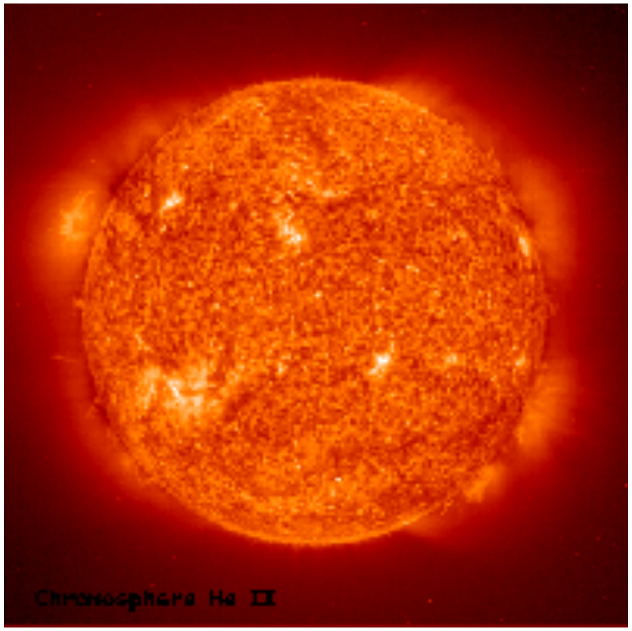
Astronomy 421



Lecture 13: Stellar Atmospheres II

Skip Sec 9.4 and radiation pressure gradient part of 9.3

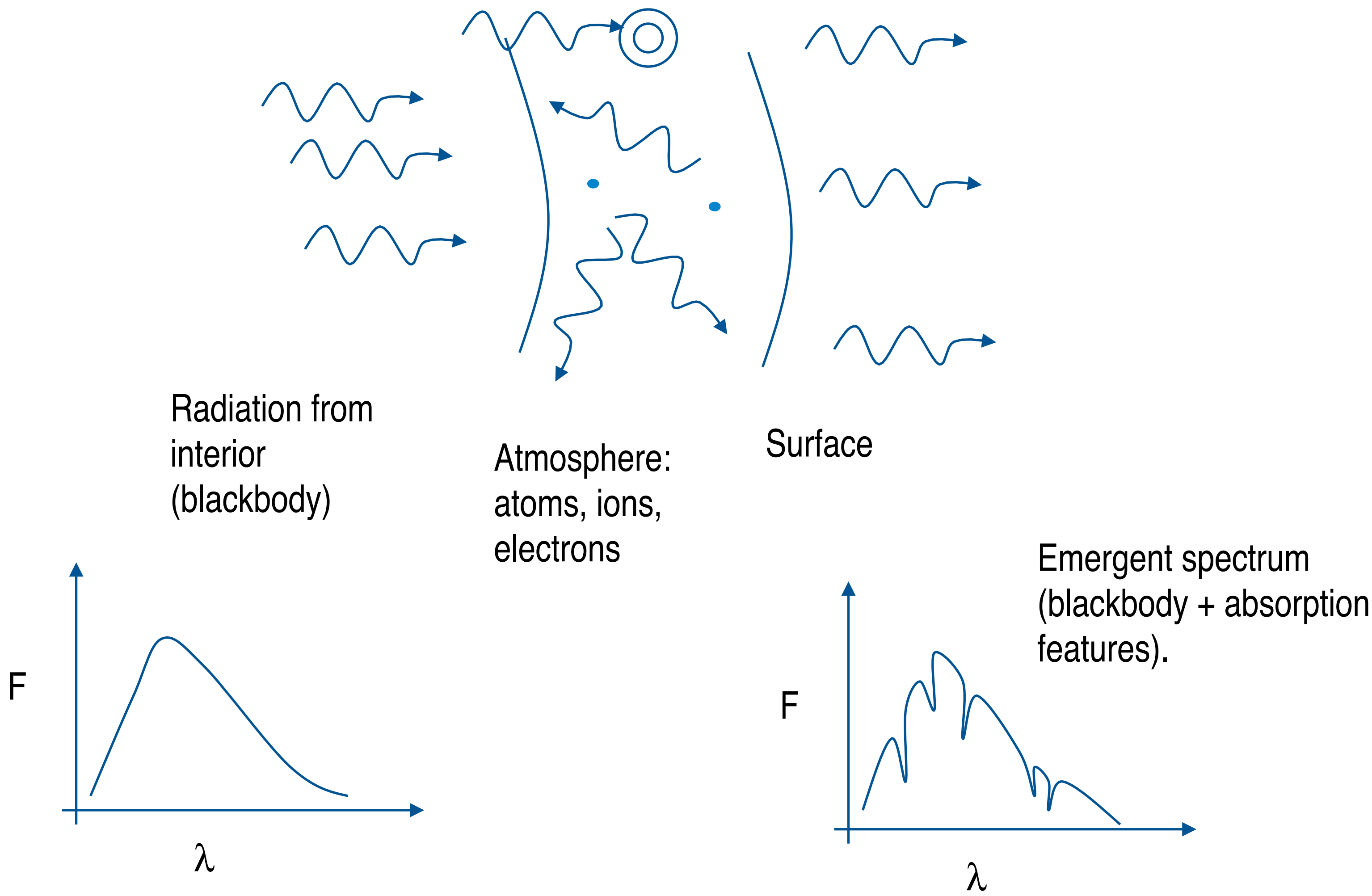
Announcements:

