Black Holes - Chapter 21

The most massive stellar cores

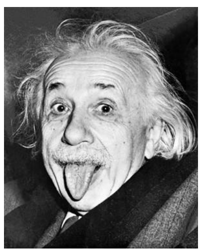
- If the core is massive enough (~3 $M_\odot$; total initial mass of star > 25 $M_\odot$ or so), even neutron degeneracy pressure can be overwhelmed by gravity. A catastrophic collapse is inevitable => black hole.
- Gravity so strong around black hole that Newton's laws no longer work. Must turn to General Relativity.
- (Fate of collapsed matter, we don't know of any pressure that can stop collapse:

  Volume $\rightarrow 0$

  Density $\rightarrow \infty$ A "singularity". We don't have the physics for this!)

Relativity

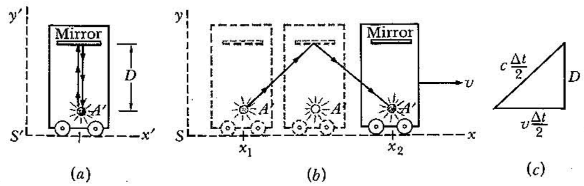
<u>Special Relativity</u>: how space and time measurements differ for observers moving at different (but constant) <u>speeds</u>. Effects only noticeable if speeds are significant fraction of c.



<u>General Relativity</u>: how space and time measurements differ depending on <u>acceleration</u>, which Einstein showed is equivalent to <u>gravity</u>. Matter distorts space and time.

Special theory of relativity

- In Newtonian physics, space and time are absolute (how they appear to us in everyday life).
- Einstein showed with this theory that this is not true: space and time measurement depends on your "frame of reference", i.e. how fast you are moving.
- Based on two principles:
  - The speed of light is the same for all frames of reference.
  - The laws of nature are the same for all frames of reference.
- First principle also explained results of Michelson-Morley experiment (1887): speed of light same both parallel and perpendicular to Earth's motion. If true, leads to strange consequences…



Light pulse leaves A', bounces off mirror, returns.
In "primed" frame (a), moving with cart, this takes time Δt' = 2D/c.

In stationary ("lab") frame (b), light travels extra distance, but at same speed. Time interval Δt must be longer.

$$\left(\frac{c\,\Delta t}{2}\right)^2 = D^2 + \left(\frac{v\,\Delta t}{2}\right)^2$$

or

$$\Delta t = \frac{2D}{\sqrt{c^2 - v^2}} = \frac{2D}{c}\,\frac{1}{\sqrt{1 - v^2/c^2}}$$

Using $\Delta t' = 2D/c$, we have

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \gamma\,\Delta t' \qquad \text{1-7}$$

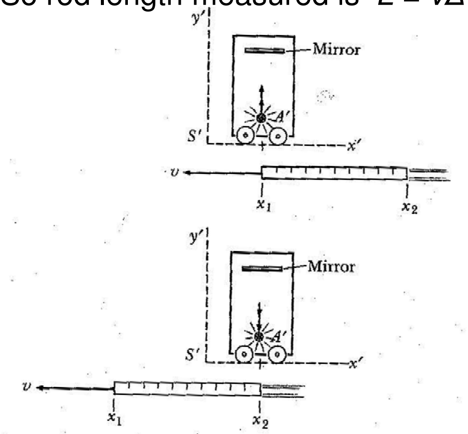
"Time dilation"

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geqslant 1 \qquad \text{1-8}$$

γ also called Lorentz factor

Likewise, lengths are contracted in direction of motion when measured from a moving frame. Consider rod stationary in unprimed (lab) frame. In primed frame, rod passes at speed *v* as light travels from A' back to A' in time *Δt'*. So rod length measured is *L' = vΔt'*.



In unprimed frame, cart travels a distance *vΔt* as it passes rod, so rod length *L = vΔt*.

$$L' = v\Delta t' = \frac{v\Delta t}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}\,L = \frac{L}{\gamma}$$

"Length contraction"

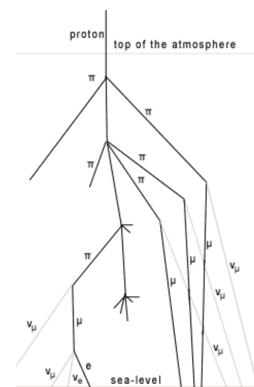
If everything is relative, which reference frame measures longer time, shorter distance?

First, "events" are things that happen at a certain spatial coordinate and time. In our example, the launching of the light pulse is an event, its reception is another.

