Parametric Model for the LWA-1 Dipole Response as a Function of Frequency

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December 20, 2011

LWA Memo #178
Version 2

Contents

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1 Introduction

This memo is an extension of [1] which provides an empirical model of the LWA-1 normalized dipole power pattern at 38 and 74 MHz. Here the nature of the power pattern will be addressed over the entire tuning range of the LWA, from 10 to 88 MHz. This is done by fitting the normalized dipole power pattern for a variety of frequencies within the tuning range and then fitting the derived parameters with a polynomial in frequency. This allows for the calculation of the power pattern for a given azimuth and zenith angle at any frequency without generating the moment data for each frequency of interest.

2 Methods

The methodology used closely follows that used in [1]. The ground is taken to be the x–y plane with the positive z-axis pointing toward zenith. The two cross-dipoles that comprise a LWA-1 stand lie in the x–z and y–z planes and are the E–W and N–S polarizations, respectively. For each dipole, the E-plane lies in the same plane as the dipole and the H-plane lies perpendicular to it and contains the positive z-axis.

The parametric model used to fit the normalized dipole power pattern, p, in any plane containing the z-axis is:

$$p(\theta) = \left[1 - \left(\frac{\theta}{\pi/2}\right)^{\alpha}\right] \cos^{\beta}\theta + \gamma\left(\frac{\theta}{\pi/2}\right)\cos^{\delta}\theta, \tag{1}$$

where θ is the zenith angle measured from the positive z-axis in radians and ranges between 0 and $\pi/2$. This model was fit to both the E and H-planes for both polarizations for 43 frequencies between 10 and 88 MHz using a least squares method. The power patterns were initially fit allowing all four parameters to vary for both the E and H-planes. However, this lead to large uncertainties in the fits which resulted in some parameters not varying smoothly with frequency. At the higher frequencies this was primarily caused by the best-fit values of the H-plane γ and δ parameters having large uncertainties. Since the uncertainties are roughly centered on zero, the γ and δ parameters were fixed at this value. This resolved the problem of the large uncertainties and yielded better fits to the moment method data.

3 Results

The results of the final fits are shown in Tables 1 and 2 for the E–W and N–S polarizations, respectively. For 38 and 74 MHz the best-fit values can be compared with the values found in [1]. Figures 1 and 2 show this comparison for the E–W polarization at 38 and 74 MHz, respectively. There is also good agreement between the moment method data from [2] and the fits derived in this memo[1].

To create the coefficients as a function of frequency, each of the eight parameters were fit using a polynomial of form:

$$c(\nu) = \sum_{n=0}^{m} a_n^c \left(\frac{\nu}{10\ \mathrm{MHz}} \right)^n, \tag{2}$$

where a_n^c is the n^{th} order coefficient of the best-fit polynomial to parameter c. The lowest-order polynomial which appeared to accurately fit the data was found to be $12th$ order. The resulting coefficients of the fits as a function of frequency for the E–W polarization are listed in Tables 3 and 4 for the E and H-planes, respectively. Similarly, Tables 5 and 6 contain the coefficients for the N–S polarization data. The fits along with the data are shown in Figures 3 (E–W) and 4 (N–S).

4 Application

The fits presented above are available in version 0.4.0 and later of the LWA Software Library[2]. To access the coefficients in Python, use:

```
>>> import os
>>> import numpy
>>> from lsl.common.paths import data as dataPath
>>> dataDict = numpy.load(os.path.join(dataPath, 'lwa1-dipole-emp.npz')
>>> fitX = dataDict['fitX']
>>> def planeE(theta, freq=10e6):
...    alpha = numpy.polyval(fitX[0,0,:], numpy.array(freq))
...    beta  = numpy.polyval(fitX[0,1,:], numpy.array(freq))
...    gamma = numpy.polyval(fitX[0,2,:], numpy.array(freq))
...    delta = numpy.polyval(fitX[0,3,:], numpy.array(freq))
```

[1] All of the NEC files and scripts needed to reproduce the values found in this memo can be found at http://fornax.phys.unm.edu/lwa/subversion/trunk/DipoleResponse/