Homework #4 is due Oct 3

Outline is due October 8

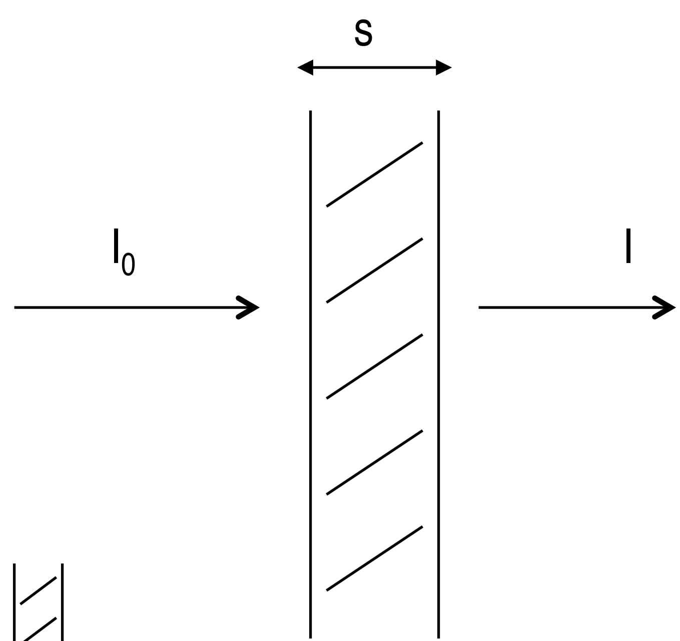
See example on the class web page

Schematic stellar atmosphere

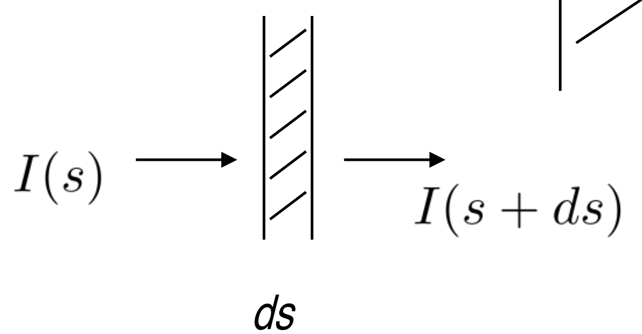


Opacity and Optical Depth

How much energy gets absorbed (or scattered) after passing through the slab?



Divide slab into many thin slabs



Change in intensity across slab: $$dI = I(s+ds) - I(s)$$

Expect:

1) $dI \propto I(s)$ (why?)

2) $dI \propto ds$ (why?)

3) $dI \propto$ (absorbing ability of material at wavelength in question)

From *1)* set $$dI_\lambda = (-d\tau_\lambda) I_\lambda(s)$$

proportionality constant TBD

2) $$d\tau_\lambda \propto ds$$

3) $$d\tau_\lambda \propto \kappa_\lambda \rho$$

ρ = density of material in slab

κ_λ= *absorption coefficient =* cross section for absorbing + scattering photons per unit mass of material (units $m^2\ kg^{-1}$ or $cm^2\ g^{-1}$).

$$d\tau_\lambda = \kappa_\lambda \rho ds$$

Then $$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds$$

or $$\frac{dI_\lambda}{I_\lambda} = -\kappa_\lambda \rho ds$$

So $$\int_{I_{o,\lambda}}^{I_\lambda} \frac{dI_\lambda}{I_\lambda} = -\kappa_\lambda \rho \int_0^s ds$$

Thus $$I_\lambda = I_{0,\lambda} e^{-\kappa_\lambda \rho s} = I_{0,\lambda} e^{-\tau_\lambda}$$

