Powers of Ten, Angles, Units, Mechanics

Chapters 1, 4

How does astronomy work?

In astronomy, we make observations and measurements:

Angles
Motions
Morphologies
Brightnesses
Spectra
Etc.

We interpret and explain in terms of physics:

Mechanics
Atomic and molecular processes
Radiation properties
Thermodynamic properties
Etc.

Robust theories in turn motivate the next observations, thus our understanding is continually being refined.

Powers-of-ten notation

- Astronomy deals with very big and very small numbers – we talk about galaxies AND atoms.
- Example: distance to the center of the Milky Way can be inefficiently written as about 25,000,000,000,000,000,000 meters.
- Instead, use powers-of-ten, or exponential notation. All the zeros are consolidated into one term consisting of 10 followed by an exponent, written as a superscript. Thus, the above distance is 2.5 x 10^{19} meters.

Examples of powers-of-ten notation (Ch. 1.6):

One hundred	= 100 = 10^2	
One thousand	= 1000 = 10^3	kilo
One million	= 1,000,000 = 10^6	mega
One billion	= 1,000,000,000 = 10^9	giga
One one-hundredth	= 0.01 = 10^{-2}	centi
One one-thousandth	= 0.001 = 10^{-3}	milli
One one-millionth	= 0.000001= 10^{-6}	micro
One one-billionth	= 0.000000001= 10^{-9}	nano

Examples of power-of-ten notation

$150 = 1.5 \times 10^{2}$
$84,500,000 = 8.45 \times 10^{7}$

$0.032 = 3.2 \times 10^{-2}$
$0.0000045 = 4.5 \times 10^{-6}$

The exponent (power of ten) is just the number of places past the decimal point of the first non-zero digit.

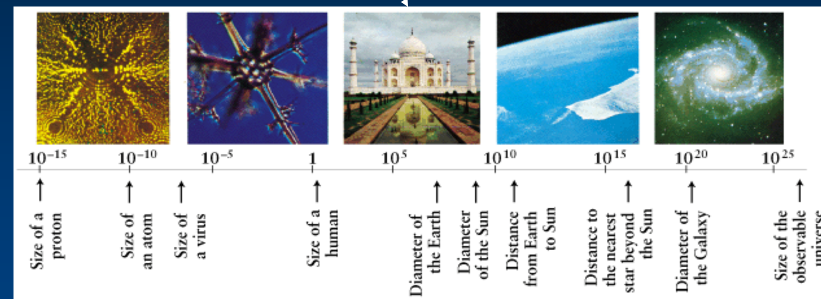
We can conveniently write the size of anything on this chart! (Sizes given in meters):

An atom

Cell is about 10^{-4}m

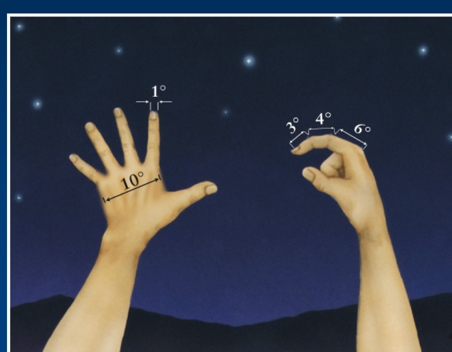
Taj Mahal is about 60 meters high

Earth diameter is about 10^{7} m

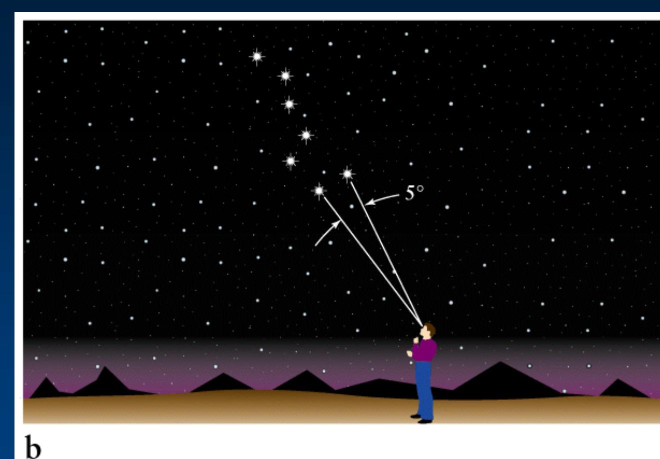


Angles

- We must determine positions of objects on the sky (even if we don't know their distances) to describe:
  - The apparent size of a celestial object
  - The separation between objects
  - The movement of an object across the sky
- You can estimate angles, e.g. the width of your finger at arm's length subtends about 1 degree



Example of angular distance: the "pointer stars" in the big dipper



The Moon and Sun subtend about one-half a degree

How do we express smaller angles?

