Performance of Simple Delay-and-Sum Beamforming Without Per-Stand Polarization Corrections for LWA Stations

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Contents

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1 Introduction

It has been known since LWA Memo 67 [1] that the antennas within an LWA station are likely to experience significant levels of mutual coupling. However, it is not yet clear if any special measures are required or useful in the per-stand processing prior to "full RF" beamforming to account for this. The station electronics can be much simpler if the differences in the responses of antennas can be neglected; in particular, if the differences in polarization response as a function of frequency can be neglected [2].

1.1 Background

LWA Memo 140 [3] considered the impact of mutual coupling on the ability to perform per-stand calibration prior to beamforming, using the scheme discussed in LWA Memos 106 [4], 107 [5], and 138 [6]. It is found that for a stand near the center of the array, the requirements in terms of the length of FIR filters is about the same, and that the mutual coupling does not seem to have much effect on the ability to convert the "raw" voltage signals provided from each dipole into standard left- and right-circular polarizations. However, this is not the same as saying that mutual coupling does not significantly affect the polarization response; Memo 140 only shows that mutual coupling does not have much impact on the performance of this calibration. Furthermore, only one stand near the center of a 64-stand version of the array was considered. Thus, Memo 140 does not have much to say about the performance of a "full RF" beamforming scheme in which the differences in per-stand polarization response are neglected.

Memo 147 [7] showed some initial results from an attempt to characterize the stand-to-stand variations in the response of antennas in a full (256-stand) LWA station array. This study was limited to one frequency, 38 MHz, using the old (100 m diameter circular, 4 m minimum inter-stand separation) station footprint. After some steps to demonstrate the efficacy/validity of the numerical analysis, the H-plane patterns were examined. It was found that the H-plane co-pol patterns vary dramatically from stand to stand, with gain differences ranging from +2 dB to −6 dB relative to a single stand isolated from the rest of the array. The cross-pol patterns were also seen to have significant variations.

1.2 This Memo

Having determined that mutual coupling does in fact significantly affect stand *in situ* responses, this memo attempts to follow up Memo 147 to answer the "bottom line" question of whether these variations matter in the sense that per-stand, frequency-dependent polarization and dispersion corrections are required to achieve acceptable beamforming performance. First, it should be noted that an important difference between this memo and Memo 147 is that we now assume the new 110 m × 100 m elliptical station array footprint, with 5 m minimum inter-stand separation.[1]

It is found that despite the large stand-to-stand variations that exist over the entire frequency range of interest, simple beamforming – that is, delay-and-sum beamforming for each of the two "raw" polarizations neglecting stand-to-stand variations other than geometrical/propagation delay – is nearly optimal in terms of main lobe gain. The coupling-induced differences in stand responses appear to "average out" with very low bias, such that near-optimum array gain can be achieved without per-stand corrections for polarization or antenna dispersion.

Although not quantified in this memo, it is important to note that these results *do not* imply that mutual coupling has negligible effect on array gain. Although the effect of coupling on main beam gain is very small for the two pointings considered in this memo, it is known from previous

[1]This new geometry was obtained by email from A. Cohen on Dec 18, 2008 and does not appear to be documented anywhere at present. For this reason, the stand position data are included in this memo as Appendix A.

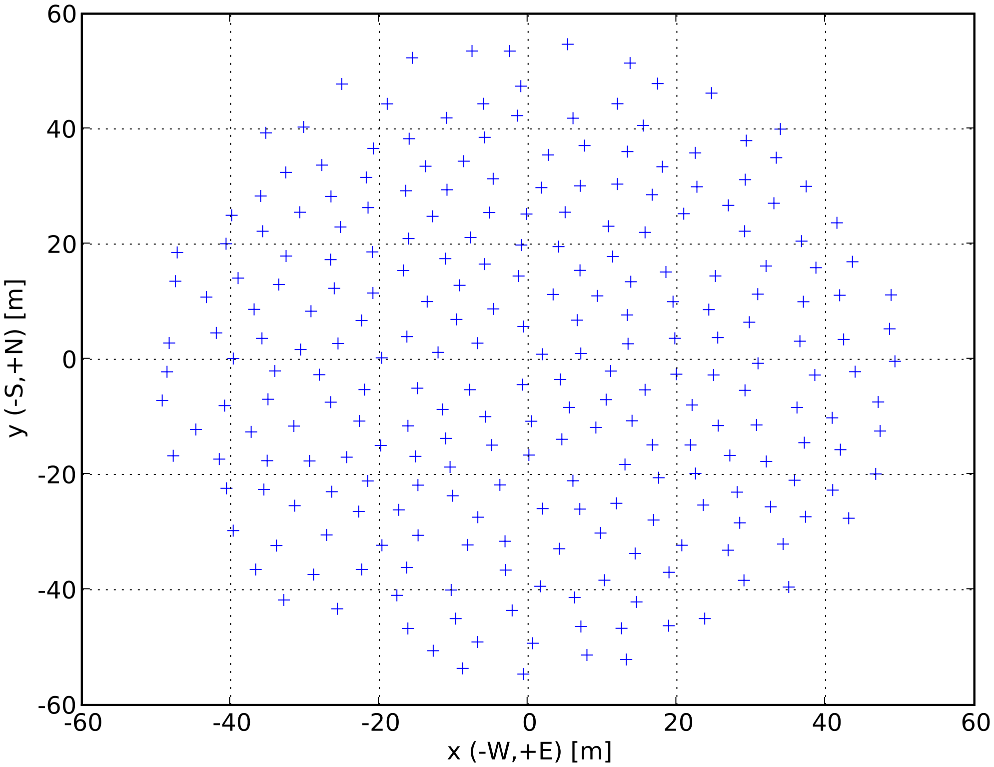


Figure 1: Array geometry. The exact locations of the stands are given in Appendix A.

