Parametric Model for the LWA-1 Dipole Response as a Function of Frequency

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Contents

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1 Introduction

This memo is an extension of [1] which provides an empirical model of the LWA-1 normalized dipole power pattern at 38 and 74 MHz. Here the nature of the power pattern witll be addressed over entire tuning range of the LWA, from 10 to 88 MHz. This is done by determining fitting the normalized dipole power pattern for a variety of frequencies within the tuning range and then fitting the derived parameters with a polynomial in frequency. This allows for the calculation of the power pattern for a given azimuth and zenith angle at any frequency without generating the moment data for each frequency of interest.

2 Methods

The methodology used closely follows that used in [1]. The ground is taken to be the x–y plane with the positive z-axis pointing toward zenith. The two cross-dipoles that compromise a LWA-1 stand lie in the x–z and y–z planes and are the E–W and N–S polarizations, respectively. For each dipole, the E-plane lies in the same plane as the dipole and the H-plane lies perpendicular to it and contains the positive z-axis.

The parametric model used to fit the normalized dipole power pattern, p, in any plane containing the z-axis is:

$$p(\theta) = \left[1 - \left(\frac{\theta}{\pi/2}\right)^{\alpha}\right] \cos^{\beta}\theta + \gamma \left(\frac{\theta}{\pi/2}\right) \cos^{\delta}\theta, \quad (1)$$

where θ is the angle measured from the positive z-axis, i.e., the zenith angle, in radians and ranges between 0 and $\pi/2$. This model was fit to both the E and H-planes for both polarizations for 21 frequencies between 10 and 88 MHz using a downhill simplex method. The power patterns were initially fit allowing all four parameters to vary for both the E and H-planes. However, this lead to some parameters not varying smoothly with frequency. Fixing the γ and δ values for the H-plane fits to zero alleviated this problem.

3 Results

The results of the final fits are shown in Tables 1 and 2 for the E–W and N–S polarization, respectively. For 38 and 74 MHz the best-fit values can be compared with the values found in [1]. Figures 1 and 2 show this comparison for the E–W polarization at 38 and 74 MHz, respectively. There is good

