Secondary anisotropies

The Sunyaev-Zel'dovich effect, predicted in the 1970s.

CMB photons
\[ T = (1 + z) \times 2.725K \]

galaxy cluster with hot ICM
\[ z \sim 0 - 3 \]

scattered photons (hotter)

last scattering surface
\[ z \sim 1100 \]

CMB photons have a \(~1\%) chance of inverse Compton scattering off of the ICM electrons; photon *number* is conserved
• This wavelength shift translates into a temperature shift

• Thus, can be seen as a higher order effect in the CMB

• The shift is independent of redshift

\[ \frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} = f(x) y = f(x) \int n_e \frac{k_B T_e}{m_e c^2} \sigma_T \, d\ell, \]

• Since clusters of galaxies collapse from large volumes (1000 Mpc^3), the ratio of baryons to dark matter should be representative of the universe as a whole. Observations of SZ determine the baryon fraction in the universe.
Lecture 21: Big Bang Nucleosynthesis
Astro 422 Presentations:

Thursday April 28:
- 9:30 – 9:50  _Isaiah Santistevan__________
- 9:50 – 10:10 _Cameron Trapp__________
- 10:10 – 10:30 _________________________

Tuesday May 3:
- 9:30 – 9:50  __Chris Quintana____________
- 9:50 – 10:10 __Austin Vaitkus___________
- 10:10 – 10:30 _________________________

Thursday May 5:
- 9:30 – 9:50  _Montie Avery_______________
- 9:50 – 10:10 _Andrea Tallbrother________
- 10:10 – 10:30 _Veronica Dike_____________
- 10:30 – 10:50 _________________________

Send me your preference. First come, first served.
Key concepts:

General Relativity and the Robertson-Walker metric
Proper distance
Luminosity distance
Angular diameter distance
Big Bang Nucleosynthesis
Gravitation and cosmology

• In 1916, Einstein replaced Newton's conception of gravitation as a force, with GR.
  – Gravitation is dynamics of spacetime.

• 1917 Einstein applied GR to the universe.
  – Universe homogenous on average, and static
  – It is closed on itself (curved volume of space with no boundary)

• Einstein found no solutions to his equations unless he inserted an extra term, acting as a repulsion to offset the gravitational attraction of matter.
  – Known as the cosmological constant, $\Lambda$.

How does gravity affect spacetime, and geometry of universe?
How do we understand geometrical properties of the universe?

How do we measure distances and *spacetime intervals*?

- **If the universe is closed**, light rays from opposite sides of a hot spot bend toward each other...
- **If the universe is flat**, light rays from opposite sides of a hot spot do not bend at all...
- **If the universe is open**, light rays from opposite sides of a hot spot bend away from each other...

... and as a result, the hot spot appears to us to be larger than it actually is.

... and so the hot spot appears to us with its true size.

... and as a result, the hot spot appears to us to be smaller than it actually is.
Testing Euclid with measurements of Baryon Acoustic Oscillations (BAO)
Testing Euclid with measurements of Baryon Acoustic Oscillations (BAO)
Robertson-Walker Metric:

Describes distances in curved 3-d space.

We cannot visualize space curvature in 3-d since we are not 4-d animals. Let's start with a curved 2-d surface - the ant analogy of a closed universe.

Ant cities (=galaxies) are distributed homogeneously on average on the curved surface of the ant world.

A finite surface area, but no edge
No center on the surface.
A radius of curvature $R$ for the surface of the sphere lies in the third dimension. This direction doesn't mean anything for ants!

