Beam Dwell and Repointing

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1 Summary

Conventional mechanically-steered reflector antennas can continuously track a source using a known, constant beam pattern as the source moves across the sky. The same will not be possible for LWA stations. The discrete-time nature of array digital signal processing means the beam cannot track continuously, but rather only in discrete steps. Further, unlike reflector antennas, the LWA beam pattern changes with pointing direction. Although it is tempting to try to emulate the continuous source-tracking paradigm of conventional reflector antennas, these two considerations seem to make this a bad idea for LWA. Repointing the LWA station beam as quickly possible will result in a subtle "rumbling" of both the time-domain output and the antenna pattern, in a way that may be hard to quantify and that is likely to compromise instrumental stability. A better idea for tracking is simply to hold the station beam utterly constant for as long as possible, and then to "jump" to a new pointing only when necessary; i.e., as needed to maintain an acceptable level of sensitivity and coherence over bandwidth. Such a scheme will also dramatically simplify requirements on digital signal processing and monitor/control systems.

2 Analysis

The first consideration in determining a maximum reasonable time between repointings is that the pattern gain in the direction of the source should not degrade too much. The time taken by the source to drift through the beam is smallest when the beam is narrowest. The beam is narrowest for zenith ($\theta = 0$) pointing at the highest frequency ($\nu = 88$ MHz). The theoretically-derived normalized voltage pattern from a uniformly-illuminated circular aperture is given by [1]

$$f(\theta) = \frac{2J_1\left(\frac{\pi D}{\lambda}\sin\theta\right)}{\frac{\pi D}{\lambda}\sin\theta} \tag{1}$$

where $J_1(\cdot)$ is the Bessel function of the first kind and first order, D is the diameter of the aperture (in this case, the station), and λ is wavelength. The above expression is normalized in the sense that the maximum value, at $\theta = 0$, is unity. For $D = 100$ m and $\lambda = 3.4$ m (88 MHz), one finds that voltage pattern is down from 100% to 99.5% (99% in terms of power) at $\theta = 0.121°$. Thus, there is a $0.242°$ span over which the source can drift with less than 1% degradation in gain. Given the maximum apparent sky rate of rotation of $360°$ day^{-1}, we find the minimum time the source can be expected to remain in this region of the beam is 58 s; i.e., about 1 min.

The second consideration in determining a maximum reasonable time between repointings is that coherency should not be excessively degraded. This is a potential issue because LWA stations will use "delay-and-sum" beamforming in order to obtain beams with large bandwidth. Consider a plane wave incident from direction in which $\hat{\mathbf{r}}$, a unit vector with tail fixed to the origin, points. Let the position of the n^{th} antenna in the array be $\mathbf{p}_n$, a vector whose tail is also fixed at the origin. Then $\tau_n(t)$ is the geometrical delay (with respect to the origin) associated with $\mathbf{p}_n$ and is given by:

$$\tau_n(t) = \frac{-\hat{\mathbf{r}}(t)\cdot\mathbf{p}_n}{c} \tag{2}$$

where c is the speed of light and we explicitly show the time dependence of $\hat{\mathbf{r}}$ to underscore the fact this is time-varying since the sky appears to move relative to the array. The delay $\tau_n(t)$ can be expressed in terms of a Taylor series expansion around time $t = t_0$ as follows:

$$\tau_n(t) = \tau_n(t_0) + \left[\frac{d}{dt}\tau_n(t)\right]_{t_0}(t-t_0) + \frac{1}{2}\left[\frac{d^2}{dt^2}\tau_n(t)\right]_{t_0}(t-t_0)^2 + \ \ldots \tag{3}$$

To compute the first derivative of $\tau_n(t)$ we note that $\mathbf{p}_n$ can be expressed as

$$\mathbf{p}_n = \hat{\mathbf{x}}d_n\cos\phi_n + \hat{\mathbf{y}}d_n\sin\phi_n \tag{4}$$

where d_n is the distance of the associated antenna from the origin, and ϕ_n is the associated radial coordinate in the $x-y$ plane. Similarly,

$$\hat{\mathbf{r}}(t) = \hat{\mathbf{x}}\cos\phi(t)\sin\theta(t) + \hat{\mathbf{y}}\sin\phi(t)\cos\theta(t) + \hat{\mathbf{z}}\cos\theta(t) \quad (5)$$