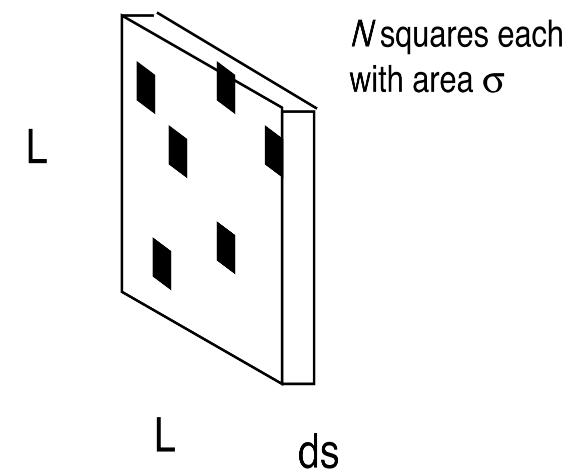
τ_λ is the *optical depth,* and is dimensionless.

A simple way to illustrate: imagine an opaque material consisting of little black squares embedded in clear plastic.

The absorption coefficient depends on projected area of light-absorbing squares and on how many there are per unit mass.

Take a thin slice, so none of the opaque squares overlap.

How much light is lost?



dI/I_0=(total projected area of squares)/(total projected area of slab)

$$\frac{dI}{I_0} = \frac{N\sigma}{L^2}$$

Recall κ = cross section per mass so:

$$\kappa = \frac{N\sigma}{\text{mass of slab}} = \frac{N\sigma}{\rho V} = \frac{N\sigma}{\rho L^2 ds}$$

$$\Rightarrow \frac{N\sigma}{L^2} = \kappa\rho ds$$

$$|\frac{dI}{I_0}| = \kappa\rho ds \text{ and } dI = -I_0\kappa\rho ds$$

$$I_\lambda = I_{0,\lambda} e^{-\kappa_\lambda \rho s} = I_{0,\lambda} e^{-\tau_\lambda}$$

More on optical depths:

Consider $\tau_\lambda = 1$

Then

κ_λ is cross-section per mass, but σ_λ is cross-section per particle. Thus

$$\frac{1}{\kappa_\lambda \rho} = \frac{1}{n\sigma_\lambda} = l$$ mean free path!

Intensity falls by 1/e over one mean free path at λ

$$\tau_\lambda \gg 1 \text{ optically thick}$$

$$\tau_\lambda \lesssim 1 \text{ optically thin}$$

Example: Typically, in the Earth's atmosphere κ = 0.0001 cm^2/g, ρ=0.001 g/cm^3. Look through 1 km long slab. What is the optical depth?

$$\tau_\lambda = \kappa_\lambda \varrho s = 0.0001 \times 0.001 \times 10^5 = 0.01$$

(cm^2/g) (g/cm^3) (cm)

What fraction of a light beam with intensity *I* will be blocked and escapes, respectively?

$$I_\lambda = I_{0,\lambda} e^{-\tau} \rightarrow dI_\lambda = I_{0,\lambda} - I_\lambda = I_{0,\lambda}(1 - e^{-\tau_\lambda}) = 0.00995 I_0$$

(what is e^{-x} for small x)?

1% absorbed, 99% transmitted.

Double the length of the slab, what happens to the fraction?

2% absorbed, 98% transmitted.

What if κ_λ = 0.001 cm^2/g (a smoggy day in Albuquerque)? What are τ_λ, I_λ?

Example: Look through atmosphere in a city like LA, where κ = 0.1 cm^2/g, ρ=0.001 g/cm^3 . Again for 1km, what is the optical depth?

$$\tau = \kappa\rho ds = 0.1 \times 0.001 \times 10^5 = 10$$

What fraction of a light beam with intensity *I* will be blocked and escapes, respectively?

$$dI = I_0 - I = I_0(1 - e^{-\tau}) = 0.99995 I_0$$

Completely absorbed!

Double the length of the slab, what happens to the fraction?

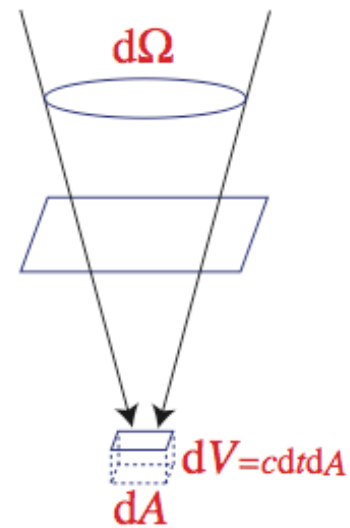
I_λ can vary with direction. So define

$$\text{mean intensity } = \frac{\int I_\lambda d\Omega}{4\pi} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda \sin\theta d\theta d\phi = < I_\lambda >$$

