Chapter 38

Problem 10E. a) This is an easy problem to start. Here the $\gamma$ factor is $\gamma = 2$. Then $\beta = 0.866$ and $v = 2.6 \times 10^8$ m/s.
b) The time dilation factor is, of course, also 2 so this is the factor by which the clock’s run slowly.

Problem 11E. The $x$ component of the stick will be changed while the $y$ component will not be changed. The $y$ component is $y = 0.5$ meters while $x = 0.866$ meters. This component is shortened by the factor $\gamma = 2.29$ so that $x' = 0.378$ meters. Then the total length is $L' = \sqrt{x'^2 + y'^2} = 0.63$ meters.

Problem 15P. Here we have a much more complicated (i.e. interesting) problem.
a) An Earth observer says the star is 26 light years distant and the rocket ship is traveling in our reference frame at nearly $c$ so that it will take approximately 26 years to get there. The actual time will be $26/0.99c = 26.3$ years.
b) It takes 26.3 years for the trip and 26 years for the light (radio) signal to get back to Earth, for a total of 52.3 years.
c) The dilation factor is $\gamma = 7.09$. The age of the Earth observer as measured by the rocket observer will be $t = 26.3/\gamma = 3.71$ years.

Of course, this rocket observer also thinks that the distance to the star is only $26/\gamma = 3.67$ light years.

Problem 16P. The velocity $v << c$ so that all relativistic corrections will be very very small.
a) We have $\beta = 2.1 \times 10^{-6}$. Since this is so small, we can also use approximations for $\gamma$ as well. Since

$$\gamma = \left(1 - \beta^2\right)^{-1/2} \approx 1 + \frac{\beta^2}{2}$$

for $\beta << 1$ which is certainly true here. Then $\gamma \approx 1 + 2.21 \times 10^{-12}$. This is the length-shortening factor, so that

$$L' = \frac{L}{\gamma} = \frac{L}{1 + 2.21 \times 10^{-12}} \approx L \left(1 - 2.21 \times 10^{-12}\right)$$

The change of length is $\Delta L = 2.21 \times 10^{-12}L$ and the fractional change is given by (in absolute magnitude)
\[
\frac{\Delta L}{L} = 2.21 \times 10^{-12}
\]

b) The time dilation factor is the same tiny \( 1 + 2.21 \times 10^{-12} \). Then we know that \( t = \gamma \tau = (1 + 2.21 \times 10^{-12}) \tau = \tau + 2.21 \times 10^{-12} \tau \). We want \( t - \tau = 1 \times 10^{-6} = 2.21 \times 10^{-12} \tau \). Therefore \( \tau = 2.53 \times 10^6 \) seconds = 5.25 days!