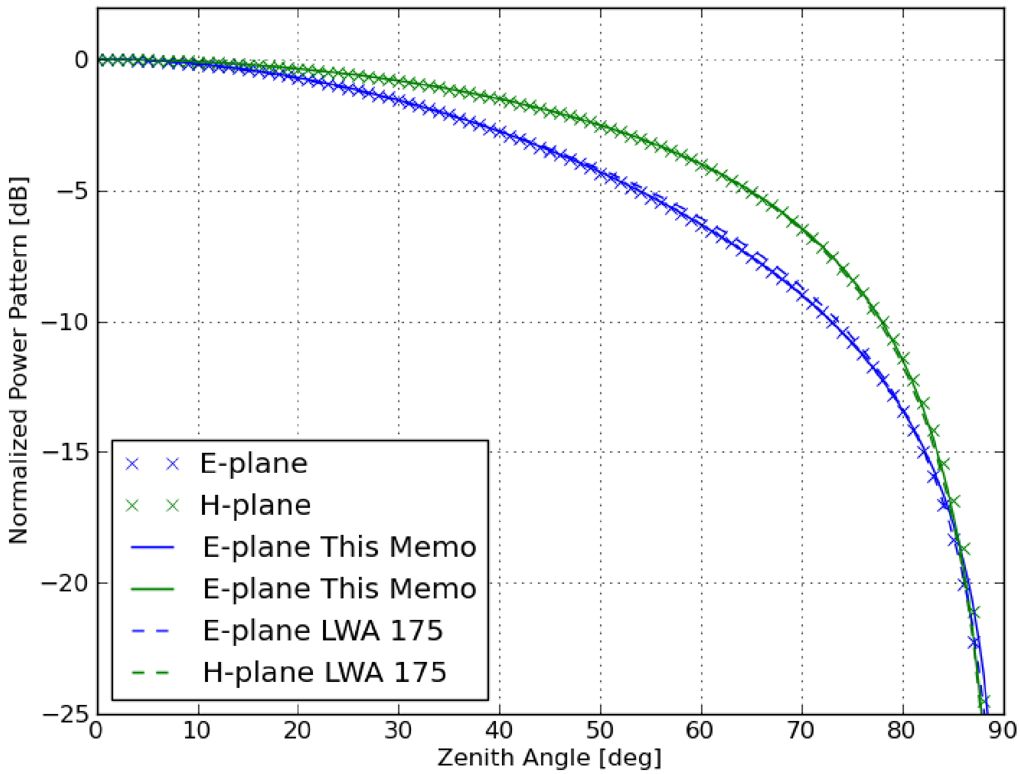

Figure 1: Comparison between the moment method data from [2] (crosses), the fits presented in this memo (solid lines), and the fits presented in [1] at 38 MHz.

agreement between the moment method data from [2] and the fits derived in this memo.

To create the coefficients as a function of frequency, each of the eight parameters were fit using a polynomial of form:

$$c(\nu) = \sum_{n=0}^{m} a_n^c \left(\frac{\nu}{10 \text{ MHz}} \right)^n, \tag{2}$$

where a_n^c is the n^{th} order coefficient of the best-fit polynomial to parameter c. The lowest-order polynomial which appeared to accurately fit the data was found to be $11th$ order. The resulting coefficients of the fits as a function of frequency for the E–W polarization are listed in Tables 3 and 4 for the E and H-planes, respectively. Similarly, Tables 5 and 6 contain the coefficients for the N–S polarization data. The fits along with the data are shown in

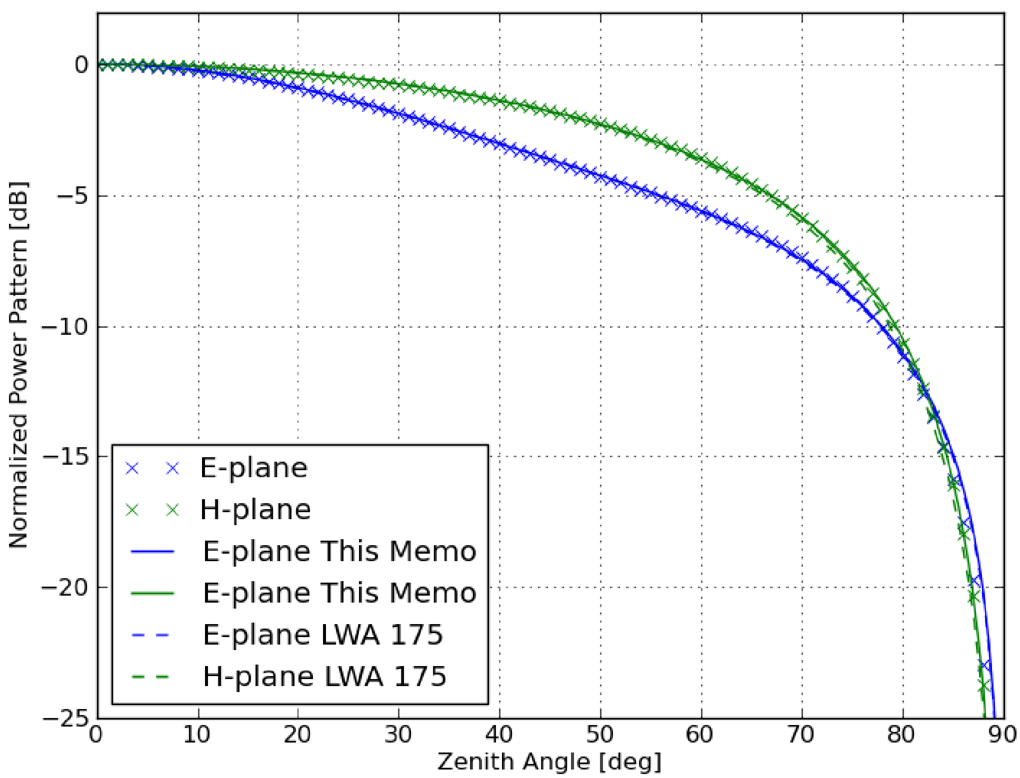

Figure 2: Similar to Figure 1 but for 74 MHz. The labeling is the same as in Figure 2.