[2] http://fornax.phys.unm.edu/lwa/trac/

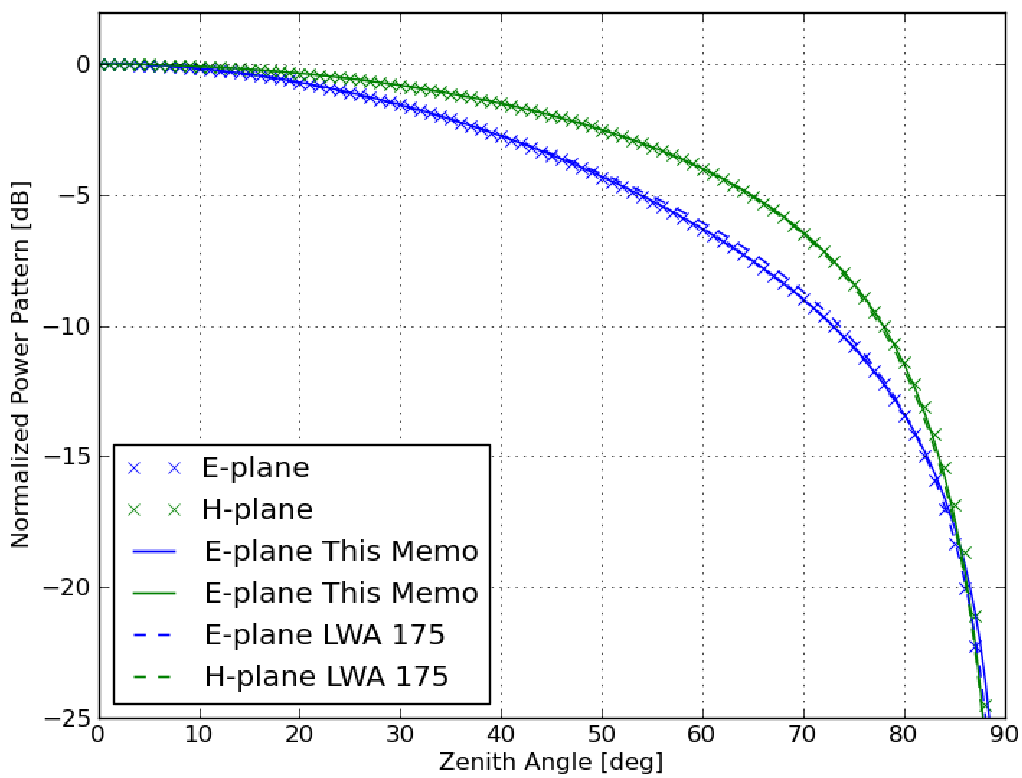

Figure 1: Comparison between the moment method data from [2] (crosses), the fits presented in this memo (solid lines), and the fits presented in [1] at 38 MHz.