Energy density

Let's consider the energy detected:

$$dE = I_\lambda dt dA d\Omega d\lambda$$



Light travels at *c*, and dE is the energy in the volume

$$dV = c dt dA$$

This gives an expression for *dE:*

$$dE = \frac{I_\lambda}{c} dV d\Omega d\lambda$$

The energy per unit volume of radiation with wavelength λ to $\lambda + d\lambda$ is the *specific energy density*, which then must be defined as

$$\frac{dE}{dV d\lambda} = du_\lambda \equiv \frac{I_\lambda}{c} d\Omega$$

The energy density is then the integral across all solid angles:

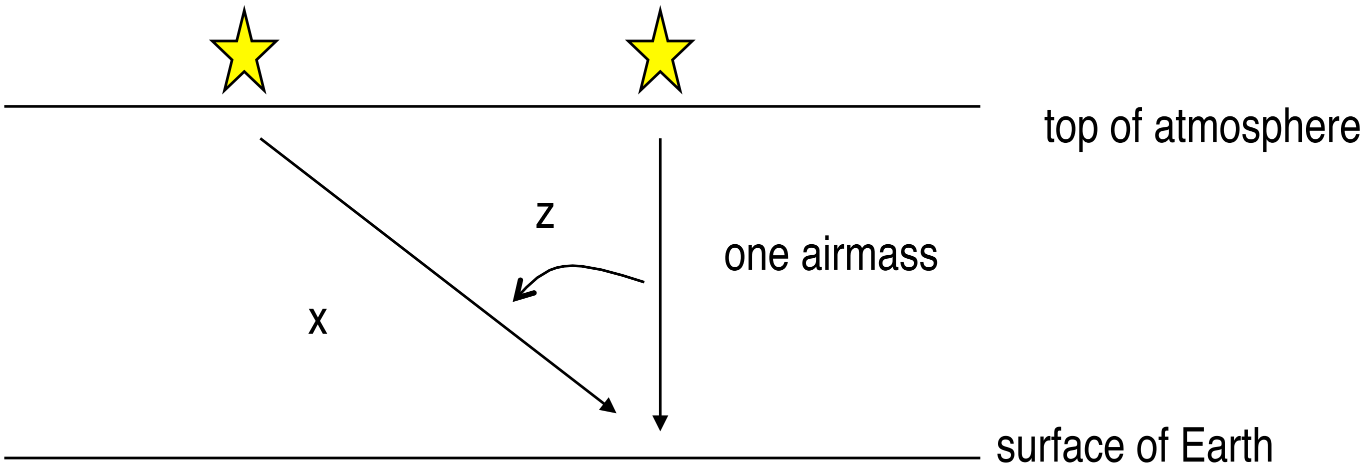
$$u_\lambda = \int \frac{I_\lambda}{c} d\Omega$$

In the case of an isotropic radiation field, *I* independent of solid angle:

$$u_\lambda = \frac{4\pi}{c} < I_\lambda > \qquad\qquad u_\lambda d\lambda = \frac{4\pi}{c} < I_\lambda > d\lambda$$

Example: extinction in the atmosphere of the Earth

Light from star to an observer at the surface of the Earth. Photons may be absorbed or scattered by the atmosphere = *dimming.* The amount of dimming must depend on the amount of atmosphere. The term *airmass* is used to describe this.



one airmass: amount of air directly ahead.

$$\text{airmass}\, x = \frac{\text{one airmass}}{\cos z} = (\text{one airmass}) \sec z$$

Usually, fraction of light loss small and $I(X) = I_0 e^{-cX}$

or, in magnitudes $m(X) = m_0 + kX$

where k= *first order extinction coefficient,* depends on the properties of local atmosphere and wavelength. Typical values for k:

U 0.6

B 0.4

V 0.2

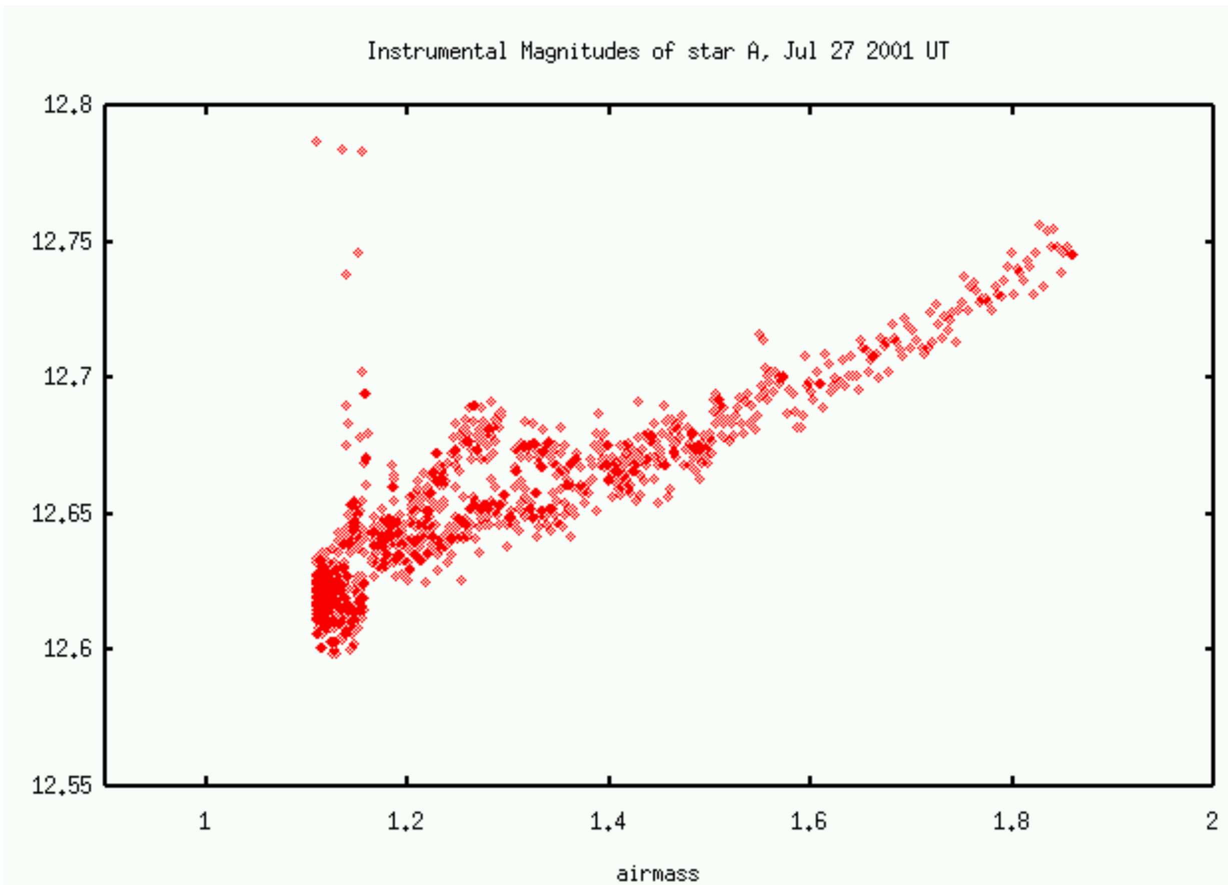
R 0.1

I 0.08

Note: blue light more extinguished than red. Observed especially when sun is setting and rising.

Good observing locations have a small extinction coefficient, and good nights also have small extinction coefficient.

Air changes from night to night, so to correct for extinction we must determine the first order coefficient.



Emergent flux reminder

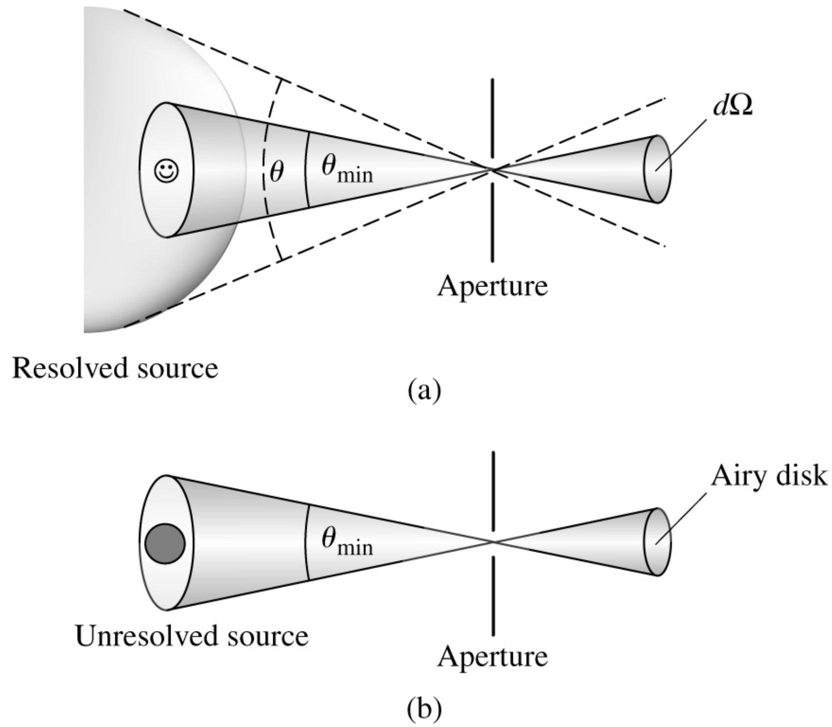
This is the intensity of radiation passing through *dA*, integrated over all angles:

$$F_\lambda d\lambda = \int_\Omega I_\lambda d\lambda \cos\theta d\Omega$$

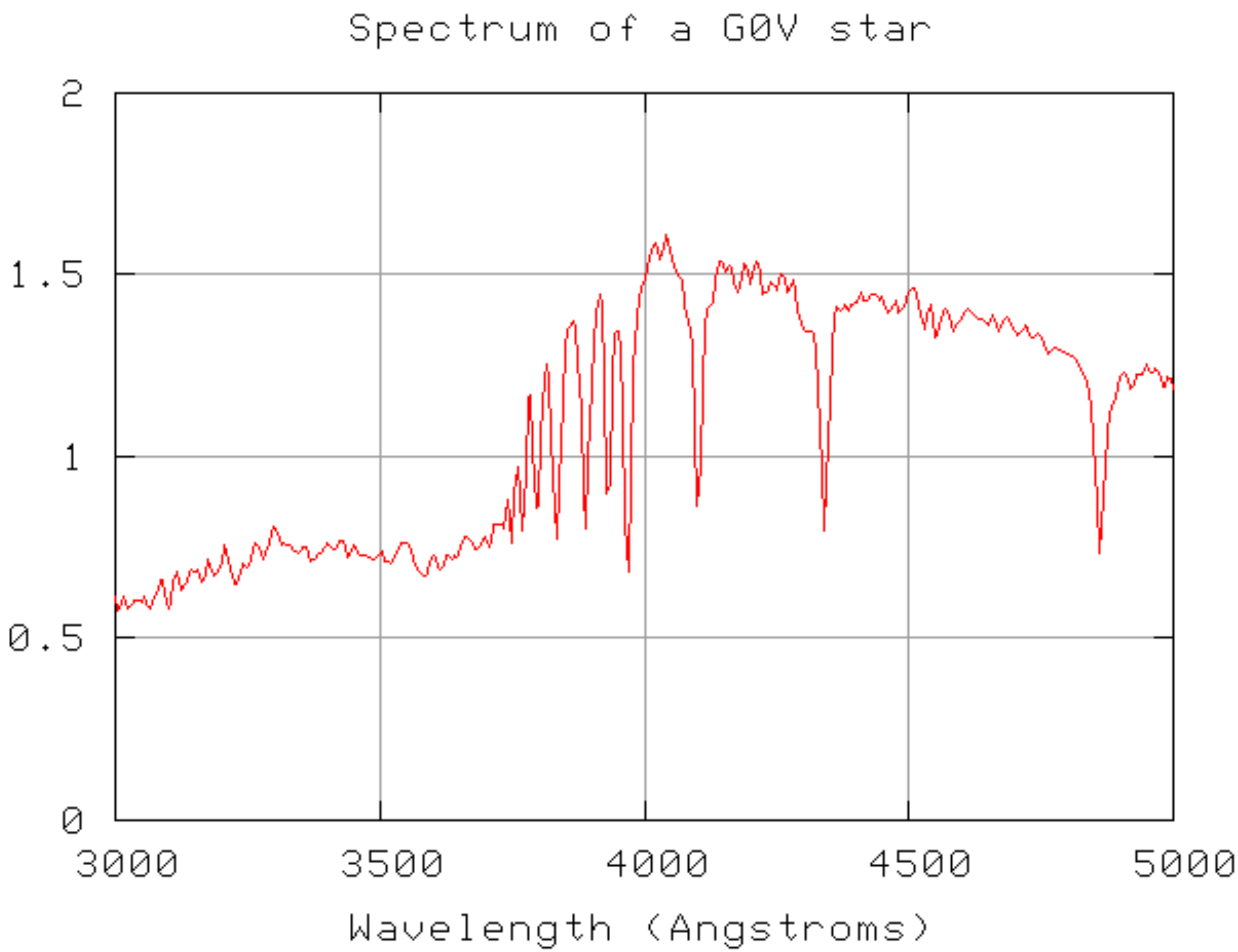
$$\int_\lambda F_\lambda d\lambda = \sigma T^4 \text{ for blackbody}$$

What do we measure with a telescope, *F* or *I*?

Depends on whether the source is resolved or not:



In upper plot we measure the specific intensity, while in the lower figure we measure the flux.



Sources of Opacity - How many can you find?