Figures 3 (E–W) and 4 (N–S).

4 Application

The fits presented above are available in version 0.4.0 and later of the LWA Software Library[1]. To access the coefficients in Python, use:

```
>>> import os
>>> import numpy
>>> from lsl.common.paths import data as dataPath
>>> dataDict = numpy.load(os.path.join(dataPath, 'lwa1-dipole-emp.npz')
>>> fitX = dataDict['fitX']
>>> def planeE(theta, freq=10e6):
...   alpha = numpy.polyval(fitX[0,0,:], numpy.array(freq))
...   beta  = numpy.polyval(fitX[0,1,:], numpy.array(freq))
...   gamma = numpy.polyval(fitX[0,2,:], numpy.array(freq))
...   delta = numpy.polyval(fitX[0,3,:], numpy.array(freq))
...   out = (1-(2*theta/numpy.pi)**alpha)*numpy.cos(theta)**beta
...   out += gamma*(2*theta/numpy.pi)*numpy.cos(theta)**delta
...   return out
```

It should be noted that the coefficient values for the N–S polarization stored in the `lwa-dipole-emp.npz` are swapped between the E and H-planes so that same azimuth interpolation code can be used (see [1] for a recommended interpolation scheme).

References

[1] S. Ellingson, LWA Memo 175, December 2010.

[2] S. Ellingson *et al.*, "The Long Wavelength Array," Proc. IEEE, Vol. 97, No. 8, pp. 1421-1430, Aug 2009. Also available as LWA Memo 157.

