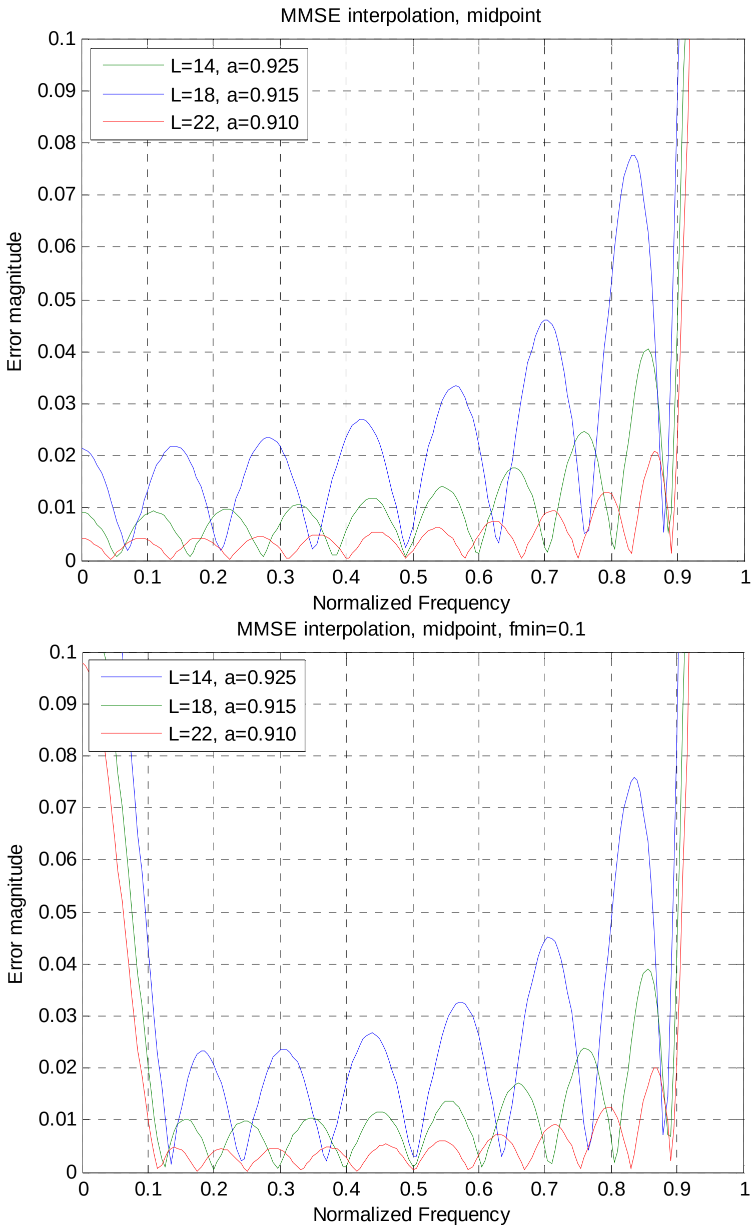

Figure 1: Interpolation error magnitude vs. frequency for MMSE error interpolation filters of length 14, 18, and 22 taps. Top: mean square error minimized over normalized frequencies 0 to approximately 0.9. Bottom: error minimized over normalized frequencies 0.1 to approximately 0.9. Interpolation is to the midpoint between samples; the error is smaller for all other fractional-sample interpolation

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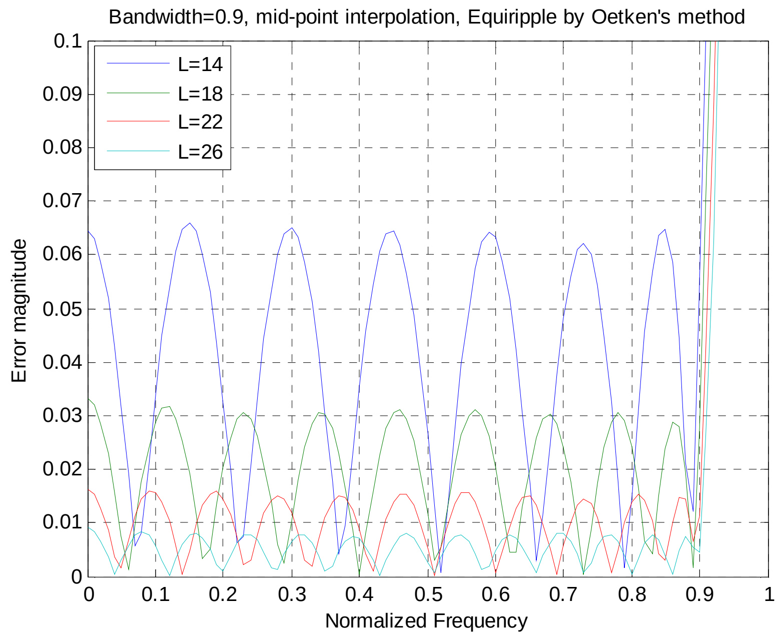

Figure 2: Error magnitude vs. frequency for minimax filters of length 14, 18, 22, 26 taps, based on code from [10]. Maximum error is minimized over normalized frequencies 0 to 0.9.