Ants draw a circle walking around with a string of length $D$ attached to a point on the sphere:

\[ \text{Circumference} = 2\pi r = 2\pi D \text{ when } l \ll R \]
Ant walking around measures

\[ C_{meas} = 2\pi R \sin \theta \]

The ant expects to measure

\[ C_{exp} = 2\pi D = 2\pi R \theta \]

They are the same when the angle is small - in the *nearby universe*. 
Very large radius (in ant world): $D \Rightarrow \pi R$.
Circle of large "radius" and small circumference:

Circumference $C = 2\pi r \Rightarrow 0$ when $D \Rightarrow R$. 
Curvature of space, $K$, is defined as:

$$K = 6\pi \frac{C_{exp} - C_{meas}}{C_{exp}A_{exp}}$$

C&O shows that:

$$K = 6\pi \frac{C_{exp} - C_{meas}}{C_{exp}A_{exp}} = \frac{1}{R^2} + O^2 + ...$$

Where $A_{exp} = \pi D^2$ is the expected area

What does it mean that $K=0$?
What does it mean that $K<0$?
The curvature $K$ proportional to $R^{-2}$, ie large sphere appears locally flat.

$k=0$: flat

$k<0$: negative curvature

$k>0$: positive curvature

Looking ahead: in 3d $R$ is the scale factor. Curvature will vary on small scales due to gravity, but cosmological principle says it is constant on large scales.
Closed ant world:
Since light travels on geodesics (great circles), can an ant see its rear by looking straight ahead?
Closed ant world:
Since light travels on geodesics (great circles), can an ant see its rear by looking straight ahead?

Problems:
1) An ant does not live long enough for light to go all the way around.
2) Even if it would, the world may not be old enough for light to go around.
How to measure distances on a spherical surface

\[(dl)^2 = (Rd\theta)^2 + (rd\phi)^2\]

\[r = R \sin(\theta)\]

\[dr = R \cos(\theta) d\theta\]

\[\cos(\theta) = \frac{\sqrt{R^2 - r^2}}{R}\]
Thus \[ (dl)^2 = \left( \frac{dr}{\sqrt{(R^2 - r^2)/R^2}} \right)^2 + (rd\phi)^2 \]

For a sphere \( k = R^{-2} \). Generally for any curved surface the differential distance on the surface:

\[ (dl)^2 = \left( \frac{dr}{\sqrt{1 - kr^2}} \right)^2 + (rd\phi)^2 \]

To go from a 2-d to 3-d, we move from polar to spherical coordinates:

\( (r, \phi) \) to \( (r, \theta, \phi) \)
This yields:

\[(dl)^2 = \left( \frac{dr}{\sqrt{1 - Kr^2}} \right)^2 + (rd\theta)^2 + (r \sin \theta d\phi)^2\]

Now recall the metric:

\[(ds)^2 = (cdt)^2 - (dl)^2\]

\[\Rightarrow (ds)^2 = (cdt)^2 - \left( \frac{dr}{\sqrt{1 - Kr^2}} \right)^2 - (rd\theta)^2 - (r \sin \theta d\phi)^2\]
interval between two events:

\[ ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2 dt^2 \]
Now we'd like to express this in terms of the scale factor, $R(t)$.

$$r(t) = R(t)\tilde{\omega}$$

then $K(t) \equiv \frac{K}{R(t)^2}$

(for sphere $K \equiv \frac{1}{R^2}$)

$$(ds)^2 = (c dt)^2 - R^2(t) \left[ \left( \frac{d\omega}{\sqrt{1 - K\omega^2}} \right)^2 + (\tilde{\omega} d\theta)^2 + (\tilde{\omega} \sin \theta d\phi)^2 \right]$$

*Robertson-Walker metric* determines the spacetime interval between two events in an isotropic, homogeneous universe.
Einstein Field Equations

\[ G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} \]

\( G \) is the *Einstein* tensor and describes time and space coordinates and their derivatives (i.e. how geometry evolves).

\( T \) is the *stress-energy tensor* and describes mass, energy and momentum of matter and radiation.

Calculates geometry of spacetime produced by a given distribution of mass/energy.
The Friedmann Equation

Solving Einstein's field equations for a space that is described by the Robertson-Walker metric you end up with the Friedmann Equation. It will be identical to the Newtonian case with \( k = K(t)R^2(t) \).

\[
\left[ \left( \frac{1}{R} \frac{dR}{dt} \right)^2 - \frac{8}{3} \pi G \rho \right] R^2 = -kc^2
\]

This is for a non-static universe.