[1] http://fornax.phys.unm.edu/lwa/trac/

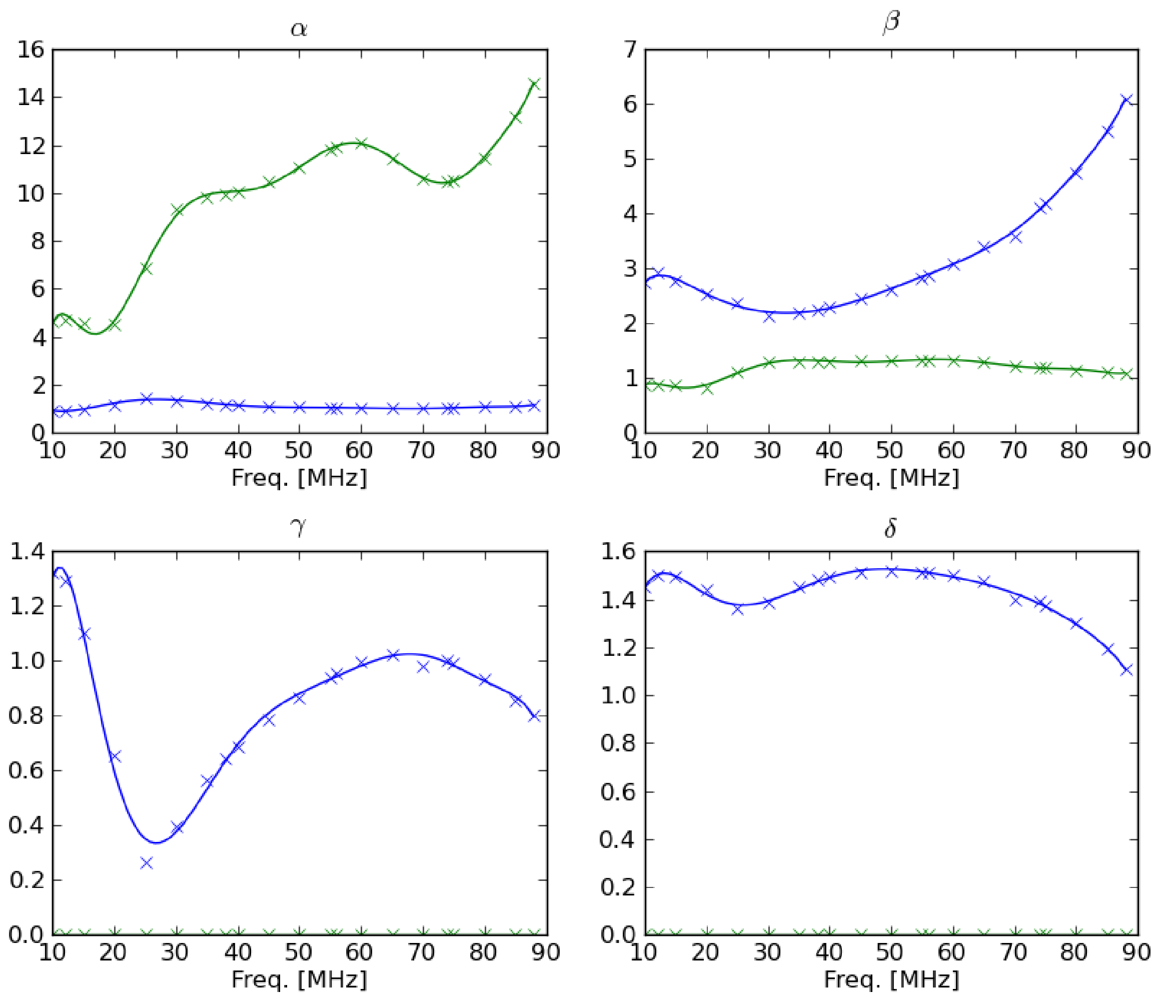

Figure 3: Best-fit polynomials (solid lines) to the four model parameters (crosses) for the E–W polarization for the E-plane (blue) and H-plane (green). In general, the polynomials are in good agreement with the data. The exception is for the E-plane γ parameter near 25 MHz.