work (Memo [8] and [9]) that the difference for some pointings can be on the order of 10's of percent. Instead, the above findings indicate that simple delay-and-sum beamforming of the raw dipole polarizations is nearly optimal in the sense that it provides the maximum possible gain, whatever that gain may be. Furthermore, it should be noted that these results do *not* imply that the shape of the main lobe or the levels and positions of sidelobes are not significantly affected by mutual coupling. To the contrary, the large variations in per-stand responses noted in this memo and in Memo 147 mean that it is quite likely that mutual coupling has a significant effect on sidelobe levels and positions. The effect of mutual coupling on the main beam shape is probably less significant – "small", but difficult to characterize in a meaningful way without further study.[2] Finally, it should be noted that the fence and shelter are not yet considered in the results presented here.

2 Array Model

The LWA station array consists of 256 stands, each consisting of two perpendicularly-arranged dipoles, with irregular spacings within an ellipse with major and minor axes of diameter 110 m aligned North-South and 100 m aligned East-West, respectively. This is shown in Figure 1. The minimum spacing between stands is 5 m.

[2]Slide 6 of reference [9] gives an idea of what to expect; i.e., a very small difference in beamwidth but with an offset in the direction of maximum gain as large as a degree or so.

The station array will be surrounded by a fence with 5 m minimum separation from the stands [10], and it is known from LWA Memo 129 [11] that the fence can have a significant impact on the antenna patterns. It is also known from LWA Memo 141 [12] that the shelter can also have a significant effect on the antenna patterns. However, the fence and the shelter are not included in the model used to generate the results presented in this memo. Determining the effect of the fence and shelter is left for future work.

Stand design is not completely determined at present. This is actually not of much consequence in this study since the proposed "tied fork" dipole design, when replicated 512 times, results in a model with prohibitively-large computational burden and high potential for subtle numerical difficulties when analyzed using the moment method (e.g., NEC2 or NEC4). Therefore, a simple dipole model is used instead. In this model, each dipole is a perfectly-conducting cylinder 3.947 m long and 6 cm in diameter. This is roughly equivalent to a "blade"-type antenna having blade width of 12 cm [13]. The center 15 cm is the feedpoint region, and is horizontal to the ground. The "arms" on either side bend downward at a 45° angle. In the coordinate system used in this memo, dipoles are aligned parallel to the x and y axes, with z pointing toward the zenith. The feedpoint heights of x- and y-aligned dipoles is 2.019 m and 1.929 m respectively; that is, the entire dipole is shifted up or down slightly to prevent feedpoint wires from intersecting or becoming too close to be properly modeled. The ground is assumed to be an infinite flat surface of perfectly-conducting material.

NEC2 is used to perform the analysis. The 15-cm-long feedpoint wire is divided into three segments, and the center segment is loaded with a series impedance of 100 Ω, modeling the FEE input. Each dipole arm is divided into 10 segments, which is appropriate for frequencies up to 88 MHz. The design of the dipole and its segmentation were carefully reviewed with respect to known NEC design guidelines and limitations, and the model was analyzed in a preliminary study to confirm it's numerical stability over the frequency range 10–88 MHz. The model was further validated at 38 MHz, as explained in LWA Memo 147 [7]. The complete array model (but not yet including the shelter and fence) uses about 12,000 segments and requires roughly one hour to run (on a 2007-vintage dual-core Centrino-based laptop, running Ubuntu Linux) for each frequency of interest.

A reasonable question to ask is "How well does this model describe arrays consisting of stands based on the "tied fork" dipole concept, with each stand over a small ground screen?" The answer, unfortunately, is extremely difficult to answer in a satisfying way. Qualitatively, the behavior is expected to be very similar with similar trends. Quantitatively, based on experience, the results are expected to agree "sufficiently well" over most of the frequency range of interest, since the antennas are very similar from an electromagnetic perspective. The tied-fork dipole is likely to have somewhat broader impedance bandwidth, so the greatest discrepancies in impedance are likely to occur at the highest and lowest frequencies, and the greatest discrepancies in pattern will occur at the highest frequencies. Regardless of all of these points, the issue is essentially moot as the tied-fork design would require about 3 times as many segments, resulting in computational burden which is roughly $3^3 = 27$ times greater. Changing from an infinite perfectly-conducting ground screen to 256 separate ground screens with intervening exposed earth makes the computation altogether intractable without extraordinary measures; e.g., moving to a large, high-performance computer cluster. An additional issue favoring the use of the simple dipole + infinite ground model over a closer-to-true model is that conventional tests for model "reasonableness" and numerical stability are correspondingly difficult to carry out for the latter. Thus, we are essentially trading off decreased model detail for both speed and increased confidence in the results.[3]

[3]A possible topic for future work be to identify a simple frequency-dependent model for the production stand design. That is: For each frequency, these is a simple model model which best approximates the production stand response. Then, in future analyses, the simple stand model would be varied with frequency in order to better predict the performance of the actual stand.