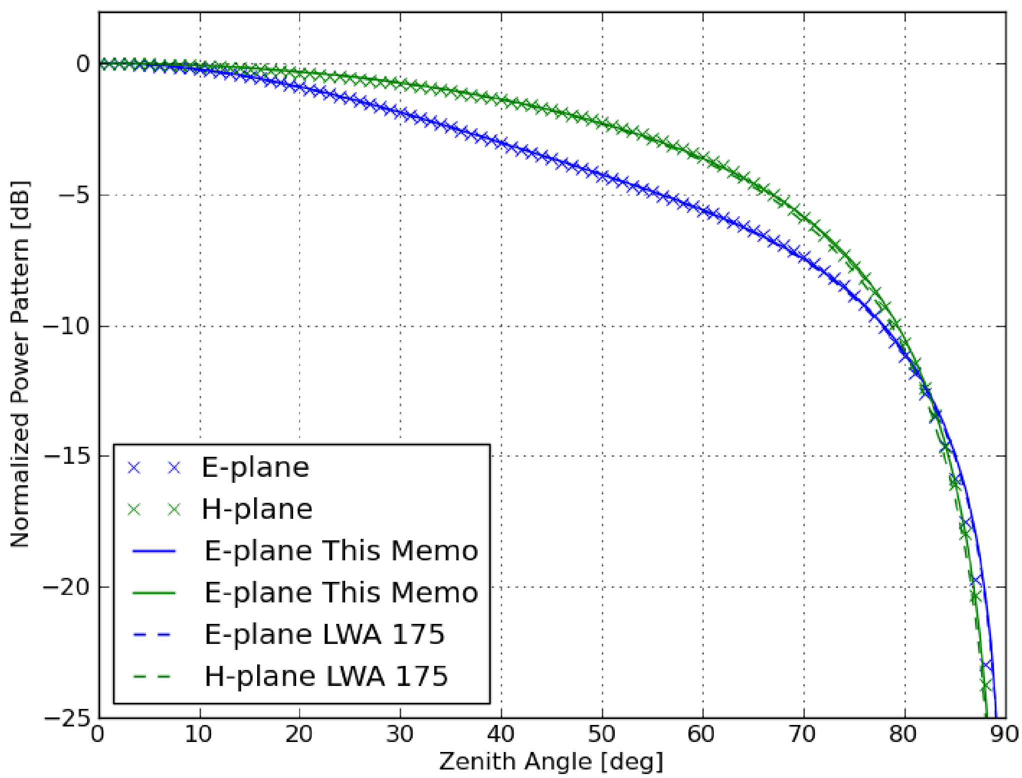

Figure 2: Similar to Figure 1 but for 74 MHz. The labeling is the same as in Figure 1.

