Chapter 35

Problem 20P. These are refraction problems so we are dealing with a different formula. We have, as our starting point,

\[
\frac{n_1}{p} + \frac{n_2}{i} = \frac{(n_2 - n_1)}{r}
\]

a) we have

\[
\frac{1}{10} + \frac{1.5}{i} = 0.5 \frac{30}{30}
\]

This is easily solved to give \( i = -18 \) a virtual image.

b) These are all the same but I will not just list the answers. I will do the work so that, if you have had problems, you can see what is happening.

Here we are asked for the radius \( r \). But this is not hard. Again we start from

\[
\frac{n_1}{p} + \frac{n_2}{i} = \frac{(n_2 - n_1)}{r}
\]

and plug in the numbers we are given

\[
\frac{1}{10} - \frac{1.5}{13} = \frac{0.5}{r}
\]

The only unknown is the radius and we can quickly solve and get \( r = -33 \). The image is virtual and hence erect because the image distance is negative.

c) Here we solve for the object distance \( p \). This is silly of course but we do this only for the sake of exercise. If you don’t know the position of your object then ...

\[
\frac{1}{p} + \frac{1.5}{600} = \frac{0.5}{30}
\]

This gives us \( p = 71 \) cm. The image is real and hence inverted because the image distance is positive.

d) To round out a complete set, we solve for the unknown index. This is not so silly because it represents one way of actually measuring the index of refraction of a material.

\[
\frac{1}{20} - \frac{n}{20} = \frac{n - 1}{-20}
\]
This problem is a lot easier because every denominator has a factor of 20 in it. Therefore, canceling these we get $1 - n = -(n - 1) = 1 - n$. But this is not an equation but a tautology. $n - 1 = n - 1$?? This is solvable for ANY value of the index. Any value is possible!!!! The image is virtual and hence erect.

e) We have (going from glass into air)

\[
\frac{1.5}{10} - \frac{1}{6} = \frac{-0.5}{r}
\]

which gives me $r = +30$. The image distance is negative, therefore the image is virtual and erect.

f) We have

\[
\frac{1.5}{p} - \frac{1}{7.5} = \frac{-0.5}{-30}
\]

Signs are tough here. Be careful!

This solves to give me $p = +10$. The image is virtual and hence erect.

g) We have

\[
\frac{1.5}{70} + \frac{1}{i} = \frac{-0.5}{30}
\]

Again this is easy to solve (I know most students are addicted to calculators and do these problems by dividing. I am addicted to lowest common denominators (after all, I worked like hell in 3rd and 4th grades on them) and use these to solve instead. Then I get $i = -26$ cm (I had to use 4200 as my LCD) and the image is virtual and hence erect.

h) Finally,

\[
\frac{1.5}{100} + \frac{n}{600} = \frac{n - 1.5}{-30}
\]

While not super easy to solve it is not hard and (not surprisingly) gives me $n = 1$. The image is real and hence inverted.

Whew.

**Problem 21P.** This requires a drawing to aid the book’s drawing.
The book’s drawing marks the relevant distances $d$ and $d_a$. What I was interested in seeing in this drawing are the angles. Snell’s Law then tells us that $n \sin \theta_i = \sin \theta_r$ because we are going from material to air. But trigonometry tells us that $y = d \tan \theta_i$ and, if we look carefully, it also tells us that $y = d_a \tan \theta_r$.

Therefore $d \tan \theta_i = d_a \tan \theta_r$. But if we can substitute $\sin \theta$ for $\tan \theta$, which we are allowed to do for small angles, we have $d \sin \theta_i = d_a \sin \theta_r$. But we also have that $\sin \theta_r = n \sin \theta_i$ so I will substitute and get $d \sin \theta_i = d_a n \sin \theta_i$. Therefore, the incident angle cancels out (unless it is zero degrees which I ignore as there is no refraction there). Thus I am left with $d = d_a n$ which is my desired result. That is

$$d_a = \frac{d}{n}$$

and the apparent depth is less than the real depth. This is why the bottoms of swimming pools look closer than they are and why your legs look shorter when under water.

**Problem 29E.** I will start by writing out the lens makers formula

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} = (n - 1) \left[ \frac{1}{r_1} - \frac{1}{r_2} \right]$$

The first thing I want to do is to show that the focal length is the same no matter which face is turned to the light. In other words, for the two orientations
We know that \( r_1 \) is the first surface that the light sees when it comes in from the left and that, of course, \( r_2 \) is the second surface.

From the first drawing we see that \( r_1 = \infty \) and that \( r_2 = -20\). It is MINUS 20 because the center of curvature lies to the LEFT of the vertex. For a lens and for any refracting surface, the center of curvature must lie to the RIGHT of the vertex to be positive!!!!

Then we have

\[
\frac{1}{f} = (n - 1) \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] = 0.5 \left[ 0 - \frac{1}{-20} \right] = +\frac{1}{40}
\]

Now take the second diagram. We have here \( r_1 = +20 \) (the center is to the right of the vertex) and \( r_2 = \infty \). Then we have

\[
\frac{1}{f} = (n - 1) \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] = 0.5 \left[ \frac{1}{20} - 0 \right] = \frac{1}{40}
\]

Either orientation of the lens gives us a focal length of \( f = +40 \text{ cm} \), a positive value!!!!

b) Now this is easy

\[
\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{40} - \frac{1}{40} = 0
\]

Thus \( i = \infty \) which is, after all, what we expect when we place at object at the focal point of a lens!