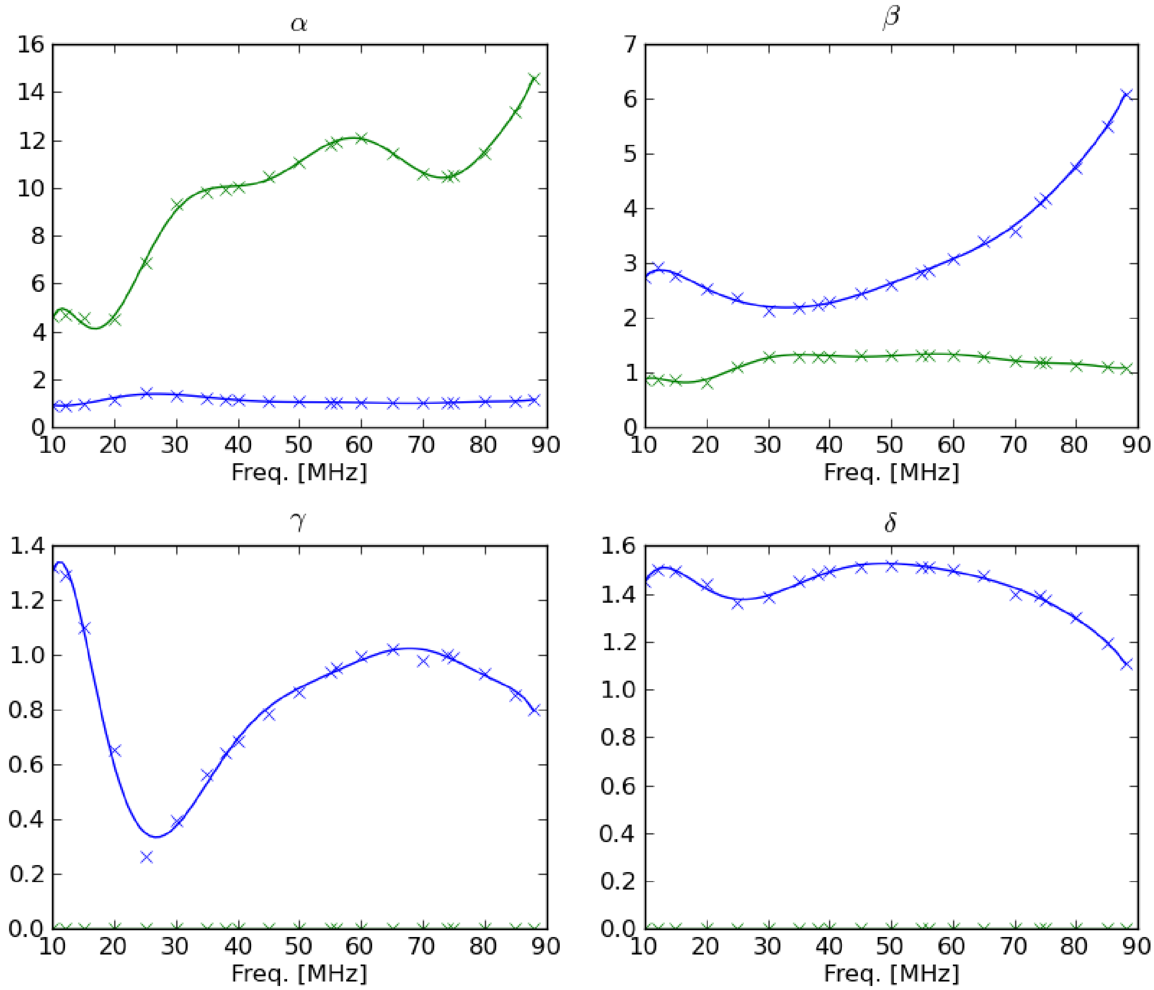

Figure 4: Similar to Figure 3, but for the N–S polarization. The labeling is the same as in Figure 3. Again, there is good agreement between the polynomial and the the data except around 25 MHz for the E-plane parameter γ.

Frequency	*Plane*	α	β	γ	δ
10 MHz	E	0.91	2.73	1.32	1.45
	H	4.67	0.88	0.00	0.00
20 MHz	E	1.18	2.53	0.65	1.44
	H	4.51	0.83	0.00	0.00
30 MHz	E	1.34	2.14	0.40	1.39
	H	9.35	1.29	0.00	0.00
38 MHz	E	1.18	2.25	0.64	1.48
	H	9.94	1.30	0.00	0.00
40 MHz	E	1.15	2.30	0.69	1.49
	H	10.06	1.30	0.00	0.00
50 MHz	E	1.07	2.61	0.87	1.52
	H	11.09	1.32	0.00	0.00
60 MHz	E	1.02	3.09	0.99	1.50
	H	12.13	1.32	0.00	0.00
70 MHz	E	1.02	3.59	0.98	1.40
	H	10.60	1.23	0.00	0.00
74 MHz	E	1.03	4.10	1.00	1.39
	H	10.48	1.19	0.00	0.00
80 MHz	E	1.06	4.75	0.94	1.30
	H	11.43	1.15	0.00	0.00
88 MHz	E	1.16	6.09	0.80	1.11
	H	14.60	1.09	0.00	0.00

Table 1: Parameters for the E–W Polarization

Frequency	*Plane*	α	β	γ	δ
10 MHz	E	0.90	2.80	1.35	1.46
	H	4.69	0.88	0.00	0.00
20 MHz	E	1.17	2.51	0.66	1.45
	H	4.60	0.85	0.00	0.00
30 MHz	E	1.31	2.14	0.43	1.40
	H	9.77	1.31	0.00	0.00
38 MHz	E	1.16	2.27	0.68	1.49
	H	10.58	1.33	0.00	0.00
40 MHz	E	1.14	2.32	0.72	1.50
	H	10.77	1.34	0.00	0.00
50 MHz	E	1.06	2.63	0.89	1.52
	H	12.42	1.36	0.00	0.00
60 MHz	E	1.02	3.09	1.00	1.50
	H	14.53	1.39	0.00	0.00
70 MHz	E	1.01	3.73	1.03	1.44
	H	13.13	1.32	0.00	0.00
74 MHz	E	1.03	4.05	1.01	1.40
	H	13.15	1.29	0.00	0.00
80 MHz	E	1.06	4.67	0.95	1.32
	H	14.68	1.26	0.00	0.00
88 MHz	E	1.14	5.92	0.83	1.14
	H	19.11	1.21	0.00	0.00