Recall from the Newtonian case that we used the density parameter to re-write the Friedmann equation:

\[
H_o^2(\Omega_0 - 1) = kc^2
\]

- \( \Omega_0 = 1, \ k = 0 \) flat universe
- \( \Omega_0 > 1, \ k > 0 \) closed universe
- \( \Omega_0 < 1, \ k < 0 \) open universe
Cosmological constant

Added by Einstein to allow a static universe. Solutions to the Einstein equations then becomes:

\[
\left[ \left( \frac{1}{R} \frac{dR}{dt} \right)^2 - \frac{8}{3} \pi G \rho - \frac{1}{3} \Lambda c^2 \right] R^2 = -k c^2
\]

\( \Lambda \) leads to a force:

\[
\vec{F}_\Lambda \propto \Lambda \hat{r}
\]

What does it mean if \( \Lambda > 0 \)?

**A repulsive force.**
We need to revisit what the term \( \rho \) means!
More generally, we must deal with a 3-component universe:

- mass (baryonic + dark)
- relativistic particles \((\gamma, \nu)\)
- "dark energy" (whatever is causing \( \Lambda \))

Friedmann equation then is written as:

\[
\left[ \left( \frac{1}{R} \frac{dR}{dt} \right)^2 - \frac{8}{3} \pi G (\rho_m + \rho_{rel} + \rho_\Lambda) \right] R^2 = -kc^2
\]

\[
\rho_\Lambda \equiv \frac{\Lambda c^2}{8\pi G}
\]
Recall that

\[ H(t) = \frac{1}{R(t)} \frac{dR(t)}{dt} \]

\[ \rho_c = \frac{3H^2}{8\pi G} \]

Then the Friedmann equation can be written as:

\[ H^2[1 - (\Omega_m + \Omega_{rel} + \Omega_{\Lambda})]R^2 = -kc^2 \]

where

\[ \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_c} \]
Thus, the *total density parameter* is expressed as

\[
\Omega \equiv \Omega_m + \Omega_{rel} + \Omega_\Lambda
\]

\[
1 = 0.27\pm0.04 + 8.24 \times 10^{-5} + 0.73 \pm 0.04
\]

When \( \Lambda > 0 \), the equivalent mass density of dark energy \( \rho_\Lambda \) adds to the other densities' effect on \( k \), the curvature of the universe.

But: a positive \( \rho_\Lambda \) acts like the repulsive force, so the presence of dark energy *decouples* the geometry of the universe (open, closed, flat) from its dynamics, which is determined by the relative importance of \( \rho_m, \rho_{rel} \) and \( \rho_\Lambda \).

Best observations indicate that \( \Omega = 1, \ k = 0, \ \rho_\Lambda > 0 \) (accelerating) universe.
Measuring Cosmological Parameters

We would like to know the scale factor $R(t)$ for the universe.

- For a model universe, where we know exactly the composition, we can compute $R(t)$ from the Friedmann equations. That's what we did in a previous lecture.

- The real $R(t)$ is a bit trickier to find. $R(t)$ is not a direct observable, it has to be derived from other observations.

- Determining $H_0$ should be easy: For small redshift, $cz = H_0 d$, so if we measure $d$ and $z$ we can make the linear plot.
• Measuring \( z \) is relatively easy. The distance is much harder!

• It is also hard to define: we have defined the *proper distance* to be the length of a spatial geodesic between two points when the scale factor is fixed at value \( R(t) \).

• The relation between scale factor and the proper distance is

\[
r = c \int_{t_e}^{t_0} \frac{dt}{R(t)}
\]

where light is emitted at time \( t_e \) and observed at \( t_0 \). Here, \( r = d_p(t_0) \).
• With a little bit of work, and a Taylor expansion the proper distance to a galaxy can be written as

\[ d_p(t_0) \approx c(t_0 - t_e) + \frac{cH_0}{2}(t_0 - t_e)^2 \]

• The first term is what the proper distance would be in a static universe: the lookback time times the speed of light.