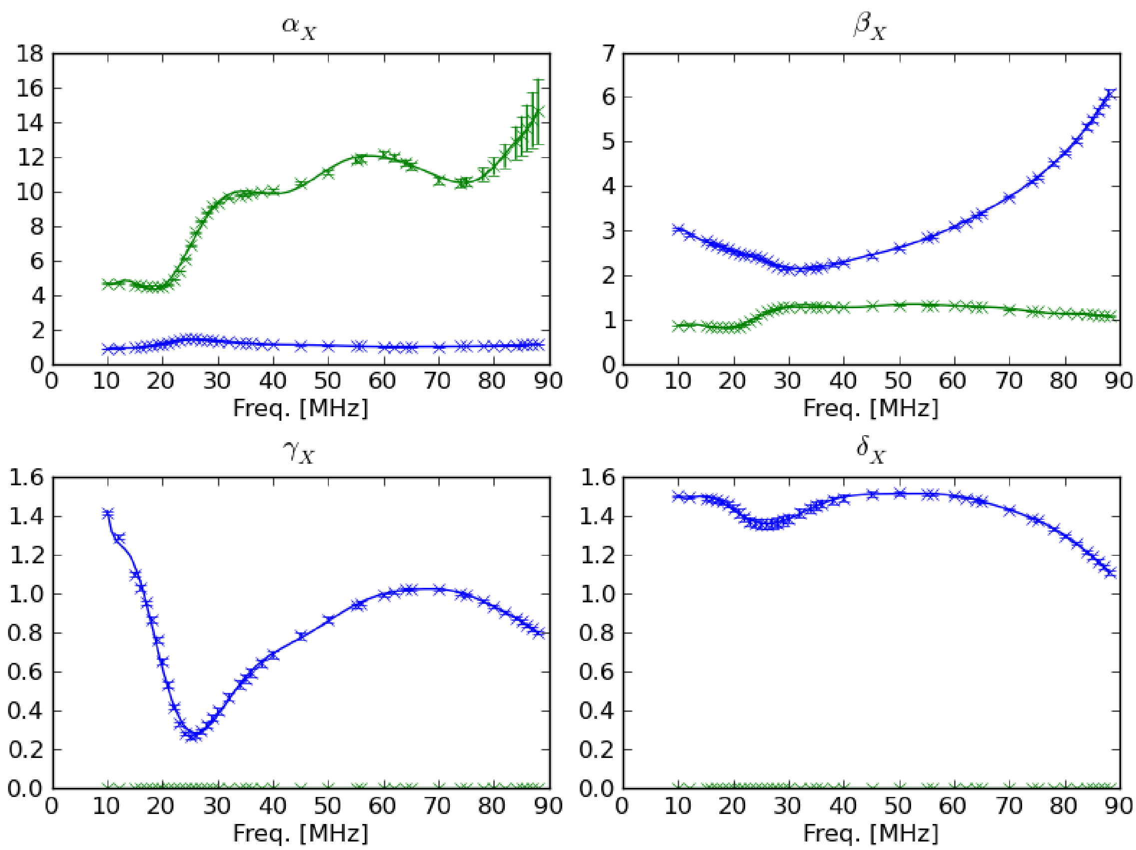

Figure 3: Best-fit polynomials (solid lines) to the four model parameters (crosses) for the E–W polarization for the E-plane (blue) and H-plane (green). The error bars represent the 1σ parameter uncertainties from the least squares fits. In general, the polynomials are in good agreement with the data.

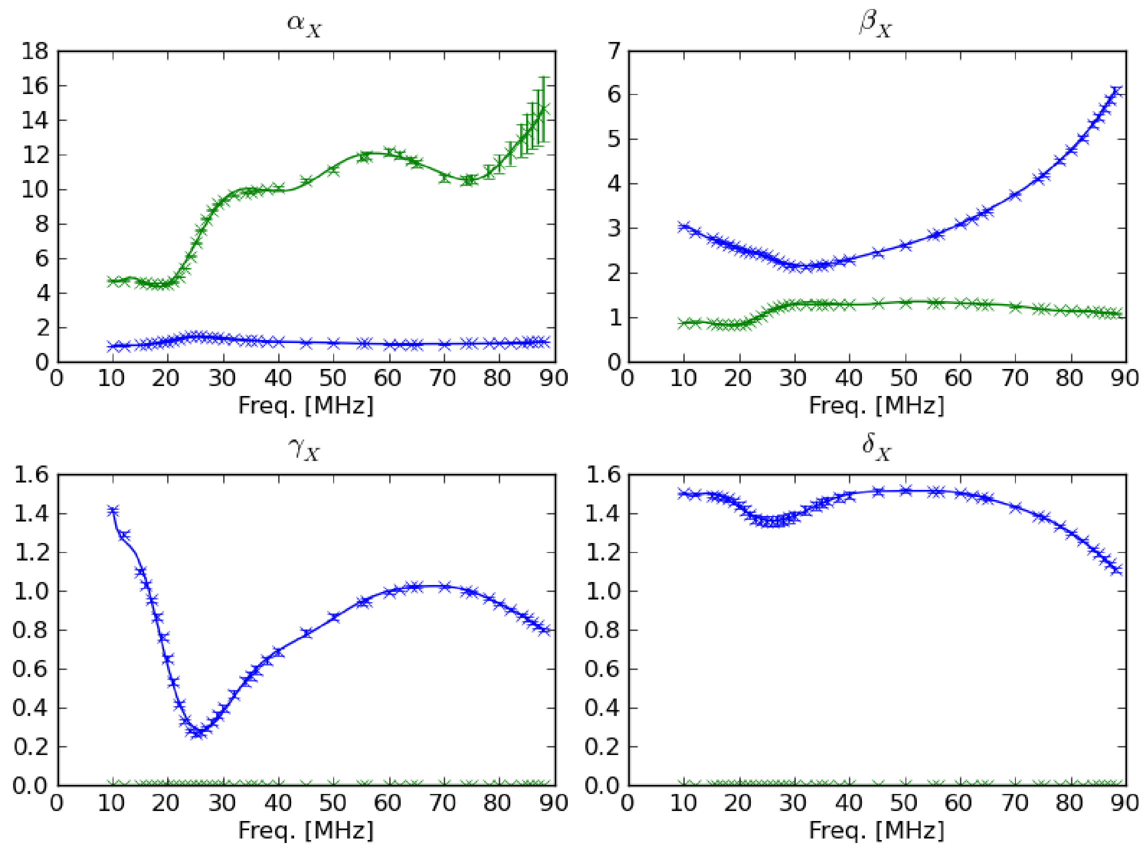

Figure 4: Similar to Figure 3, but for the N–S polarization. The labeling is the same as in Figure 3. Again, there is good agreement between the polynomial and the the data.

```
...    out = (1-(2*theta/numpy.pi)**alpha)*numpy.cos(theta)**beta
...    out += gamma*(2*theta/numpy.pi)*numpy.cos(theta)**delta
...    return out
```

It should be noted that the coefficient values for the N–S polarization stored in the `lwa-dipole-emp.npz` are swapped between the E and H-planes so that the same azimuth interpolation code can be used (see [1] for a recommended interpolation scheme).

