HOMEWORK X

Chapter 16

Question 6. These problems are meant as reviews of the concepts involved in oscillations, our newest topic.

In figure (a) the two blocks are at rest at their extreme positions. But they are the opposite positions, one with the spring maximally extended, the other with the spring maximally compressed. Thus these masses are one-half cycle apart, a phase difference of $\pi$ radians.

In case (b) both blocks are at their equilibrium positions BUT they are moving in opposite directions. This means, yet again that they are $\pi$ radians out of phase.

Finally, part (c). There is a phase difference of $\pi/2$ radians in this case, or $1/4$ of a cycle.

Problem 17E. We need to write the formula for $x(t)$. To do so we need the angular frequency, the amplitude, and the phase.

$$x = A \cos(\omega t + \varphi)$$

We are given $f = 0.25$ so $\omega = 2\pi f = 1.57 \text{ rad/s}$. When $x = 0.37$ we have $v = 0$. We call this the zero of time.

Since $v = -\omega A \sin(\omega t + \varphi)$ we then have $0.37 = A \cos \varphi$ and $0 = -\omega A \sin \varphi$

Thus we have two equations in two unknowns. We can divide the second equation through by $-\omega$ giving us $0.37 = A \cos \varphi$ and $0 = A \sin \varphi$. This second equation is satisfied by $\varphi = 0$ and then the first equation is satisfied by $A = 0.37 \text{ cm}$. Thus $x(t) = 0.37 \cos(1.57t)$

a) Because $f = 0.25 \text{ Hz}$, we have $T = 1/f = 4 \text{ seconds}$.

b) We have $\omega = 1.57 \text{ rad/s}$

c) The amplitude is $A = 0.37 \text{ cm}$

d) $$x(t) = 0.37 \cos(1.57t)$$

e) $$v = -\omega A \sin(\omega t + \varphi)$$

f) The maximum (we don’t care about the sign for maximum or minimum) we have $v_{\text{max}}$ occurring when the $\sin(\omega t) = 1$ which means $v_{\text{max}} = \omega A = 0.58 \text{ cm/s}$

g) The acceleration as a function of time is given by $a = -\omega^2 A \cos(\omega t + \varphi)$ and the maximum value occurs when the cosine is one. Therefore $a_{\text{max}} = \omega^2 A = 0.91 \text{ cm/s/s}$

h) At $t = 3 \text{ seconds}$ we get $x = 0.37 \cos(1.57 \times 3) = -8.83 \times 10^{-4} \text{ cm}$

i) At $t=3 \text{ seconds}$ we get $v = -\omega A \sin(\omega t) = -0.581 \text{ m/s}$
While the answer given in the back of the book for part i) is correct, I believe the answer on WebAssign is incorrect. I have informed the people at NCSU

Problem 21P. This is quite similar to the problem I made up for Homework XI. We can do it simply by realizing that

\[ T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \]

Then

\[ T_1 = 2 = 2\pi \sqrt{\frac{m}{k}} \]

and

\[ T_2 = 3 = 2\pi \sqrt{\frac{m + 2}{k}} \]

If we then take ratios, we get

\[ \frac{T_2}{T_1} = 1.5 = \sqrt{\frac{m + 2}{m}} \]

This is one equation in one unknown which is easily solved to give us \( m = 1.6 \) kg

Problem 43E. This is a very standard spring-mass problem. Hanging the mass at rest from the spring results in an equilibrium problem. Gravity pulls the mass down with a force \( mg \) while the spring pulls the mass up with a force \( kx \). Thus \( kx = mg \) and, for us then

\[ k = \frac{mg}{x} = \frac{1.3 \times 9.8}{0.096} \]

I must be consistent with the units. \( k = 133 \) N/m

b) The displacement of 5 cm gives us the amplitude. We have

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{133}{1.3}} = 10.1 \]

This means that

\[ T = \frac{2\pi}{\omega} = 0.662 \text{ seconds} \]

c) \( f = 1/T = 1.6 \) Hz
d) We already have $A = 5$ cm.

e) We also know that $v_{\text{max}} = \omega A = 50.5$ cm/s

**Problem 49P.** We are given the mass and the spring constant. This gives us, of course, the value

$$\omega = \sqrt{\frac{k}{m}} = 9.75$$

If the system were allowed to come to equilibrium we would have the distance from unstretched to the equilibrium point. This is $A$. It is equal to

$$A = \frac{mg}{k} = .10 \text{ meters}$$

a) The lowest distance below the highest point is just $2A = 0.20$ meters

b) The angular frequency is $9.75 \text{ rad/s}$ so $f = 1.55 \text{ Hz}$