3 Optimal vs. "Simple" Beamforming

In this memo we are concerned with the performance of "simple" beamforming; in particular, what is lost with respect to "optimal" beamforming. Thus, we need to define these terms.

3.1 Optimal Beamforming

Optimal beamforming is defined as beamforming which results in maximum signal-to-noise ratio (SNR) in the specified pointing direction, subject to no other constraints (such as sidelobe levels/positions, etc.), and assuming spatially-white noise[4]. Assuming a single monochromatic plane wave incident on the array, this is achieved by multiplying the signal measured at each dipole by it's conjugate value. In the signal processing literature, this is commonly known as *maximal ratio combining* (MRC). MRC has the effect of weighting dipoles producing signals with larger SNR more heavily than dipoles producing signals with smaller SNR, and it can be shown that the result has the maximum possible SNR.

An informal proof of this is as follows: Consider a monochromatic plane wave incident on a dipole array, and assume that the noise is spatially white. In this case, the power collected by each dipole is equal to $|I|^2R$, where I is the (complex-valued RMS) current at the dipole terminals, and R is the load across the dipole terminals. The maximum power that can be collected by the array is simply the sum of the powers collected at the dipole terminals. In this case, we can interpret I as the dipole signal and I^*R, where I^* is the conjugate of I, as the associated beamforming coefficient. Scaling all the beamforming coefficients by the same amount does not change the SNR; therefore using I^* as the beamforming coefficient yields output with the same SNR. Thus, I^* is the (non-unique) MRC beamforming coefficient for a dipole producing the output current I.

Optimal beamforming over a non-zero bandwidth requires frequency-dependent coefficients, where the coefficients for each frequency are determined using the same criteria. The inverse Fourier transform of these coefficients yields the impulse response of a filter which applies the correct coefficient to all frequencies simultaneously. If the polarization response of each stand (pair of orthogonal dipoles) varies from stand to stand, then the polarizations must be converted to a common pair of orthogonal polarizations (possibly, but not necessarily, left- and right-circular) before the coefficients (or filters) are calculated and applied, so that there is no loss due to imperfect correlation between similarly-oriented dipoles.

3.2 Simple Beamforming

There are many technical challenges that must be overcome to implement optimal beamforming. As mentioned in Section 1, it is desirable to avoid per-stand polarization corrections so as to simplify the design. In fact, it is desirable to avoid stand-specific processing for anything other than implementing the propagation delays required for "delay-and-sum" beamforming. In this approach, we simply delay the signal from each dipole according to the appropriate geometrical (propagation) delay, and sum the results. Any variations in response between stands is ignored. Further, we wish to generate the two polarizations which are output from the beamformer by simply combining all similarly-oriented dipoles, regardless of any variations in polarization response from stand to stand. Henceforth, we refer to this approach as "simple" beamforming. Simple beamforming is optimal only if the stand responses are identical. In the presence of mutual coupling, simple beamforming cannot not be optimal, since the coupling-induced variations in patterns between dipoles are ignored (i.e., it is not MRC), as are variations in polarization response. However, if these variations are small, or if they large but "average out", then simple beamforming may approach the performance of optimal beamforming. We shall see in the next section that this is indeed the case, for the latter reason.

[4]Of course the noise is not spatially white for LWA, but the actual situation introduces complications which are not relevant to the purpose of this study.

4 Results

In this memo, we consider the response of the array for two scenarios: (1) A single plane wave incident from the zenith ($\theta = 0$), with a linearly-polarized electric field oriented in the x direction; and (2) A single plane wave incident from $\theta = \phi = 45^\circ$ (ϕ measured from the x axis toward the y axis), with a linearly-polarized electric field oriented parallel to the ground.

4.1 Effect of Mutual Coupling on Collecting Area Generated by Optimal Beamforming

Figure 2 shows the collecting area of the array in each case, assuming MRC beamforming. Since there is only one incident plane wave and we assume the noise is spatially white, this result can be determined by computing the collecting area for each dipole individually, and then summing the results (as discussed in Section 3.1). The power collected by each dipole is simply the absolute value of the RMS current at the antenna terminals (determined by NEC) squared, times the load impedance (100Ω). The collecting area can then be obtained by dividing by the incident flux density, which is known since the incident electric field is known. The result is tallied separately for the x- and y-oriented dipoles; thus this is optimal only for the raw polarizations treated individually, and is not guaranteed to be optimal for any particular pair of orthogonal polarizations. Also shown is the result for a single ("standalone") dipole multiplied by 256, which can be compared to the above result to assess the impact of mutual coupling. It is clear from Figure 2(a) and (b) that the impact is very small. Thus, mutual coupling has negligible effect on collecting area in these two scenarios, assuming the collecting area is obtained by optimal combining of the raw polarizations.