One circle has 2π radians = 360°

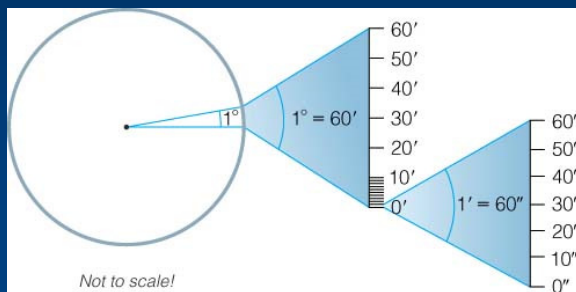
One degree has 60 arcminutes (a.k.a. minutes of arc):

1° = 60 arcmin = 60'

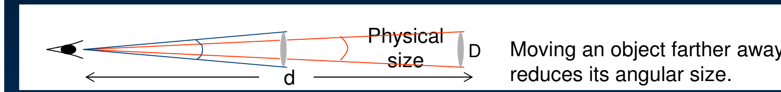
One arcminute has 60 arcseconds (a.k.a. seconds of arc):

1' = 60 arcsec = 60"

One arcsecond has 1000 milli-arcseconds (yes, we need these!)



Angular size - linear size - distance



The angular size depends on the linear (true) size AND on the distance to the object. See Box 1-1.

Use the *small-angle formula*:

$$D = \frac{\alpha d}{206{,}265}$$

where D = linear size of an object (any unit of length),
d = distance to the object (*same* unit as D)
α = angular size of the object (in arcsec, useful in astronomy),

206,265 is the number of arcseconds in a circle divided by 2π (i.e. it is the number of arcseconds in a radian).

Where does this formula come from?

Examples

1. The Moon is at a distance of about 384,000 km, and subtends about 0.5°. From the small-angle formula, its diameter is about 3400 km.

2. M87 (a big galaxy) has angular size of 7', corresponding to diameter 40,000 pc (1 pc = about 300 trillion km) at its large distance. What is its distance?

3. The resolution of your eye is about 1'. What length can you resolve at a distance of 10 m?

Important note on Significant Figures

If you are given numbers in a problem with a certain degree of precision, your answer should have the same degree of precision. e.g. if you travel 1.2 m in 1.1 sec, what is your speed? 1.2/1.1 = 1.1, even though calculator says 1.090909090909090909…, the input numbers were only given to 2 sig fig, so the answer is too.

Units in astronomy

Every physical quantity has units associated with it (don't ever leave them off!).

Astronomers use the metric system (SI units) and powers-of-ten notation, plus a few "special" units.

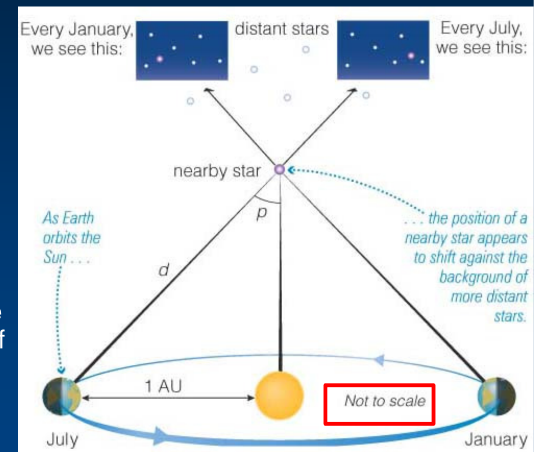
Example: Average distance from Earth to Sun is about 1.5×10^{11} m = 1 Astronomical Unit = 1 AU

Used for distances in the Solar system.