Table 2: Parameters for the N–S Polarization

n	a_n^α	a_n^β	a_n^γ	a_n^δ
0	-29.1171	-37.4913	28.1047	6.1240
1	111.7311	139.6610	-104.2538	-18.9478
2	-174.5775	-206.6546	170.0110	31.0437
3	151.1112	173.6406	-152.4836	-27.1886
4	-80.8344	-92.5111	83.6969	14.3324
5	28.3137	32.8787	-29.8477	-4.8506
6	-6.6878	-7.9743	7.1374	1.0900
7	1.0738	1.3243	-1.1554	-0.1643
8	-0.1156	-0.1481	0.1250	0.0164
9	0.0080	0.0107	-0.0087	-0.0010
10	-0.0003	-0.0004	0.0003	0.0000
11	0.0000	0.0000	-0.0000	-0.0000

Table 3: Parameters for the E–W Polynomial Fit, E-Plane

n	a_n^α	a_n^β	a_n^γ	a_n^δ
0	-24.4342	-15.0726	0.0000	0.0000
1	40.9295	51.7252	0.0000	0.0000
2	49.1505	-68.2541	0.0000	0.0000
3	-142.6709	48.3015	0.0000	0.0000
4	128.2364	-20.4400	0.0000	0.0000
5	-61.6535	5.4472	0.0000	0.0000
6	18.0709	-0.9260	0.0000	0.0000
7	-3.3874	0.0974	0.0000	0.0000
8	0.4088	-0.0056	0.0000	0.0000
9	-0.0308	0.0001	0.0000	0.0000
10	0.0013	0.0000	0.0000	0.0000
11	-0.0000	-0.0000	0.0000	0.0000

Table 4: Parameters for the E–W Polynomial Fit, H-Plane

n	a_n^α	a_n^β	a_n^γ	a_n^δ
0	-35.8559	-15.8955	0.0000	0.0000
1	83.6072	54.9819	0.0000	0.0000
2	-18.8464	-73.6632	0.0000	0.0000
3	-81.1572	53.2765	0.0000	0.0000
4	92.8221	-23.2555	0.0000	0.0000
5	-47.9284	6.4846	0.0000	0.0000
6	14.4091	-1.1822	0.0000	0.0000
7	-2.7154	0.1402	0.0000	0.0000
8	0.3258	-0.0104	0.0000	0.0000
9	-0.0242	0.0004	0.0000	0.0000
10	0.0010	-0.0000	0.0000	0.0000
11	-0.0000	-0.0000	0.0000	0.0000

Table 5: Parameters for the N–S Polynomial Fit, E-Plane

n	a_n^α	a_n^β	a_n^γ	a_n^δ
0	-32.0398	15.2639	44.6701	15.1992
1	121.7969	-39.9753	-160.9161	-49.8539
2	-189.3479	52.0476	252.1002	75.5694
3	163.3863	-35.8860	-219.4249	-63.2826
4	-87.2855	14.2674	118.0633	32.7558
5	30.5719	-3.3143	-41.5807	-11.1081
6	-7.2269	0.4032	9.8711	2.5416
7	1.1619	-0.0072	-1.5923	-0.3955
8	-0.1253	-0.0050	0.1722	0.0413
9	0.0087	0.0007	-0.0120	-0.0028
10	-0.0003	-0.0000	0.0005	0.0001
11	0.0000	0.0000	-0.0000	-0.0000

Table 6: Parameters for the N–S Polynomial Fit, H-Plane