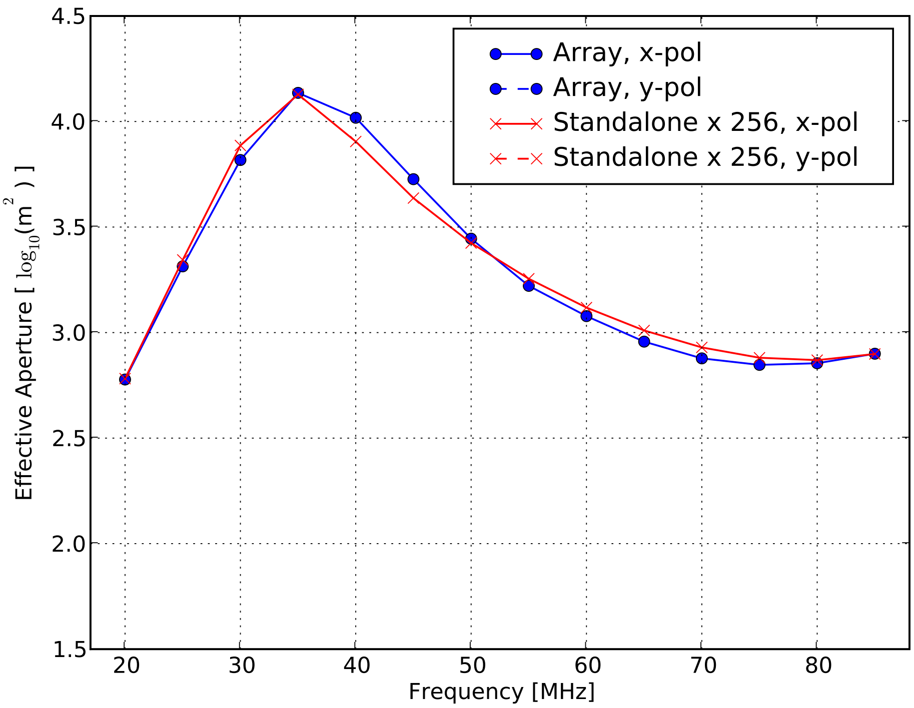
The above result is optimal for the raw polarizations individually, but not necessarily optimal for fully-polarimetric beamforming, since per-stand polarization corrections were not performed. However, it is clear that per-stand polarization correction would not have made a significant difference, since even when we ignore these corrections we get nearly the same result obtained in the "standalone $\times$ 256" case for which the stand polarization responses are identical. Thus, we shall henceforth assume that MRC beamforming for the raw polarizations individually is essentially optimal even if the complete polarimetric response is of interest, and even though per-stand polarization corrections are not performed.

The above results are a bit surprising, since previous studies (see Section 1) make it clear that the individual element responses vary dramatically in response to mutual coupling. In fact, that is also the case here, as demonstrated in Figure 3. In this figure, zero mutual coupling corresponds to all the markers being located exactly at $(1, 0)$, whereas significant and disorderly mutual coupling manifests as perturbation of the markers from that position. It is clear that the mutual coupling is in fact significant and disorderly for various combinations of frequencies and incident directions. However, the center of distribution always remains quite close to $(1, 0)$, which probably explains the close agreement in Figure 2. In other words, it appears that the mutual coupling-induced "errors", although large, tend to average out leaving only a very small bias.

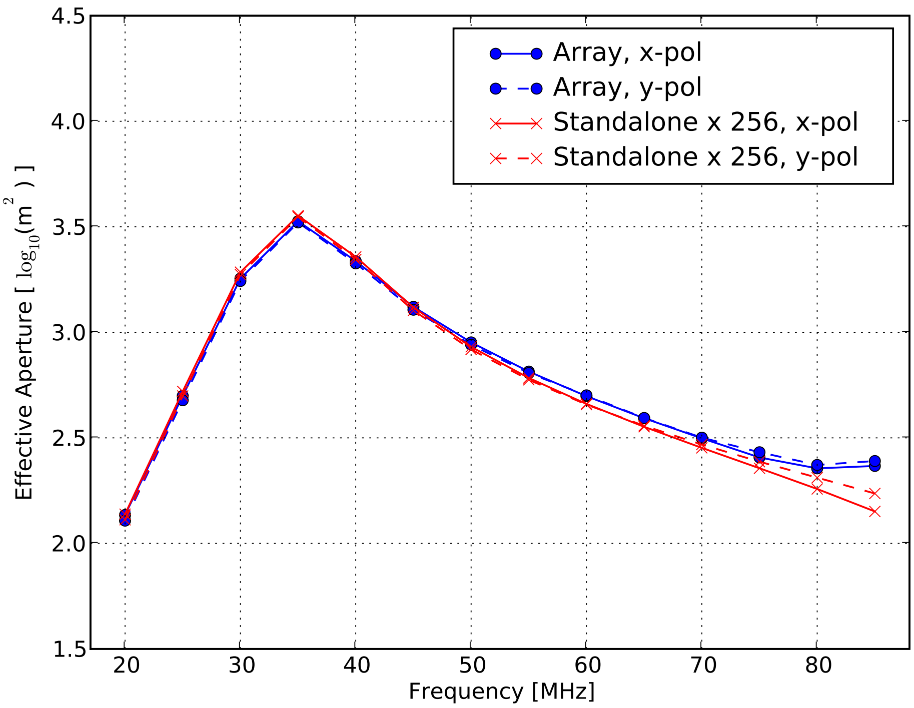
4.2 Performance of Simple Beamforming Compared to Optimal Beamforming