"Proper time": time interval between events in frame where they both occur at same place. This is shorter time. This was the primed frame in our experiment. Time is dilated in any other frame.

"Proper length": length between two spatial coordinates measured in frame where they are at rest. This is longer length. This was the unprimed frame in our experiment. Length is contracted in any other frame.

Example: muon decay in Earth's atmosphere.
Muon half-life in its rest frame: 2 μs. Typically created at 9000 m with speed 0.998c.



Given number measured at 9000 m, might expect few at sea-level: only travel 600 m in 2 μs. But in our frame, lifetime is γ 2 μs ~ 30 μs. So we get many.

In muon's frame, height of atmos. contracted to 9000m/γ = 600 m. Again, same large number will reach surface.

General Relativity

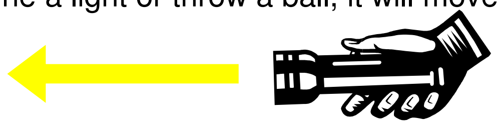
General Relativity: Einstein's (1915) description of gravity (extension of Newton's). It begins with:

The Equivalence Principle

Demonstrated by either of two thought experiments:

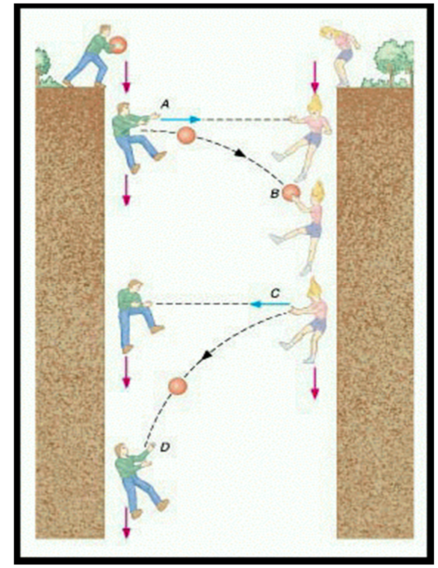
1) Freefall and weightlessness are equivalent

a) Imagine you are far from any source of gravity, thus weightless. If you shine a light or throw a ball, it will move in a straight line.

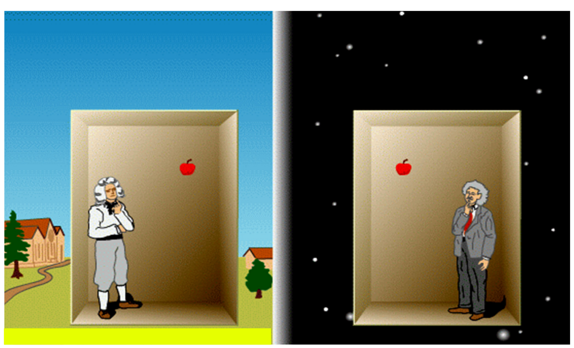


b) If you are in freefall (due to gravity), you are also weightless. Einstein says these are equivalent. So in freefalling reference frame, light and ball also travel in straight lines.

c) Now imagine two people in freefall on Earth, passing a ball. From their perspective, they pass it in a straight line. From a stationary perspective, it follows a curved path. So will a flashlight beam. But curvature of light path small because light is fast and Earth's gravitational acceleration is small.



2) Gravity and acceleration are equivalent

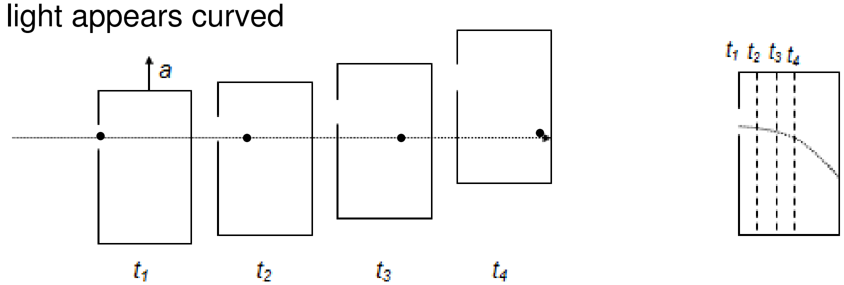


An apple falling in Earth's gravity is the same as one falling in an elevator accelerating upwards in free space.

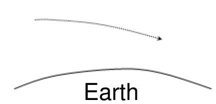
All effects you would observe by being in an accelerated frame of reference you would also observe when under the influence of gravity.