**Problem 34P.** Another (the last) one of these.

a) The lens is a converging lens which makes the focal length \( f = +10 \). Then this is easy.

\[
\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}
\]

The image is real and is 20 cm to the right of the lens. We have no information about the radii. Since the image is real, it is inverted. The magnification is
\[ m = - \frac{i}{p} = - \frac{20}{20} = -1 \]

The size of the image equals the size of the object but is inverted with respect to the object.

b) We have virtually the same problem. Since \( f = +10 \) the lens is converging. Then

\[ \frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10} - \frac{1}{5} = \frac{-1}{10} \]

The image distance is -10 cm which is what we EXPECT when we put an object between the lens and its focal point. The image is virtual and erect. The magnification is \( m = +2 \).

No information on radii.

c) The image magnification is \( m > 1 \) which means it is POSITIVE. Thus the image is virtual and erect. Can we tell whether it is a converging or diverging lens yet? Let’s see.

Assume \( f = +10 \) and I get

\[ \frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10} - \frac{1}{5} = \frac{-1}{10} \]

as we did above. The image is virtual but the lens is converging. It is just that we are between the lens and the focal point.

Now try \( f = -10 \) and I see that

\[ \frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10} - \frac{1}{5} = \frac{-3}{10} \]

or \( i = -3.33 \) cm. What does this say. It tells us first that the image is still virtual and erect. Its size however is

\[ m = \frac{i}{p} = \frac{-3.33}{10} = -.33 < 1 \]

So this does NOT work. Thus the lens is CONVERGING, the focal length is \( f = +10 \) cm and the magnification is \( m = +2 \).

d) Here we are again. We try \( f = +10 \) first and we get

\[ \frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10} - \frac{1}{5} = \frac{-1}{10} \]

as we did above. But we know this gives us a magnification of \( m = +2 \).

Therefore, this cannot be correct and we MUST have \( f = -10 \) cm. But we have also done this problem and we know it gives us \( i = -3.33 \) cm and \( m = 0.33 < 1 \). The image is virtual and erect (and smaller). Since the focal length is negative, the lens must be diverging.
e) Here we must calculate the focal length from the lens makers formula. We have

$$\frac{1}{f} = (n-1) \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] = 0.5 \left[ \frac{1}{30} - \frac{1}{-30} \right] = + \frac{1}{30}$$

Therefore we find that $f = +30$ cm and the lens is, therefore, converging.

Now with $p = +10$ we can predict that the image will be virtual!! The object distance $p < f$ the focal length. Therefore, we know that a converging lens will give a virtual image. Let us see this algebraically.

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{30} - \frac{1}{10} = -\frac{1}{15}$$

The image distance is $i = -15$ cm and is, of course, virtual and erect. We see quickly that $m = +1.5$.

It is nice to predict a priori what is going to happen.

f) Since this is the same exact problem with the signs of the radii reversed, we know that we will have a diverging lens of $f = -30$ cm.

Then

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = -\frac{1}{30} - \frac{1}{10} = -\frac{4}{30}$$

or $i = -7.5$ cm. The magnification is $m = +0.75$. The image is virtual and erect.

These are getting more complicated and more useful.

g) We recalculate

$$\frac{1}{f} = (n-1) \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] = 0.5 \left[ \frac{1}{-30} - \frac{1}{-60} \right] = -\frac{1}{120}$$

The focal length is $f = -120$ cm and the lens is a diverging one. The image distance is (the image will surely be virtual for a diverging lens)

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{1}{-120} - \frac{1}{10} = -\frac{13}{120}$$

Therefore $i = -9.2$ cm. The image is virtual. The magnification is $m = +0.92$ and therefore the image is erect.

h) Here we are given the magnification but we are NOT told if $m = \pm 0.5$. We must, therefore, try both varieties. Try $m = +0.5$ first. This is a virtual and erect image where

$$m = \frac{i}{p} = +0.5 = -\frac{i}{10}$$
This gives us \( i = -5 \) cm and is certainly virtual. The focal length of such a lens would be found from

\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10} = \frac{1}{f}
\]

Therefore \( f = -10 \) cm and the lens is diverging. All of this fits well.

Now try \( m = -0.5 \) which gives us \( i = +5 \) cm and is a real and inverted image.

We find

\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{10} + \frac{1}{5} = \frac{3}{10}
\]

The focal length \( f = +3.33 \) cm and the lens is converging.

Which one of these solutions fits?? Uh oh, we are told that the image is NOT inverted whereas the second case (converging lens) is indeed an inverted image. Therefore only the FIRST case is suitable here and \( f = -10 \), a diverging lens, \( i = -5 \) and \( m = +0.5 \)

i) Ah, here we have the EXACT same problem as our second case in part h) above. So we have done this.

We have a converging lens of \( f = +10 \) cm, \( i = +5 \) cm, \( m = -0.5 \). The image is real and inverted (and smaller).