Figures 5 and 6 apply the interpolation scheme suggested in [1] to both the parametric models presented here and those presented in [1]. In general, the interpolation scheme provides a good approximation to the azimuthal variations in the moment method data. The greatest discrepancies occur at the higher zenith angles (lower elevation angles) where the dipole response is lower.

References

[1] S. Ellingson, LWA Memo 175, December 2010.

[2] S. Ellingson *et al.*, "The Long Wavelength Array," Proc. IEEE, Vol. 97, No. 8, pp. 1421-1430, Aug 2009. Also available as LWA Memo 157.

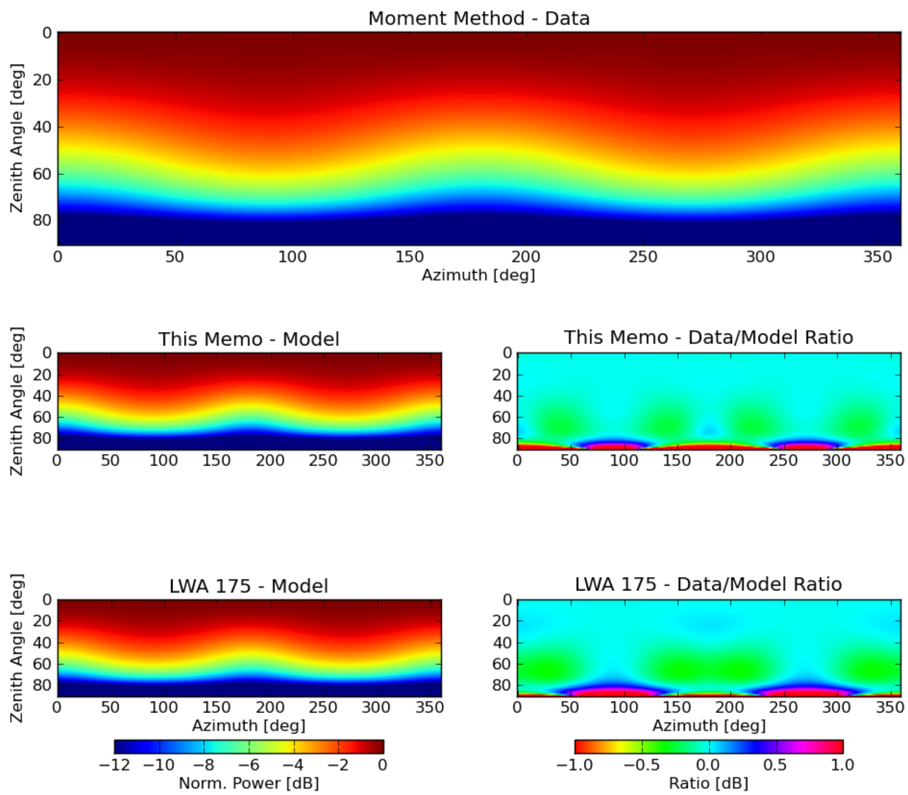

Figure 5: Comparison between the moment method data from [2] (upper panel), the azimuth interpolated fits presented in this memo (middle panel series), and the interpolated fits presented in [1] at 38 MHz (lower panel series). The interpolation scheme suggested in [1] provides a good representation of the azimuthal dependence in the data.