Bending of light in this case:

In an accelerating elevator in free space, straight path of light appears curved



(equal time intervals)

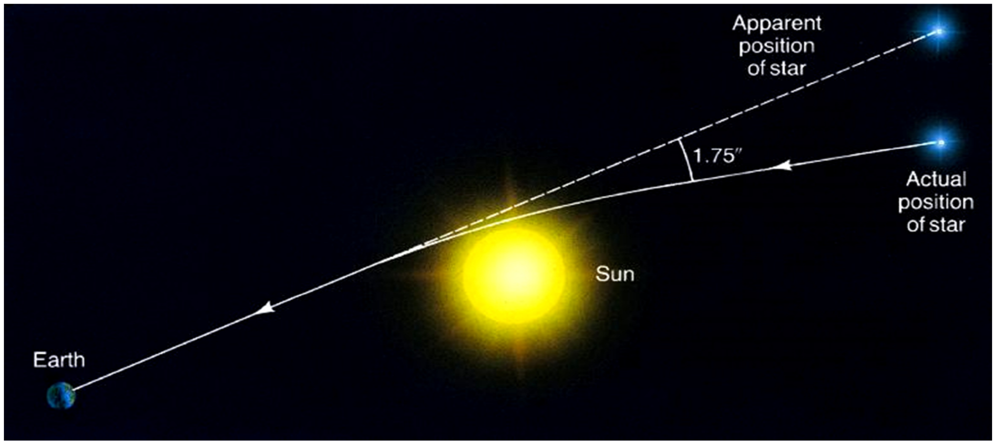


Same thing must happen in a gravitational field.

Testable Consequences of General Relativity:

1. Bending of light (just discussed)

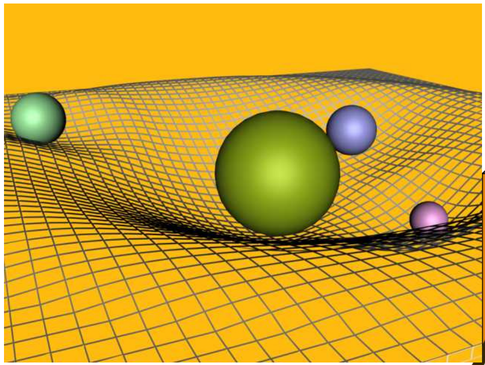
Observed! In 1919 eclipse.

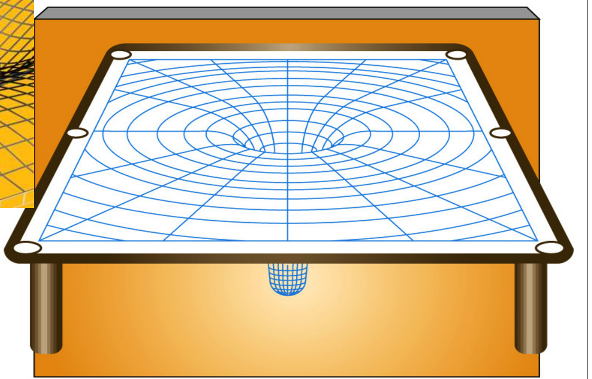


Gravitational lensing. The gravity of a foreground cluster of galaxies distorts the images of background galaxies into arc shapes.

Saturn-mass black hole

Einstein showed how the gravity of an object distorts, or curves, space around it, analogous to a rubber sheet in 2D. Freely falling objects passing through this curved space are forced to follow curved paths – they can't go in straight lines. True even for massless particles.

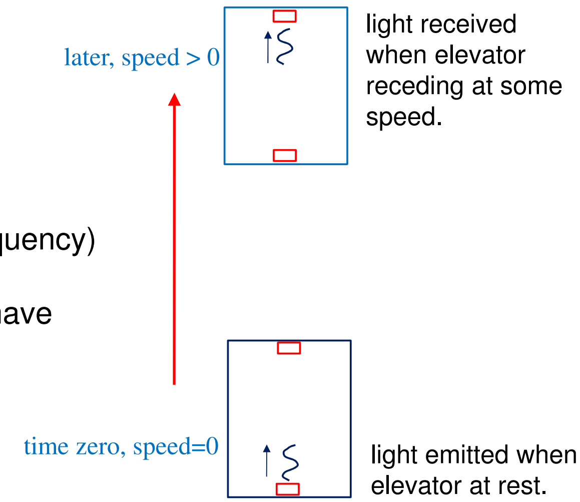


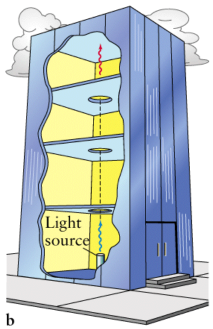


2. <u>Gravitational Redshift</u>

Consider accelerating elevator in free space (no gravity).