Sources of stellar opacity and emissivity (we won't write eqns for all of these – too complex!):

1) <u>Bound-bound absorption</u>

- When e^- makes upward transition in atom or ion. Subsequent downward transition either:
  - back to initial orbit (effectively a scattering process)
  - back to different orbit (true absorption process for original λ)
  - $\geq$ 2 transitions back to lower levels (true absorption, degradation of average photon energy)
- Call this $\kappa_{\lambda,bb}$. Recall mks units are $m^2\ kg^{-1}$. Is zero except at wavelengths capable of producing upward atomic transitions => absorption lines in stellar spectra. Depends on temperature, abundances, QM transition probabilities. No simple function.

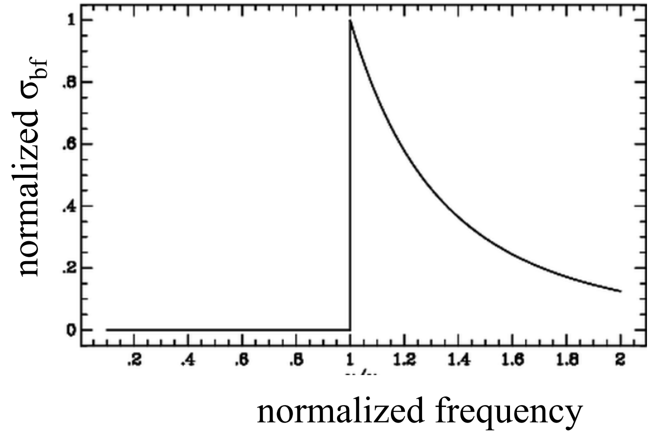
2) <u>Bound-free absorption = photoionization</u>

- $\kappa_{\lambda,bf}$ is a source of continuum opacity. Any photons with $\lambda < hc/\chi_n$ (where χ_n is the ionization potential of n^{th} orbital) can cause ionization. Inverse process: recombination - also degrades photon energies.

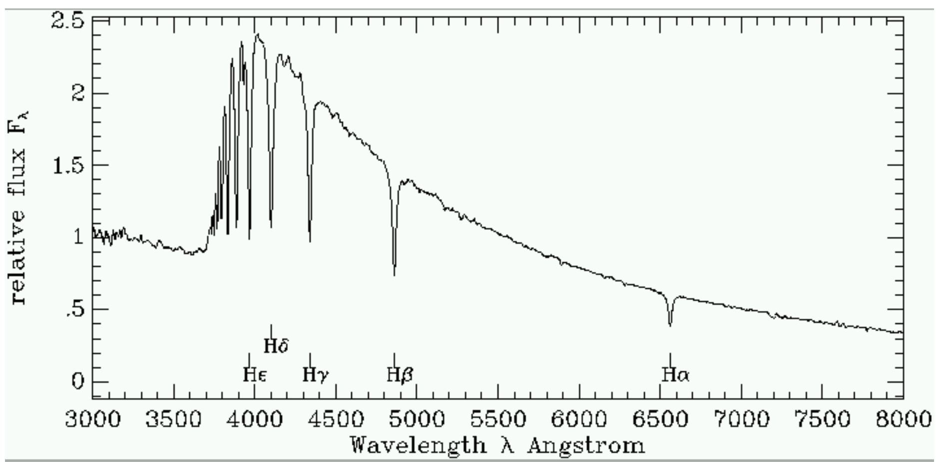
Example: H atom in level *n*

$$\sigma_{bf} = 1.31 \times 10^{-19} \frac{1}{n^5} \left(\frac{\lambda}{500\,\mathrm{nm}} \right)^3 \ \mathrm{m}^2$$

(only for $\lambda \leq \frac{hc}{\chi_n}$ $\sigma_{bf} = 0$ for $\lambda > \frac{hc}{\chi_n}$)

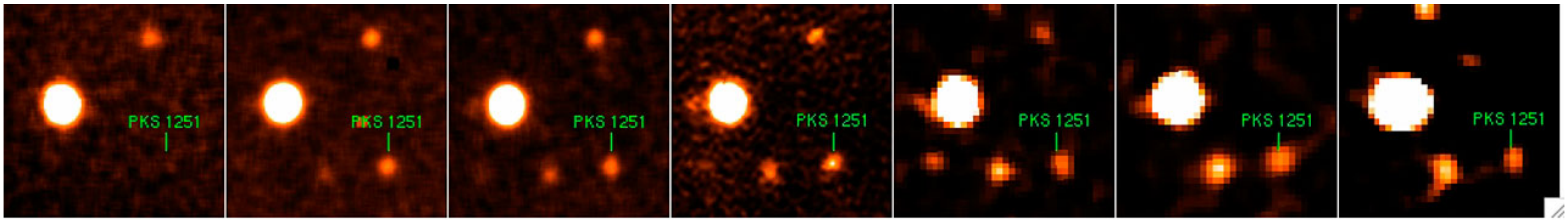


So for level *n*, $\kappa_{\lambda,bf} = \sigma_{bf}$ times the number of atoms or ions in that level per kg