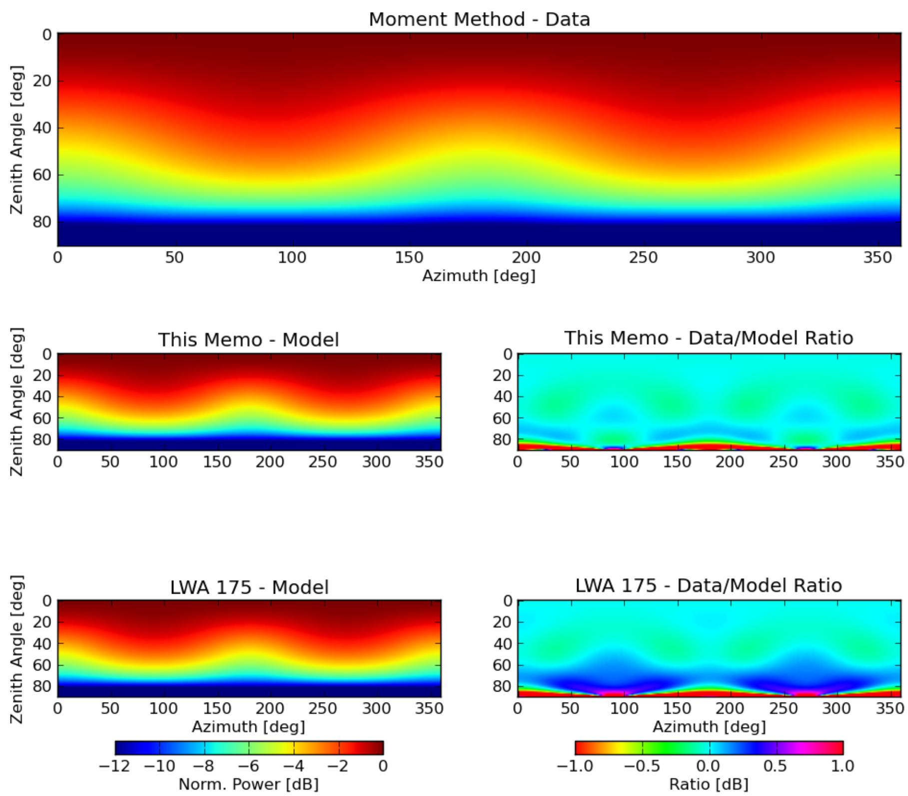

Figure 6: Similar to Figure 5 but for 74 MHz. The layout and labeling are the same as in Figure 5.

Frequency	*Plane*	α	β	γ	δ
10 MHz	E	0.89	3.05	1.42	1.50
	H	4.67	0.88	0.00	0.00
20 MHz	E	1.18	2.53	0.65	1.44
	H	4.51	0.83	0.00	0.00
30 MHz	E	1.34	2.14	0.40	1.39
	H	9.35	1.29	0.00	0.00
38 MHz	E	1.18	2.25	0.64	1.48
	H	9.94	1.30	0.00	0.00
40 MHz	E	1.15	2.30	0.69	1.49
	H	10.06	1.30	0.00	0.00
50 MHz	E	1.07	2.62	0.87	1.52
	H	11.09	1.32	0.00	0.00
60 MHz	E	1.02	3.09	0.99	1.50
	H	12.13	1.32	0.00	0.00
70 MHz	E	1.02	3.76	1.02	1.43
	H	10.60	1.23	0.00	0.00
74 MHz	E	1.03	4.10	1.00	1.39
	H	10.48	1.19	0.00	0.00
80 MHz	E	1.06	4.75	0.94	1.30
	H	11.43	1.15	0.00	0.00
88 MHz	E	1.16	6.09	0.80	1.11
	H	14.60	1.09	0.00	0.00

Table 1: Parameters for the E–W Polarization

Frequency	*Plane*	α	β	γ	δ
10 MHz	E	0.89	3.05	1.42	1.50
	H	4.69	0.88	0.00	0.00
20 MHz	E	1.17	2.51	0.66	1.45
	H	4.60	0.85	0.00	0.00
30 MHz	E	1.31	2.14	0.43	1.40
	H	9.77	1.31	0.00	0.00
38 MHz	E	1.16	2.27	0.68	1.49
	H	10.58	1.33	0.00	0.00
40 MHz	E	1.14	2.32	0.72	1.50
	H	10.77	1.34	0.00	0.00
50 MHz	E	1.06	2.63	0.89	1.52
	H	12.42	1.36	0.00	0.00
60 MHz	E	1.02	3.09	1.00	1.50
	H	14.53	1.39	0.00	0.00
70 MHz	E	1.01	3.73	1.03	1.44
	H	13.13	1.32	0.00	0.00
74 MHz	E	1.03	4.06	1.01	1.40
	H	13.15	1.29	0.00	0.00
80 MHz	E	1.06	4.67	0.95	1.32
	H	14.68	1.26	0.00	0.00
88 MHz	E	1.14	5.92	0.83	1.14
	H	19.11	1.21	0.00	0.00