Next, we consider whether we can achieve results similar to those of Figure 2 using the "simple" beamforming scheme, as opposed to the optimal (MRC) beamforming scheme. A comparison is shown in Figure 4 for a zenith-pointing beam (another pointing is considered below). In this case, the results are expressed in terms of gain in dB relative to an arbitrary (but common) reference.[5] Note that the results are nearly identical – i.e., that optimal and simple beamforming yield very

[5] This is a convenience that allows the coefficients for simple beamforming to have magnitude equal to one. As explained in Section 3.1, the SNR is not affected by a constant scaling in coefficients.

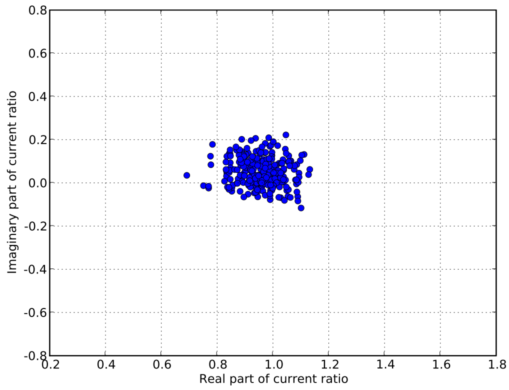


(a) $\theta = 0$. The cross- (y-) polarized result is out of range of this plot; see Figure 4(a).

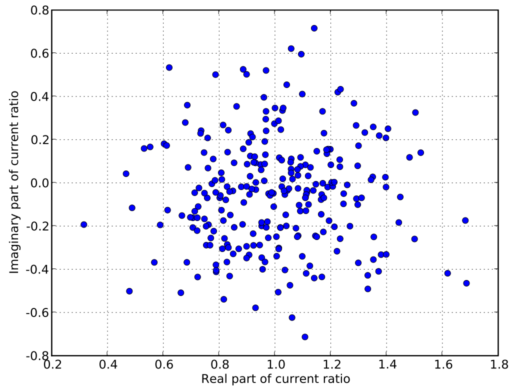


(b) $\theta = \phi = 45°$

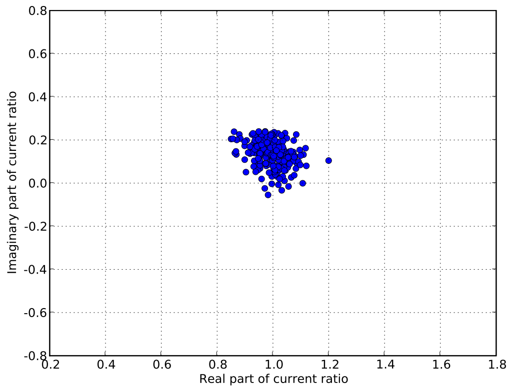
Figure 2: Collecting area assuming MRC beamforming (that is, beamforming which is optimal for the raw polarizations processed individually) in the indicated direction.