• The second term is a correction due to the expansion of the universe during the time the light was traveling.

• This equation would be useful if photons carried information about \((t_0 - t_e)\). Instead they carry \(R(t_e)\) information, via \(R(t) = 1/(1+z)\)
• So unfortunately the proper distance is not a measurable property.

• What can we measure?
  – Radar distances
  – parallax distances etc

• We can also measure the flux $F$ from an object. then we can apply the standard candle method.

• You know $L$, and we derive the so called luminosity distance.
  – Called distance because it has those units.

\[ d_L^2 = \frac{L}{4\pi F} \]
Angular-diameter distance

The luminosity distance is not the only distance measure that can be estimated from observable properties of objects. We also have *standard meter sticks.*

This you can use if the size of the object is known.

\[ d_A = \frac{l}{\theta} \]

\( d_A \) is called the *angular-diameter* distance. This will be the same as the proper distance if the universe is static and Euclidean.
• If the universe is expanding or is curved, this is not true. If you are in a universe described by the Robertson-Walker metric, then assume you are at origin.

• The meter stick is at a comoving distance \( r \), and at a time \( t_e \) is emits light that you observe at time \( t_0 \). The comoving coordinates at the ends of the meter stick were

\[
(r, \theta_1, \phi), (r, \theta_2, \phi)
\]

• As the light from the meter stick moves towards the origin, it travels via geodesics with \( \theta, \phi \) being constant.
  – Thus the angular size you measure is constant

\[
\theta = \theta_2 - \theta_1
\]
• The distance $d_s$ between the two ends on the stick measured at time $t_e$ can be found from the R-W metric.

• After some re-writing we find a relation between the luminosity distance and the angular-diameter distance:

$$d_A = \frac{l}{\theta} = \frac{d_L}{(1 + z)^2}$$

• Thus, if you observe an object that is both a standard candle and a standard meter stick, the angular-diameter distance will be smaller than the luminosity distance!
• Also, if the universe is spatially flat:

\[ d_A(1 + z) = d_p(t_0) = \frac{d_L}{(1 + z)} \]

• So, the angular-diameter distance is not the same as the current proper distance.
  – It is equal to the proper distance at the time when the light from the object was emitted:

\[ d_A = \frac{d_p(t_0)}{(1 + z)} = d_p(t_e) \]
Big Bang Nucleosynthesis

- Early universe was radiation dominated, with \( T(t) \propto t^{-1/2} \)

- At high temperatures, photon-photon collisions could produce material particles.

- Can happen when \( E_{\text{phot}} = h\nu \sim kT > m_0c^2 \)

\[ T_{\text{threshold}} > \frac{m_0c^2}{k} \]

The threshold temperature for creation of particles of rest mass \( m_0 \).
For example:

<table>
<thead>
<tr>
<th>Particle</th>
<th>$m_0c^2$ (MeV)</th>
<th>$T_{thr}$ ($10^9$ K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>0.51</td>
<td>5.93</td>
</tr>
<tr>
<td>muon</td>
<td>105.66</td>
<td>1226</td>
</tr>
<tr>
<td>pi-mesons</td>
<td>135-140</td>
<td>~1600</td>
</tr>
<tr>
<td>proton</td>
<td>938.26</td>
<td>10,888</td>
</tr>
<tr>
<td>neutron</td>
<td>939.55</td>
<td>10,903</td>
</tr>
</tbody>
</table>
Baryon and anti-baryon $\leftrightarrow \gamma + \gamma$ in TE

\[
T > 10^{12} \text{ K} \\
t < 10^{-4} \text{ s}
\]

There is an asymmetry between number of baryons and anti-baryons of 1 part in $10^9$.

E.g., for every $10^9$ antiprotons, there would be $10^9+1$ protons.

As the temperature drops, the threshold temperature is not reached for those particles, and they recombine:

baryon + antibaryon $\rightarrow \gamma + \gamma$

All but 1 in $10^9$ baryons were annihilated!
$T < 10^{10} \text{ K}, t \sim 1 \text{s}$

When $T < T_{\text{thr}}$ for $e^{-}$ and $e^{+}$ creation, $e^{-}$ and $e^{+} \rightarrow \gamma + \gamma$ with slight excess of $e^{-}$.