Table 2: Parameters for the N–S Polarization

n	a_n^α	a_n^β	a_n^γ	a_n^δ
0	$-4.52993117 \times 10^{1}$	$-3.06669173 \times 10^{1}$	7.11119215×10^{1}	1.13133864×10^{1}
1	1.72359627×10^{2}	1.37253656×10^{2}	$-2.66450447 \times 10^{2}$	$-3.49394214 \times 10^{1}$
2	$-2.72231145 \times 10^{2}$	$-2.36812145 \times 10^{2}$	4.34395314×10^{2}	5.15975841×10^{1}
3	2.40840205×10^{2}	2.30204067×10^{2}	$-3.99317541 \times 10^{2}$	$-4.15253296 \times 10^{1}$
4	$-1.33458904 \times 10^{2}$	$-1.41481412 \times 10^{2}$	2.31201599×10^{2}	2.01675350×10^{1}
5	4.91727832×10^{1}	5.82026204×10^{1}	$-8.95219193 \times 10^{1}$	-6.17004450
6	$-1.24468312 \times 10^{1}$	$-1.65245801 \times 10^{1}$	2.39640798×10^{1}	1.18515334
7	2.19501789	3.28135779	-4.50046389	$-1.31123436 \times 10^{-1}$
8	$-2.69033938 \times 10^{-1}$	$-4.54784465 \times 10^{-1}$	$5.91991295 \times 10^{-1}$	$4.85003070 \times 10^{-3}$
9	$2.24360464 \times 10^{-2}$	$4.31144732 \times 10^{-2}$	$-5.34525601 \times 10^{-2}$	$7.05653618 \times 10^{-4}$
10	$-1.21164337 \times 10^{-3}$	$-2.66518597 \times 10^{-3}$	$3.15745524 \times 10^{-3}$	$-1.10253297 \times 10^{-4}$
11	$3.81017942 \times 10^{-5}$	$9.68125738 \times 10^{-5}$	$-1.09924322 \times 10^{-4}$	$6.25032736 \times 10^{-6}$
12	$-5.27717636 \times 10^{-7}$	$-1.56774660 \times 10^{-6}$	$1.71052917 \times 10^{-6}$	$-1.35050619 \times 10^{-7}$

Table 3: Parameters for the E–W Polynomial Fit, E-Plane

n	a_n^α	a_n^β	a_n^γ	a_n^δ
0	4.06292036×10^{2}	3.03871307×10^{1}	0.00000000	0.00000000
1	$-1.70684537 \times 10^{3}$	$-1.33721722 \times 10^{2}$	0.00000000	0.00000000
2	3.09543805×10^{3}	2.56701705×10^{2}	0.00000000	0.00000000
3	$-3.16451420 \times 10^{3}$	$-2.75753883 \times 10^{2}$	0.00000000	0.00000000
4	2.03500849×10^{3}	1.85099885×10^{2}	0.00000000	0.00000000
5	$-8.71389346 \times 10^{2}$	$-8.22785548 \times 10^{1}$	0.00000000	0.00000000
6	2.56447752×10^{2}	2.50247905×10^{1}	0.00000000	0.00000000
7	$-5.26225167 \times 10^{1}$	-5.28731620	0.00000000	0.00000000
8	7.51951210	$7.75461094 \times 10^{-1}$	0.00000000	0.00000000
9	$-7.33774690 \times 10^{-1}$	$-7.74460465 \times 10^{-2}$	0.00000000	0.00000000
10	$4.66341431 \times 10^{-2}$	$5.02425427 \times 10^{-3}$	0.00000000	0.00000000
11	$-1.74000550 \times 10^{-3}$	$-1.90898917 \times 10^{-4}$	0.00000000	0.00000000
12	$2.89211689 \times 10^{-5}$	$3.22398551 \times 10^{-6}$	0.00000000	0.00000000

