Chapter 32
Problem 13E. a) The calculation of an electric field is easy.

\[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} = 9 \times 10^9 \times \frac{1.6 \times 10^{-19}}{(5.2 \times 10^{-11})^2} = 5.33 \times 10^{11} \]

The reason you have been asked to do this is to see how MASSIVE this field is. Fields of \( E = 10^7 \text{ V/m} \) are very large at human scale size. Atomic fields are huge by comparison.

b) There is NO macroscopic (our size) quantity comparable to the spin angular momentum in quantum mechanics (atomic scale size) and the manner in which such spin angular momenta behave is strange to us. We can think of this, at our early stage, as if the electron is a little ball (rather like the earth) and imagine it spinning on its axis. This is a very bad model in reality but one with which we can start to think.

Then, doing what we are told to do we have

\[ B = \frac{\mu_0 \mu}{2\pi x^3} = 2 \times 10^{-7} \times \frac{1.4 \times 10^{-26}}{(5.2 \times 10^{-11})^3} = 1.99 \times 10^{-2} \]

This is a small but not insignificant field.

c) This part is wrong. What the book has done is to take and divide the value above \( 1.4 \times 10^{-26} \) into the spin magnetic moment of the electron \( 9.27 \times 10^{-24} \) which ratio is 662. But this is wrong at least by my interpretation of the problem. The spin proton magnetic moment of the proton is

\[ \mu_P = \frac{e\hbar}{4\pi m_P} = 5.06 \times 10^{-27} \]

The ratio of the electron’s magnetic moment to the magnetic moment of the proton is then 1830 which is close to the ratio of the proton mass to the electron mass.

There are several things to notice here. Look in Appendix B and we see that

\[ \mu_P \text{ (measured)} = 1.41 \times 10^{-26} = 2.79 \mu_P \]

Thus the measured ratio is indeed 660 but the calculated ratio, given classically, is 1830. The factor of 2.79 is due to the internal structure of the proton and cannot be understood classically. It is a purely quantum mechanical phenomenon.

Problem 15E. Here is a good example of the strange nature of quantum mechanics.
a) ONLY integer values of the angular momenta are allowed. If we have an angular momentum of 3 units (the units are \( \hbar = h/2\pi \)) and we then establish a particular axis in space (the simplest way to do this is to establish an external magnetic field) then the angular momentum (a vector) can point in only certain directions - NOT any direction as it can classically. These directions are such that the projection of the angular momentum on the external axis (i.e. the value of angular momentum times the cosine of the angle between the angular momentum and the axis) will have integer values. Thus, if the maximum value of the angular momentum when it points exactly along the external axis is +3 units then the minimum value is -3 units and the allowed values are +3, +2, +1, 0, -1, -2, -3. This gives SEVEN possible value of the angular momentum.

b) Since we have (for a negatively charged particle)

\[
\vec{p}' = -\frac{e}{2m} \vec{T}
\]

and since there are seven allowed value of the projected values of \( L \) then there will be seven possible allowed values of \( \mu \).

c) The largest value of \( L_z \) is

\[
L_z = 3 \frac{e \hbar}{4\pi m}
\]

d) They clearly mean the lowest absolute value because the answer is zero.

e) Let us use the word total angular momentum not net angular momentum. Then, using the fact that angular momentum vectors add we have

\[
\vec{J} = \vec{T} + \vec{S}
\]

or the largest value is \( 3 + 1/2 \) units. (I use the symbol \( J \) for total angular momentum)

f) The allowed values will then be 3.5, 2.5, 1.5, .5, -.5, -1.5, -2.5, -3.5. There are EIGHT values.

**Problem 27P.** This is either hard or not depending somewhat on your math skills. Suppose we have a value of \( N \) which is the number of particles in a box of volume \( V = 1 \). Therefore, \( N \) is the number of particles per volume. The probability of finding a spin UP is given by

\[
P_+ = e^{-U/kT} = e^{+\mu B/kT}
\]

so the number of spins up is the number of particles (per volume) times the probability of finding a spin UP (parallel). To see this better, imagine a total of 1000 particles with a probability of 60% of having a spin up (or a probability of 40% of finding spin down). Then there will be 600 up and 400 down. This is all I have done but in symbol rather than number form.
But if I have 600 up and 400 down, then the NET effect is of 200 UP. Thus, assuming that each individual atom has a spin magnetic moment $\mu$ the total magnetic moment of the sample is

$$\mu_{total} = \mu [N_+ - N_-] = \mu [NP_+ - NP_] = \mu N [P_+ - P_-]$$

But this gives us

$$\mu_{total} = \mu N \left[ e^{+\mu B/kT} - e^{-\mu B/kT} \right]$$

Since $N$ is the number per unit volume we already have $\mu_{total}/V = M$ the magnetization. To get it into the form in the book I realize that the probability UP plus the probability DOWN must be ONE (all particles must be either UP or DOWN). But if I put ONE in the denominator as

$$M = \mu N \left[ \frac{e^{+\mu B/kT} - e^{-\mu B/kT}}{e^{+\mu B/kT} + e^{-\mu B/kT}} \right] = \mu N \tanh (\mu B/kT)$$

I get the final answer.

b) Let us call

$$\mu B/kT = x$$

and write

$$M = \mu N \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

and think about the value $x << 1$ or $\mu B << kT$. Then I can use the Taylor expansion for an exponential (see page A11 in Appendix E) where we get

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

If the exponent is negative then all the odd powers have minus signs. If $x << 1$ then $x^2 << 1$ and can be ignored. Thus for $x << 1$ we can write $e^x \approx 1 + x$ and $e^{-x} \approx 1 - x$.

Our expression then becomes

$$M = \mu N \frac{1 + x - (1 - x)}{1 + x + (1 - x)} = \mu N \frac{2x}{2} = \mu Nx = \mu N\frac{\mu B}{kT}$$

c) For $x >> 1$ or $\mu B >> kT$ we have $e^{-x} \approx 0$. Thus our expression becomes
\[ M = \mu N \left[ \frac{e^x}{e^x} \right] = \mu N \]

d) This is sort of silly, the plot of \( \tanh \) just giving the Curie curve on page 795. Here is a plot of same.

![Curie's Law](image)

Chapter 33.

**Problem 7E.**

a) This is easier.

\[ \omega = \sqrt{\frac{k}{m}} \]

But \( k = F/x = 8/2 \times 10^{-3} = 4000 \)

Then \( \omega = 89 \) radians/sec.

b) The frequency is

\[ f = \frac{\omega}{2\pi} = 14.2 \text{ Hz} \]

c) Since the case of an \( LC \) circuit is similar mathematically we have

\[ \omega = \frac{1}{\sqrt{LC}} = 89 \]

If \( L = 5 \) then \( C = 25 \mu F \)