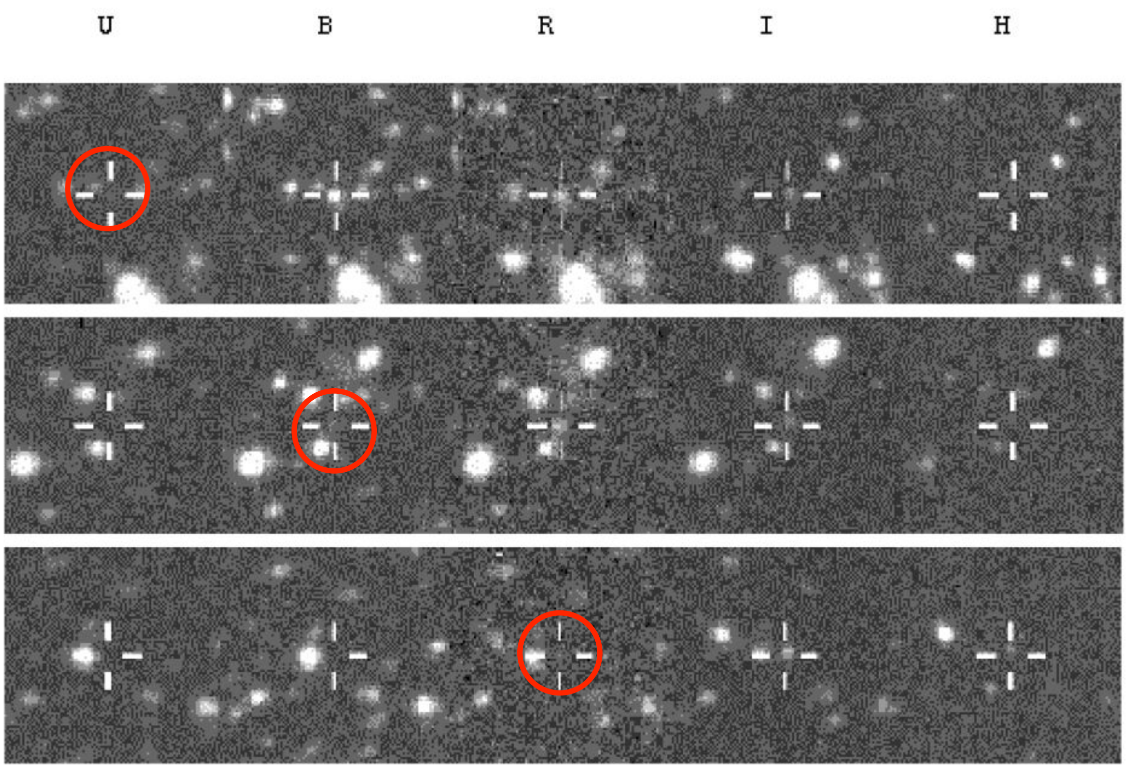


This causes the "Balmer jump". Photons lost to ionization of H from n=2 level. Requires E ≥ 13.6-10.2 = 3.4 eV, or λ ≤ 364.7 nm. Because $\sigma_{b,f} \propto \lambda^3$, spectrum gets closer to blackbody again for shorter λ's.

Similar jump at E=13.6 eV for Lyman series, but in far UV (except at high redshifts!).
Used to get redshifts and thus distances of faint galaxies.



visible → infrared



Which is the most distant object?

3) Free-free absorption

- $\kappa_{\lambda,\mathrm{ff}}$: another source of continuum opacity. Free *e^-* near ion absorbs photon and increases velocity.

Why won't isolated e^- absorb photons?

- (converse: free-free emission, or *brehmsstrahlung,* e^- loses energy passing by an ion, emits a photon)

4) Electron-scattering (Thomson scattering)

- κ_{es} : photon scatters off free e^-. Source of continuum opacity. Depends on the *Thomson cross section* of the *e^-* (relatively small):

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25}\ \mathrm{cm}^2$$

($r = e/m_e c^2$ often used as the classical 'radius' of an electron)

- But dominates at high-temperatures.

Main source of continuum opacity in stellar atmospheres of type:

F and cooler: Photoionization of H^- ions.

Any photon with

$$\lambda \leq \frac{hc}{\chi} = \frac{hc}{0.754\,\mathrm{eV}} = 1640\,\mathrm{nm} \qquad \text{(IR)}$$

B, A: Bound-free of H and free-free processes

O stars: Electron scattering and bound-free processes of He

Interiors of stars: Electron scattering