Table 4: Parameters for the E–W Polynomial Fit, H-Plane

n	a_n^α	a_n^β	a_n^γ	a_n^δ
0	-4.67361577 × 10^{1}	-2.57168497 × 10^{1}	7.34504958 × 10^{1}	1.24757325 × 10^{1}
1	1.78607828 × 10^{2}	1.19265801 × 10^{2}	-2.75675572 × 10^{2}	-3.90944288 × 10^{1}
2	-2.83829202 × 10^{2}	-2.09443099 × 10^{2}	4.49916361 × 10^{2}	5.78116652 × 10^{1}
3	2.52963340 × 10^{2}	2.07255881 × 10^{2}	-4.14005911 × 10^{2}	-4.66413118 × 10^{1}
4	-1.41420083 × 10^{2}	-1.29712536 × 10^{2}	2.39939691 × 10^{2}	2.27329249 × 10^{1}
5	5.26474299 × 10^{1}	5.43283141 × 10^{1}	-9.29836961 × 10^{1}	-6.98369609
6	-1.34868710 × 10^{1}	-1.56942887 × 10^{1}	2.49071652 × 10^{1}	1.34698313
7	2.41161359	3.16832358	-4.67973532	-1.49509056 × 10^{-1}
8	-3.00382150 × 10^{-1}	-4.46022164 × 10^{-1}	6.15741005 × 10^{-1}	5.50337619 × 10^{-3}
9	2.55284141 × 10^{-2}	4.29105053 × 10^{-2}	-5.56027389 × 10^{-2}	8.21322904 × 10^{-4}
10	-1.40995103 × 10^{-3}	-2.68962173 × 10^{-3}	3.28431557 × 10^{-3}	-1.28207933 × 10^{-4}
11	4.55549817 × 10^{-5}	9.89863473 × 10^{-5}	-1.14320770 × 10^{-4}	7.28724548 × 10^{-6}
12	-6.52303480 × 10^{-7}	-1.62285564 × 10^{-6}	1.77843040 × 10^{-6}	-1.57929065 × 10^{-7}

Table 5: Parameters for the N–S Polynomial Fit, E-Plane

n	a_n^α	a_n^β	a_n^γ	a_n^δ
0	3.74442220 × 10^{2}	2.66334909 × 10^{1}	0.00000000	0.00000000
1	-1.58363329 × 10^{3}	-1.18733375 × 10^{2}	0.00000000	0.00000000
2	2.89034779 × 10^{3}	2.31127516 × 10^{2}	0.00000000	0.00000000
3	-2.96927546 × 10^{3}	-2.51050876 × 10^{2}	0.00000000	0.00000000
4	1.91602338 × 10^{3}	1.70002572 × 10^{2}	0.00000000	0.00000000
5	-8.22272816 × 10^{2}	-7.60917207 × 10^{1}	0.00000000	0.00000000
6	2.42332563 × 10^{2}	2.32705259 × 10^{1}	0.00000000	0.00000000
7	-4.97732692 × 10^{1}	-4.93847248	0.00000000	0.00000000
8	7.11814584	7.26924548 × 10^{-1}	0.00000000	0.00000000
9	-6.95245544 × 10^{-1}	-7.28171075 × 10^{-2}	0.00000000	0.00000000
10	4.42376945 × 10^{-2}	4.73590854 × 10^{-3}	0.00000000	0.00000000
11	-1.65311917 × 10^{-3}	-1.80330479 × 10^{-4}	0.00000000	0.00000000
12	2.75295077 × 10^{-5}	3.05113715 × 10^{-6}	0.00000000	0.00000000

Table 6: Parameters for the N–S Polynomial Fit, H-Plane

A Document History

- Version 2:
  - Fixed various typos.
  - Reworked fits with a least square routine to obtain fit uncertainty estimates.
  - Increased the precision in the various tables.
  - Added text and comparisons on azimuth interpolation.
- Version 1 (February 11, 2011):
  - Initial version