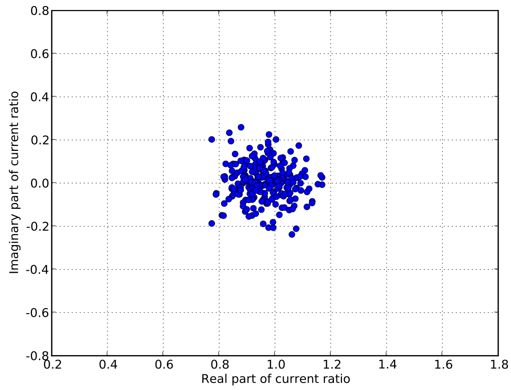
(a) $\theta = 0$, 75 MHz



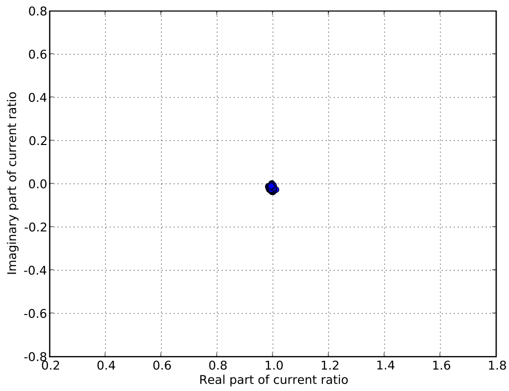
(a) $\theta = \phi = 45°$, 75 MHz



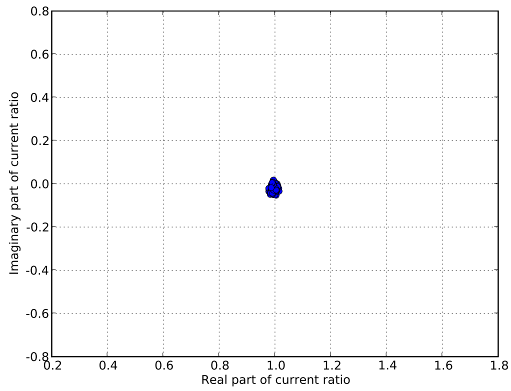
(c) $\theta = 0$, 35 MHz



(d) $\theta = \phi = 45°$, 35 MHz



(e) $\theta = 0$, 20 MHz



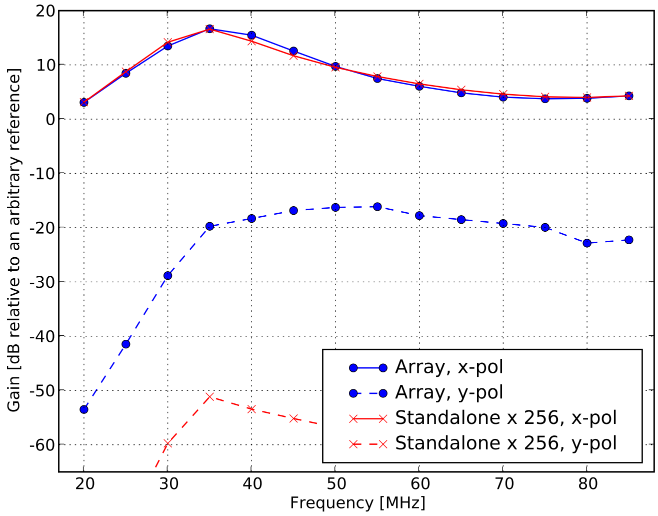
(f) $\theta = \phi = 45°$, 20 MHz

Figure 3: Terminal currents for the x-oriented dipoles. Each is divided by the corresponding current for an identical standalone dipole; thus the distance of the marker from $(1, 0)$ is an indication of the magnitude of the effect of mutual coupling.

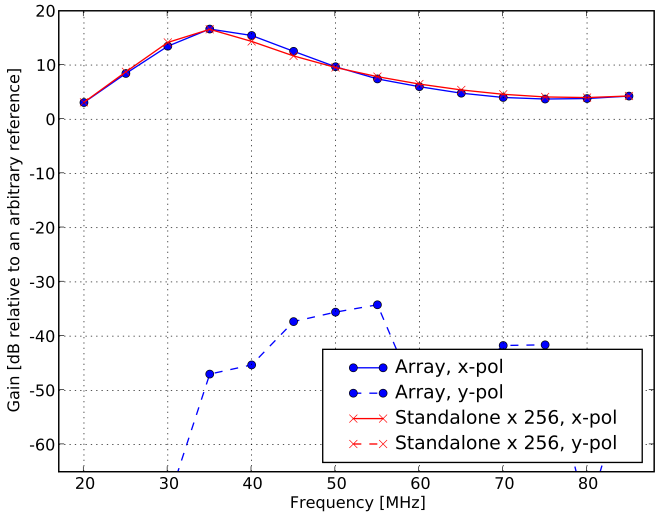
nearly the same gain – for the co-polarized dipoles. The reason for this appears to be the same as the reason why the collecting area for optimal beamforming is not significantly affected by mutual coupling; that is, because the differences in stand responses which are ignored by simple beamforming, although large, tend to average out leaving only a very small bias.

The cross- ($y-$) polarization results in Figure 4 are interesting. For optimal beamforming in the absence of mutual coupling, we see that the cross-pol is negligible; in fact, this level might not even be numerically significant. In the presence of mutual coupling, however, the cross-pol increases to a level of about -25 dB relative to the co-polarized gain above about 50 MHz. For simple beamforming, the observed cross-pol in the presence of mutual coupling is much less. A possible explanation for this is that since dipoles producing abnormally large outputs due to mutual coupling are weighted more heavily in optimal beamforming, so is the coupling-enhanced cross-pol component. After all, there is no constraint in the MRC processing that cross-pol should be low. It is possible that the result would be better (lower cross-pol for optimal beamforming) if per-stand polarization calibration was implemented, but this hardly seems necessary since we prefer simple beamforming for implementation reasons anyway.

Figure 5 compares the results for the zenith-pointing and $\theta = \phi = 45^\circ$ beams, zooming in on the x-polarization. Once again we see that simple beamforming delivers gain which is very close to that delivered by optimal (now, MRC on raw polarizations) beamforming. Combined with the finding from Section 4.1 that per-stand polarization corrections are not required to achieve nearly-optimum collecting area, this is pretty strong evidence that simple beamforming yields nearly-optimum gain and polarization characteristics, even in the presence of mutual coupling. However, the reader is once again referred to the caveats given at the end of Section 1.2.