Received light has longer wavelength (or shorter frequency) because of Doppler Shift ("redshift"). Gravity must have same effect!





Verified in Pound-Rebka experiment.

If light emitted at radius r from center of mass M with wavelength λ_0, then λ_1 measured at another radius r_1 is:

$$\frac{\lambda_1}{\lambda_0} = \frac{\sqrt{1 - \frac{2GM}{r_1 c^2}}}{\sqrt{1 - \frac{2GM}{rc^2}}}$$

At an infinite distance away: $\frac{\lambda_1}{\lambda_0} = \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}}$

Can also write left hand side as ν_0/ν_1.

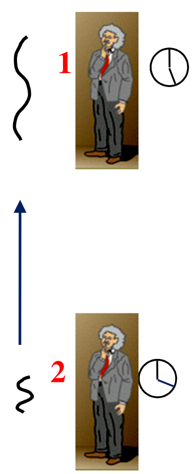
What happens when $r = 2GM/c^2$?

The photon will be redshifted to infinite wavelength or zero frequency - equivalently zero energy! It's redshifted out of existence! (this is true not only at an infinite distance away but at any distance $r_1 > 2GM/c^2$). Thus light can't escape – a **<u>black hole</u>**.

3. <u>Gravitational Time Dilation</u>

A photon moving upwards in gravity is redshifted. Since

$$\frac{c}{\lambda} = \nu = \frac{1}{T}$$



the photon's period gets longer. Observer 1 will measure a longer period than Observer 2. So they disagree on time intervals. Observer 1 would say that Observer 2's clock runs slow!

What happens to T if $r = 2GM/c^2$?

Time interval becomes infinitely long. Observer 2's time appears to stop according to Observer 1. Another way to define a **<u>black hole</u>.**

Escape Velocity

Velocity needed to escape the gravitational pull of an object, starting from a distance r from center.

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

If we set $v_{esc} = c$, then $r < 2GM/c^2$ is distance from center from which nothing can escape. Again, a **<u>black hole</u>**.

Schwarzschild Radius and Event Horizon

For an object of mass *M*, the <u>Schwarzschild Radius</u> is:

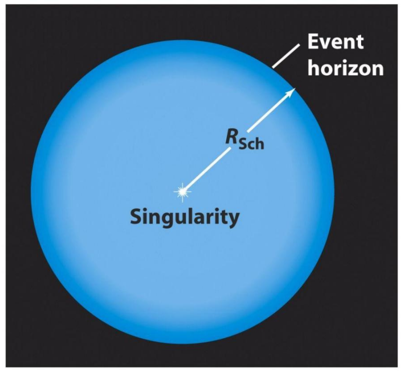
$$R_S = 2GM/c^2$$

at which $v_{esc}=c$, infinite gravitational redshift and time dilation occur.

$$R_S \text{ (km)} = 3\ M\ (M_\odot)$$

For Earth, R_S = 1 cm. If you could crush Earth to this size, it would be a black hole.

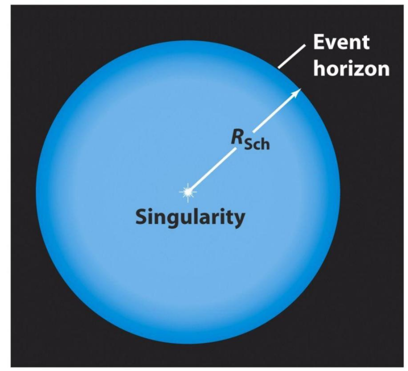
<u>Event Horizon</u> is imaginary sphere with radius R_S.



Black Holes

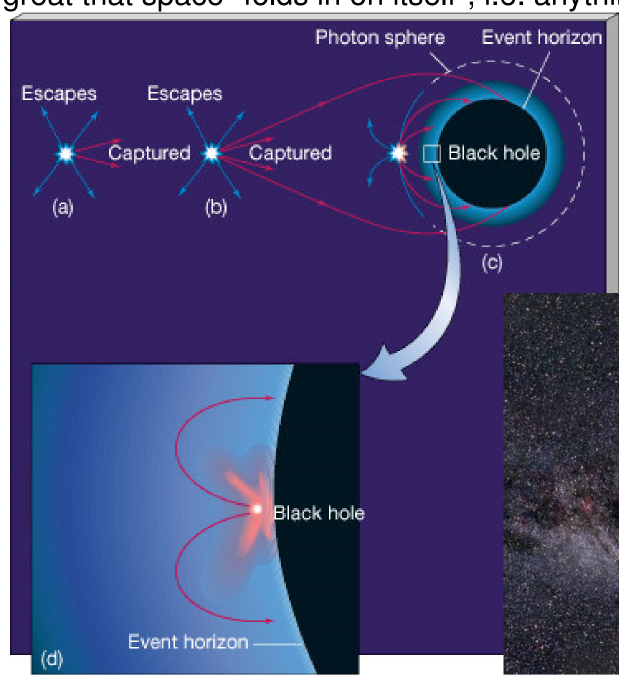
Result of collapse of core with about 3 M⊙ or more.