Thus, the universe is matter (not antimatter) and radiation, in TE since $e^{-}$ have high cross section for interaction with photons,

TE persists until $t_{\text{dec}}$.

Our understanding of this creation of particles + subsequent nuclear reactions leads to quantitative prediction of relative abundances of hydrogen and helium made right after the Big Bang.

⇒ Primordial material should be 3/4 hydrogen, 1/4 helium by mass.

See C&O for details of nuclear reactions.
Big Bang Nucleosynthesis
• As universe cools, fusion reaction starts
  – \( p+n \leftrightarrow ^2\text{H}+\gamma \)
  – deuterium starts to build up below \( 10^9 \) K
  – background photons are no longer energetic enough for back reactions

  \[
  \begin{align*}
  &^2\text{H}+p \leftrightarrow ^3\text{He} + \gamma \\
  &^2\text{H}+n \leftrightarrow ^3\text{H} + \gamma \\
  &^2\text{H}^2\text{H} \leftrightarrow ^3\text{H} + p \text{ or } ^3\text{He} + n
  \end{align*}
  \]

  – Various reactions then lead to \(^4\text{He}\) (and a bit of \(^7\text{Li}\))

• Eventually most neutrons wind up in \(^4\text{He}\) though some neutrons decay
Final yields of $^2H$, $^3He$, $^4He$ and $^7Li$ depend on:

- Neutron lifetime (measured in lab)
  - 885.7s

- Number of neutrino species
  - radiation dominated era $H^2 \propto \rho_{rel} = \rho_\gamma + N_\gamma \rho_V$
  - 3?

- H (measured by Planck)
  - 68 +/- 0.8 km/s/Mpc (vs 71 +/- 2.5 for WMAP)

- Baryon density (number density of protons and neutrons)
• Helium 4
  – measure in spectra of Pop. II stars
  – also produced in stars: big correction factors

• Helium 3
  – measured in radio (spin flip of $^3\text{He}+$ at 3.46 cm)

• Deuterium
  – lines can be separated from $^1\text{H}$
  – currently best measured isotope

• Lithium 7
  – measure in spectra
  – also produced by cosmic ray spallation, and destroyed/created by stars
  – results are currently not concordant

(Bania et al., ApJSS 1997)
Current abundances

- $D/H = (2.8\pm0.4) \times 10^{-5}$
- $^7\text{Li}/H = (1.23\pm0.68) \times 10^{-10}$

- Somewhat inconsistent:
  - $^7\text{Li}$ may be destroyed in the early universe or in stars
  - $D/H$ is consistent with WMAP $\Omega_b$
Finally, relative abundances are sensitive to density of normal (baryonic matter)

Thus $\Omega_{b,0} \sim 4\%$. So our universe $\Omega_{\text{total}} \sim 1$ with 70% in Dark Energy, 30% in matter but only 4% baryonic!
Case for the Hot Big Bang

- The Cosmic Microwave Background has an isotropic blackbody spectrum
  - it is extremely difficult to generate a blackbody background in other models

- The observed abundances of the light isotopes are reasonably consistent with predictions
  - again, a hot initial state is the natural way to generate these

- Many astrophysical populations (e.g. quasars) show strong evolution with redshift
  - this certainly argues against any Steady State models
Outstanding problems

• Why is the CMB so isotropic?
  – horizon distance at last scattering << horizon distance now
  – why would causally disconnected regions have the same temperature to 1 part in $10^5$?

• Why is universe so flat?
  – if $\Omega$ is not 1, $\Omega$ evolves rapidly away from 1 in radiation or matter dominated universe
  – but CMB analysis shows $\Omega = 1$ to high accuracy
  – so either $\Omega=1$ (why?) or $\Omega$ is fine tuned to very nearly 1

• How do structures form?
  – if early universe is so very nearly uniform

Next time we will talk about inflation.