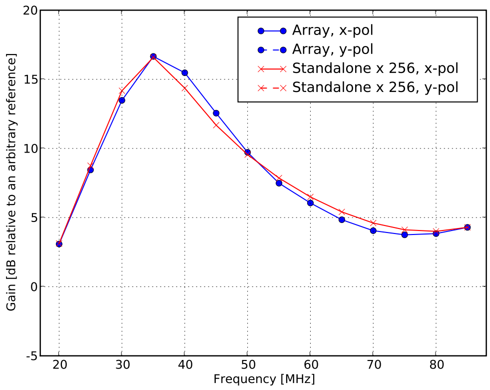


(a) Optimum Beamforming (MRC for raw polarizations individually)

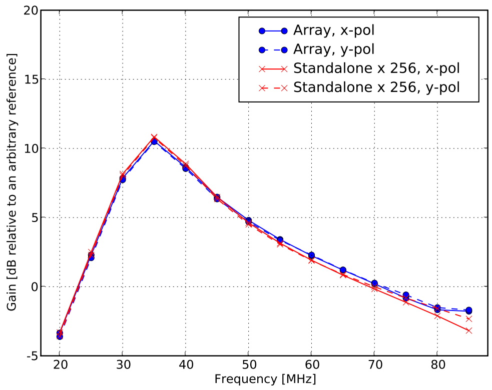


(b) Simple Beamforming

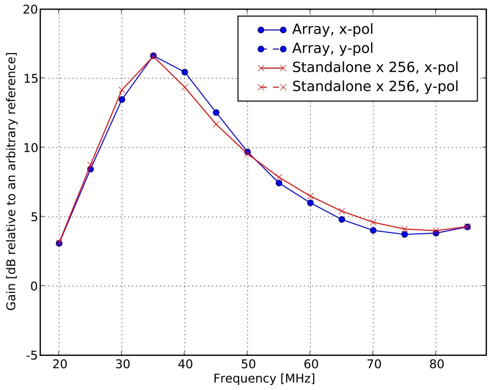
Figure 4: Main lobe gain for a zenith-pointing beam, using the indicated type of beamforming. See Figure 5(a) and (c) for a close up.



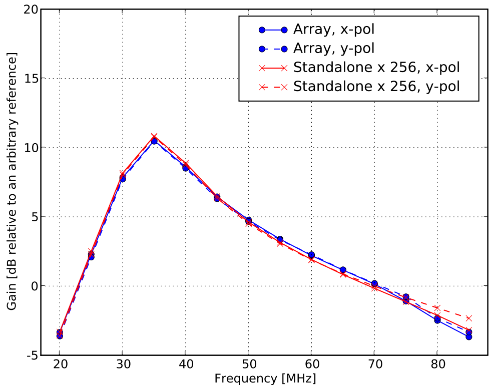
(a) Optimum Beamforming, $\theta = 0$



(b) Optimum Beamforming, $\theta = \phi = 45°$



(c) Simple Beamforming, $\theta = 0$



(d) Simple Beamforming, $\theta = \phi = 45°$

Figure 5: Main lobe gain using the indicated type of beamforming, in the indicated direction.

A Array Geometry

The following is a table of stand positions for the array considered in this memo, and shown in Figure 1. The format is x [m], y [m].

-0.6417	-54.6403
0.6188	-49.2886
-2.1446	-43.5898
1.6347	-39.4022
-3.1040	-31.5842
1.9494	-25.9196
0.1033	-16.6167
0.4292	-10.7486
-0.7357	-4.3966
1.8992	0.8836
-0.6337	5.6928
-1.2804	14.4591
-0.9083	19.8388
-0.2194	25.2222
1.7887	29.8014
2.7037	35.4848
-1.4572	42.3004
-0.9804	47.4205
-2.4760	53.5092
7.9084	-51.3329
-8.8030	-53.6532
7.0980	-46.3809
-6.8098	-49.0652
6.2519	-41.3365
-3.0192	-36.5852
4.1923	-32.8849
-8.1351	-32.2201
6.9503	-25.9969
-6.7614	-27.4078
6.0254	-21.0830
-3.7951	-21.8019
4.5376	-13.8831
-4.8877	-14.8882
5.5250	-8.3511
-5.7524	-9.9572
4.3222	-3.4923
-7.8482	-5.2768
7.0843	1.0107
-6.8086	2.8078
3.3628	11.2666
-4.6788	8.7518
6.9783	15.4447
-5.8368	16.5324
4.0877	19.5650
-7.7418	21.1581
4.9782	25.5392
-5.2158	25.4761
7.0019	30.1154
-4.6958	31.3428
7.5874	37.1024