Thus

$$\tau_n(t) = -\frac{d_n}{c}\sin\theta(t)\cos\left[\phi(t)-\phi_n\right] \quad (6)$$

$$\frac{d}{dt}\tau_n(t) = -\frac{d_n}{c}\left[\frac{d\theta}{dt}\cos\theta(t)\cos\left[\phi(t)-\phi_n\right] - \frac{d\phi}{dt}\sin\theta(t)\sin\left[\phi(t)-\phi_n\right]\right] \quad (7)$$

Let this quantity be known as α_n, which is understood to be time-varying. Note that $d\theta/dt$ and $d\phi/dt$ are both upper-bounded by the sky's apparent rate of rotation, defined earlier. Thus, we can assume the upper bound for α_n when $\phi(t) = \phi_n$ is no greater than the upper bound for any other value of $\phi(t)$; so that we need only to consider the first term to determine this bound. Assuming $d_n < 1$ km ($>> D$ in order to account for the possibility of "outriggers" being involved, larger than expected stations, and otherwise being extremely conservative), we find that $\alpha_n < 2.42 \times 10^{-10}$. Comparing the first two terms of the Taylor series for the minimum expected delay of $\approx$ (4 m)$/c$ (assuming minimum 4 m spacing between stands):

$$\frac{\alpha_n \Delta t}{\tau_n(t_0)} < \left(0.0182\ \mathrm{s}^{-1}\right)\Delta t \quad (8)$$

where $\Delta t \equiv t - t_0$. Thus, we see that under these assumptions the second term of the Taylor series becomes important at the 1% level at $\Delta t \sim 549$ ms. This can be taken as a very conservative lower bound on beam repointing time from the perspective of ensuring negligible degradation of coherency in beamforming.

A third consideration is the requirement for rapid beam-switching arising from the ionospheric calibration support scheme proposed in LWA Memo 128 [2]. This scheme requires cyclic observation of roughly 100 sources with source-dependent integration times of 10 ms to 200 ms, with the cycle repeating as rapidly as possible – within ~ 5 s in order that the ionosphere not change too much during the time required to complete the cycle. Since the longest integration time is shorter than the guidelines established above for either beam pattern (~ 1 min) or coherency (~ 500 ms), this drives the requirement for beam switching time.

3 Recommendations

Taking these considerations into account, the following scheme is proposed: First, all relevant subsystems in the station should be synchronized to a clock signal consisting of a 1 ms period, or "tic". Beam repointing would be scheduled in advance (perhaps only seconds or less in advance) to take place (that is, to begin the repointing process) on a specified clock tic. The time required to repoint a beam – including the time required for any transients or other invalid data to "flush" – should be less than or equal to 5 ms; i.e., 50% of the shortest integration time mentioned in Memo 128. It is assumed that this will not significantly slow down the time required to cycle through all pointings, as each pointing requires integration time of 10 ms or longer. However, the actual time required to cycle through sources taking into account this 5 ms "dead time" should be recalculated to verify that the resulting total cycle time is sufficiently short. If it is not, then the requirement can be scaled back from 5 ms to a smaller value. I suggest however that the dead time requirement be revised downward from 5 ms only if it is determined to be absolutely necessary, so as to avoid unnecessarily increasing the effort required to design the associated digital signal processing and control subsystems.

For other applications requiring only source tracking – and not rapid beam switching – the default interval for beam repointing would be no less than 500 ms, yielding a 99% duty cycle; i.e., only 1%

of the observing time would be possibly invalid due to the allowance of 5 ms for invalid data during repointing. If coherence is not considered an issue (it might not be...), then this interval could be increased to as much as one minute or longer, which might be desirable for very deep integrations in which instrumental/systematic effects are a primary concern.

References

[1] W. Stutzman and G. Thiele, *Antenna Theory and Design*, 2nd ed., Wiley, 1998, p. 317.

[2] A. Cohen and N. Paravastu, "Probing the Ionosphere with the LWA by Rapid Cycling of Celestial Radio Emitters," Long Wavelength Array Memo Series No. 128, Mar 12, 2008. [online] http://www.phys.unm.edu/∼lwa/memos.