This spring we are working on much larger scales. A common unit is the light-year (distance light travels in one year: 9.5×10^{15} m), but astronomers even more commonly use the "parsec"…

The parsec unit

- Basic unit of distance in astronomy. Comes from technique of trigonometric or "Earth-orbit" parallax
- Short for "parallax of one second of arc"
- Note parallax is half the angular shift of the star over 6 months
- 1 pc = the distance between Earth and a star with a parallax of 1", alternatively the distance at which the radius of the Earth's orbit around the Sun (1AU) subtends an angle of 1". 1 pc = 3.09×10^{16} m = 3.26 light years = 206,265 AU.



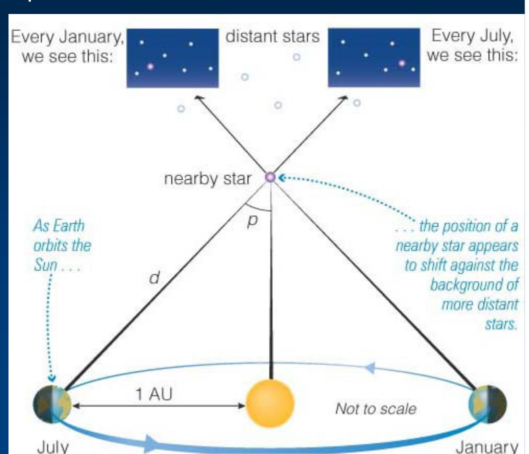
So how does trigonometric parallax relate to distance?

$$d\,(pc) = \frac{1}{p('')}$$ where p is the parallax and d is the distance.

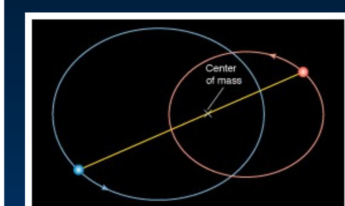
The nearest star to Sun is 1.3 pc away.

Galaxies are up to 100 kpc across.

The most distant galaxies are 1000's of Mpc away.

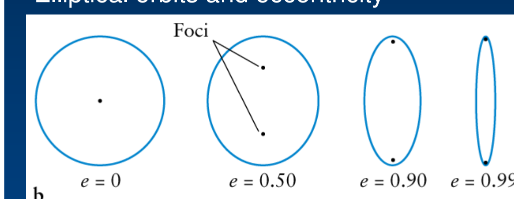


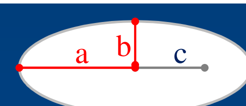
Important results from Mechanics



Two objects orbit in ellipses with the center of mass as a common focus. Ratio of distances to center of mass is always Inverse of mass ratio.

Elliptical orbits and eccentricity





$$e = \frac{c}{a}$$

$$b = a\sqrt{1-e^2}$$

Newton's Law of Gravity

$$F = G\frac{m_1 m_2}{r^2}$$

centripetal acceleration
(circular motion)

$$a = \frac{V^2}{r}$$

Newton's form of
Kepler's 3rd law

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3$$

(*a* is mean separation of the objects over an orbit)

periastron and apastron: $D_{peri} = a(1 - e)$, $D_{ap} = a(1 + e)$

Circular speed for small mass, m, orbiting large mass, M

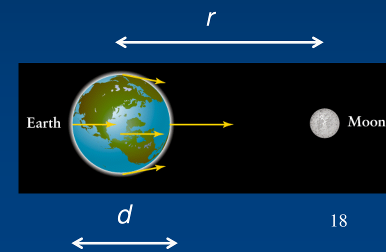
$$V_{circ} = \sqrt{\frac{GM}{r}}$$

Escape speed

$$V_{escape} = \sqrt{\frac{2GM}{r}}$$

Tidal force

$$\Delta F = 2GMm\frac{d}{r^3}$$



Circular motion in general

Understand the basic relationships between period, frequency, angular frequency and velocity. We will see these often.

Kinetic Energy and Gravitational Potential Energy

For a mass, m, moving at speed v, the KE is ½ mv^2.

For a mass m in the gravitational field of a mass M at a distance R from its center, gravitational potential energy is given by -GMm/R. Defined to be zero at R=∞. Importantly, as m falls from higher R to lower R, the gravitation PE drops and the KE increases. The sum is conserved.

We will see several examples of this conversion.