-5.8409	38.5326
6.0371	41.8597
-6.0187	44.3590
5.3172	54.6967
-7.5405	53.5187
13.1956	-52.1099
-12.7579	-50.5875
12.5562	-46.6941
-9.7426	-45.0155
10.2586	-38.3441
-10.3420	-40.0476
9.7317	-30.1566
-14.7921	-30.5523
11.8669	-24.9943
-10.1415	-23.6875
13.0254	-18.2517
-10.4922	-18.6973
9.1112	-11.8388
-11.0708	-13.7278
10.4916	-7.0014
-11.4977	-8.6987
11.1030	-2.0367
-12.1072	1.2068
6.5977	6.8001
-9.6376	6.9316
9.3057	11.0118
-9.2165	12.8474
11.3752	17.8394
-11.1262	17.4745
10.7993	23.1056
-12.8458	24.8272
11.9966	30.4219
-10.9054	29.4353
13.3505	36.0565
-8.6583	34.3964
12.0110	44.3675
-10.9647	41.9145
13.7105	51.4273
-15.5738	52.3364
18.8995	-46.2463
-16.1639	-46.7230
14.5750	-42.1156
-17.6424	-40.9739
14.3883	-33.7063
-16.3174	-36.1499
16.8614	-27.8946
-17.3944	-26.1322
17.5465	-20.5576
-14.8130	-21.8392
16.7132	-14.8541
-15.1487	-16.8466
13.9734	-10.6705
-16.1572	-11.5497
15.7265	-5.2948

-14.8887	-5.0098
13.4371	2.6973
-16.2956	3.9429
13.3268	7.6994
-13.5683	10.0224
13.8047	13.4682
-16.7780	15.4280
15.7228	22.0413
-16.0742	20.9576
16.6775	28.5732
-16.4452	29.2588
18.0523	33.4310
-13.7916	33.5207
15.4753	40.5837
-15.9914	38.2944
17.4082	47.8631
-18.9400	44.3489
23.7463	-45.0081
-25.6522	-43.3098
18.9413	-36.9731
-22.3655	-36.4806
20.6662	-32.2786
-19.6534	-32.2793
23.5470	-25.2924
-21.5648	-21.1130
22.4967	-19.8239
-19.8186	-14.9562
21.8329	-14.8647
-22.6828	-10.7151
22.0465	-7.9241
-21.9991	-5.2612
19.9318	-2.5818
-19.6855	0.2593
19.7278	3.6484
-22.3891	6.7453
19.4843	10.0060
-20.8643	11.5074
18.5294	15.1572
-20.9375	18.6428
20.9257	25.2759
-21.5189	26.3358
22.6954	29.9554
-21.7602	31.5771
22.4679	35.8366
-20.8029	36.6109
24.6508	46.2127
-25.0443	47.7928
29.0218	-38.3663
-28.8472	-37.3640
26.8890	-33.1356
-27.0893	-30.4854
28.4519	-28.3851
-22.7735	-26.4275
28.1037	-23.0527

-26.4012	-22.9848
27.1227	-16.6874
-24.3916	-16.9852
25.5481	-11.5071
-26.5409	-7.4385
24.9300	-2.7158
-28.0636	-2.6715
25.4969	3.7717
-25.5579	2.7538
24.2924	8.6263
-26.0664	12.3274
25.1775	14.4677
-26.5543	17.3048
29.1182	22.2415
-25.2262	22.9725
26.9131	26.7292
-26.5026	28.2808
29.1861	31.1853
-27.7291	33.7202
29.3327	37.9612
-30.1810	40.3269
35.0420	-39.5341
-32.8441	-41.7774
34.2805	-32.0664
-33.8331	-32.3352
32.6059	-25.6007
-31.3849	-25.4098
32.0173	-17.7290
-29.3839	-17.6411
30.7056	-11.3911
-31.4897	-11.5853
29.1609	-5.3829
-34.9724	-6.9353
30.9098	-0.6914
-30.5879	1.6933
29.7352	6.4426
-29.1909	8.3535
30.8769	11.3155
-33.5187	12.9787
31.9913	16.1965
-32.5407	17.9274
33.0469	27.0970
-30.6914	25.5317
33.3625	34.9988
-32.5725	32.4500
33.9143	39.9699
-35.2642	39.3136
-36.6268	-36.4847
37.2981	-27.3307
-39.6483	-29.7389
35.8184	-20.9876
-35.5218	-22.5997
37.1341	-14.4744
-35.0808	-17.6007

36.1501	-8.3643
-37.2341	-12.5975
38.5491	-2.7244
-34.0486	-2.0158
36.5450	3.1454
-35.7834	3.6391
37.0096	9.9968
-36.8390	8.6736
38.6967	15.9076
-38.9928	14.0988
36.7558	20.5161
-35.6935	22.2359
37.3787	30.0275
-35.9543	28.3535
43.1045	-27.5871
-40.5514	-22.3927
40.9522	-22.7011
-41.5217	-17.3201
41.9859	-15.6864
-44.6673	-12.1776
40.8872	-10.1588
-40.8047	-8.0394
43.9657	-2.1493
-39.6399	0.1358
42.4244	3.4557
-41.9223	4.5853
41.8888	11.1213
-43.2501	10.8016
43.6038	16.9351
-40.6100	20.0608
41.5201	23.6954
-39.8577	25.0047
46.7343	-19.8940
-47.7129	-16.7461
47.3268	-12.4447
-49.1936	-7.1431
47.0654	-7.4021
-48.5481	-2.1663
49.3201	-0.3418
-48.2585	2.8298
48.5971	5.2913
-47.4207	13.5602
48.8035	11.1792
-47.1802	18.5544

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