Core collapses to a point, a "singularity". As long as it shrinks to a size < R_S, it is a black hole. For a 3 M⊙ object, R_S = 9 km. (We have never resolved this distance for any BH candidate).



Anything crossing over to inside the event horizon, including light, is trapped. We can know nothing more about it after it does so.

Black holes cause enormous space curvature. At event horizon it is so great that space "folds in on itself", i.e. anything crossing it is trapped.



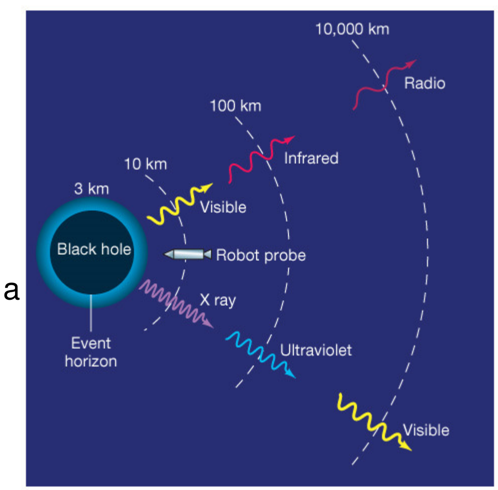
Other effects around Black Holes

1) Enormous tidal forces (Newtonian).

2) <u>Gravitational redshift</u>. Example, blue light emitted just outside event horizon may appear red to distant observer. Infinite redshift at event horizon.

3) <u>Time dilation</u>. Clock just outside event horizon appears to run slow to a distant observer. Clock approaches zero speed as it approaches event horizon.

None of these has actually been observed around a black hole, but 2) and 3) around other dense objects.

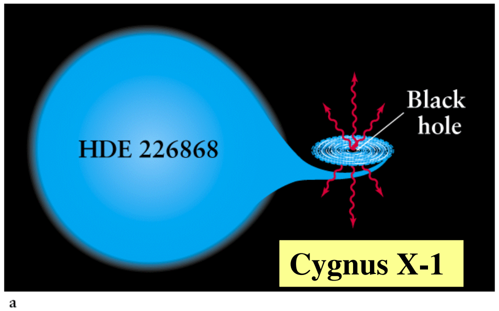


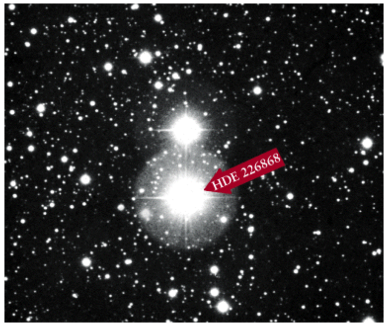
Do Black Holes Really Exist?
Good Candidate: Cygnus X-1

- Binary system: 30 $M_\odot$ star with unseen companion.

- Binary orbit => companion ~15 $M_\odot$. Neutron stars should be < 3 $M_\odot$

- X-rays => million degree gas falling into black hole.

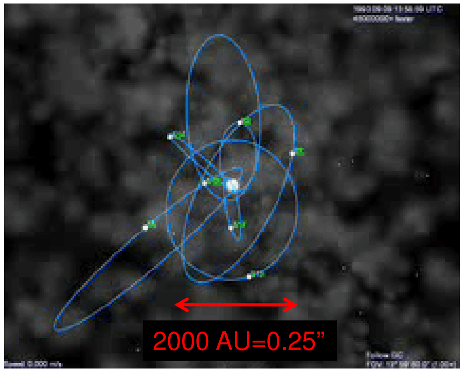




Supermassive black holes:
SgrA* at the center of the Milky Way

- The dynamical center of the Milky Way is called SgrA*, and contains a supermassive black hole.

Confirmed by measuring orbits of stars around the dense center.



Animation of stars orbiting the unseen mass